Overview

• Electricity Markets are complicated.
• Market power will be exploited (CAISO, PJM).
• Traditional concentration measures do not account for engineering constraints.
• Our approach: Examine revenue (or profit) and dispatch sensitivities to identify participants who have potential market power.
In a market setting, suppliers offer to sell energy:
- quantity
- price

Markets in a minute: generators, network, and loads

Area 1 Demand: 76.6 MW
Gen 1
Gen 2

Area 2 Demand: 83.9 MW
Gen 6

Area 3 Demand: 39.4 MW
Gen 3
Gen 4

Total System Demand: 200 MW

$/MWh

MW

12 36 60
Markets in a minute: Ideal dispatch
Markets in a minute:
Ideal dispatch
Markets in a minute: Ideal dispatch
Markets in a minute: Non-ideal dispatch

Network limitations prohibit dispatch

$/MWh

Energy Prices

dispatch

dispatch

MW
Example System

Total System Demand: 165 MW
Area 2 Demand: 49 MW
Area 3 Demand: 39.4 MW

Gen 1
Gen 2
Gen 3
Gen 4
Gen 5
Gen 6

40 $/MWh
55 $/MWh
40 $/MWh
55 $/MWh

$40.00/MWh
$55.00/MWh

Total System Demand: 165 MW
Locational Advantage and Market Power

Key Issue: Substitutability

• Can suppliers exploit locational advantages?

If so,

• Can we identify when this is possible?
Market Competitiveness: Concentration Measures

- Pivotal/Residual Supplier Indices
- Herfindahl-Hirshman Index (HHI):
  \[ \sum_{\text{participants}} \left( \% \text{ market share} \right)^2 \]

ISO Market monitors: examine concentrations in Capacity, Dispatch, and Capacity and Dispatch by Fuel Type…

FERC: evaluate concentrations in applications for “market-based rate authority.”
Market Competitiveness: Concentration Measures

- Pivotal/Residual Supplier Indices
- Herfindahl-Hirshman Index (HHI):
  \[ \sum_{\text{participants}} (\% \text{ market share})^2 \]

Concentration measures are coarse, indirect, and often inconclusive.
Market Power: Our Approach

Identify those individuals or groups with the ability to increase revenues and price.

**Competitive Environment**: Increasing (offer) price will serve to decrease a participant’s own revenues.

**Non Competitive Environment**: A participant may increase revenues and price.
Identifying Market Power: Sensitivity Approach

Examine dispatch and revenue/offer sensitivities

\[ \frac{\Delta r}{\Delta \lambda} \]

If a small set of suppliers can simultaneously raise prices and increase revenues, they have some amount of market power.
Example System

Area 1 Demand: 76.6 MW
Gen 1
Gen 2

Area 2 Demand: 49 WM
Gen 6

Area 3 Demand: 39.4 MW
Gen 3
Gen 4
Gen 5

Total System Demand: 165 MW
Six Supplier Example (Market Power)

Base Case Offers:

Generators 1-4

Generators 5-6

Higher to create load pocket
# Results (75 round experiment)

## Base Case Solution

<table>
<thead>
<tr>
<th></th>
<th>G1</th>
<th>G2</th>
<th>G3</th>
<th>G4</th>
<th>G5</th>
<th>G6</th>
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</thead>
<tbody>
<tr>
<td>Dispatch (MW)</td>
<td>31.7</td>
<td>36.0</td>
<td>34.0</td>
<td>36.0</td>
<td>17.6</td>
<td>12.0</td>
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<tr>
<td>Price ($/MWh)</td>
<td>40.0</td>
<td>40.1</td>
<td>40.0</td>
<td>40.1</td>
<td>55.0</td>
<td>54.3</td>
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</table>

## Actual Experiment*

<table>
<thead>
<tr>
<th></th>
<th>G1</th>
<th>G2</th>
<th>G3</th>
<th>G4</th>
<th>G5</th>
<th>G6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dispatch (MW)</td>
<td>37.9</td>
<td>34.9</td>
<td>30.1</td>
<td>34.9</td>
<td>14.9</td>
<td>14.6</td>
</tr>
<tr>
<td>Price ($/MWh)</td>
<td>48.5</td>
<td>48.7</td>
<td>48.5</td>
<td>48.6</td>
<td>72.0</td>
<td>70.0</td>
</tr>
</tbody>
</table>

* 80 $/MWh offer cap
Example System - Experiments

Total System Demand: 165 MW

Area 2 Demand: 49 MW

Gen 1

49 \$/\text{MWh}

Gen 2

49 \$/\text{MWh}

Area 2 Demand: 49 MW

Gen 6

70 \$/\text{MWh}

Gen 5

72 \$/\text{MWh}

Area 3 Demand: 39.4 MW

Gen 3

49 \$/\text{MWh}

Gen 4

49 \$/\text{MWh}

Total System Demand: 165 MW
Matrix of Revenue/Offer Sensitivities

Base Case Solution

\[
\begin{bmatrix}
\Delta r_1 \\
\Delta r_2 \\
\Delta r_3 \\
\Delta r_4 \\
\Delta r_5 \\
\Delta r_6 \\
\end{bmatrix} =
\begin{bmatrix}
-3298 & 3231 & 31 & 65 & 52 & -49 \\
3219 & -3695 & 244 & 263 & 315 & -310 \\
31 & 244 & -544 & 308 & -234 & 229 \\
65 & 263 & 307 & -597 & -127 & 125 \\
38 & 230 & -170 & -93 & -160 & 173 \\
-36 & -229 & 169 & 92 & 175 & -159 \\
\end{bmatrix}
\begin{bmatrix}
\Delta \lambda_1 \\
\Delta \lambda_2 \\
\Delta \lambda_3 \\
\Delta \lambda_4 \\
\Delta \lambda_5 \\
\Delta \lambda_6 \\
\end{bmatrix}
\]

Observations:
- If any supplier, acting alone, raises its (offer) price, that supplier will lose revenue.
- If all suppliers, acting together, raise (offer) prices, everyone’s revenue increases.
Matrix of Revenue/Offer Sensitivities

Base Case Solution

\[
\begin{bmatrix}
\Delta r_1 \\
\Delta r_2 \\
\Delta r_3 \\
\Delta r_4 \\
\Delta r_5 \\
\Delta r_6
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\begin{bmatrix}
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65 & 263 & 307 & -597 \\
38 & 230 & -170 & -93 \\
-36 & -229 & 169 & 92
\end{bmatrix}
\begin{bmatrix}
\Delta \lambda_1 \\
\Delta \lambda_2 \\
\Delta \lambda_3 \\
\Delta \lambda_4 \\
\Delta \lambda_5 \\
\Delta \lambda_6
\end{bmatrix}
\]

Observations:
- If the load pocket generators (5 and 6) raise their (offer) prices together, their own revenues increase with almost no effect on the other revenues.
Load Pocket Market Power

Closer examination:

\[
\begin{bmatrix}
\Delta r_5 \\
\Delta r_6
\end{bmatrix} =
\begin{bmatrix}
-160 & 173 \\
175 & -159
\end{bmatrix}
\begin{bmatrix}
\Delta \lambda_5 \\
\Delta \lambda_6
\end{bmatrix}
\]

Increased revenue area generator 5
Zero-revenue line generator 5
Load Pocket Market Power

Closer examination:

\[
\begin{bmatrix}
\Delta r_5 \\
\Delta r_6
\end{bmatrix} = \begin{bmatrix}
-160 & 173 \\
175 & -159
\end{bmatrix} \begin{bmatrix}
\Delta \lambda_5 \\
\Delta \lambda_6
\end{bmatrix}
\]

zero-revenue line generator 6

increased revenue area generator 6
Load Pocket Market Power

Generators 5 and 6 will find their win/win region.
Consider:
• both increase offers and
• generator 5 loses revenue
• generator 6 gains revenue

Then
• generator 5 will decrease its offer and
• generator 6 will increase its offer
• and they will move towards the win/win region
Load Pocket Market Power

It will take time, but
• prices will increase and
• revenues will increase.

There isn’t direct collusion – each generator acts on its own price and revenue signals.
Load Pocket Identification

How can we identify generators in a load pocket?

A property of a load pocket is that a subset of generators serve the load (in excess of limited imports).

- Examine the matrix of dispatch/price sensitivities
- Seek price increase combinations that don’t affect dispatch. (Who can print $$ ?)
Matrix of Dispatch/Price Sensitivities

Construct a matrix $M$ whose elements

$$m_{ij} = \frac{\partial g_i}{\partial \lambda_j}$$

are dispatch/price sensitivities

Then

$$\Delta g = M \Delta \lambda$$
Search for vectors

\[
\Delta \lambda = \begin{bmatrix}
0 \\
\vdots \\
0 \\
* \\
\vdots \\
*
\end{bmatrix}
\]

zero-valued elements

(strictly) positive elements

That satisfy

\[0 = M\Delta \lambda\]
A Power Flow Model

Assuming voltages are constant at 1 per unit (100% rated), line powers are related to angle differences.

\[ p_{12} = b_{12} \sin(\theta_1 - \theta_2) \]
\[ p_{13} = b_{13} \sin(\theta_1 - \theta_3) \]
\[ p_{23} = b_{23} \sin(\theta_2 - \theta_3) \]
\[ p_{24} = b_{24} \sin(\theta_2 - \theta_4) \]
\[ p_{34} = b_{34} \sin(\theta_3 - \theta_4) \]
A Power Flow Model

\[ p_{line} = B \sin(A \theta) \]

Simplified power flow equation:

\[ P = A^T B \sin(A \theta) \approx A^T B A \theta \]
Simplified OPF Model

\[
\min \sum_{i} C_i(g_i; w_i)
\]

Cost Function parameterized by \(w\).

Subject to

\[
P_{\text{branch}} = BA \theta
\]

\[
P_{\text{injected}} = \begin{bmatrix} g \\ l \end{bmatrix} = A^T P_{\text{branch}}
\]

\[
P_{\text{max}} = P_b
\]

Subset of branches at maximum flow limit.

GOAL: find sensitivities of \(g\) to price (incremental cost).

(The same fundamental structure appears in the nonlinear equations.)
Simplified OPF Model

Set up the Lagrangian,

\[ L = \sum_i \left( C_i(g_i; w_i) \right) + \mu^T (P_{\text{branch}} - BA\theta) + \sigma^T \left( \begin{bmatrix} g \\ l \end{bmatrix} - A^T P_{\text{branch}} \right) + \xi^T (P_{\text{max}} - P_b) \]

The KKT conditions require

\[
\begin{bmatrix}
\frac{\partial C(g; w)}{\partial g}(\frac{1}{g}) \\
\vdots
\end{bmatrix}
\begin{bmatrix}
I \\
\vdots
\end{bmatrix}
= \begin{bmatrix}
g \\
\theta \\
P_{\text{branch}} \\
\mu \\
\sigma \\
\xi \\
-P_{\text{max}}
\end{bmatrix}
\]

\[ \begin{bmatrix}
I \\
0 \\
\vdots
\end{bmatrix}
\begin{bmatrix}
-I \\
\vdots
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

\[ \begin{bmatrix}
\mu \\
\sigma \\
\xi \\
-P_{\text{max}}
\end{bmatrix}
\]
At this point we could solve for $\Delta g = M \Delta \lambda$
but it is instructive to examine the larger model for solutions

$$\Delta g = 0 \quad \Delta \theta = 0 \quad \Delta P_{\text{branch}} = 0$$

and work backwards to characterize $\Delta \lambda = \frac{\partial^2 C(g;w)}{\partial g \partial w}$
Zero Dispatch Change

\[
\begin{bmatrix}
\cdot & [I \ 0] & \cdot \\
-A^T B & \cdot & \cdot \\
I & -A & \cdot
\end{bmatrix}
\begin{bmatrix}
\Delta \mu \\
\Delta \sigma \\
\Delta \xi
\end{bmatrix}
= \begin{bmatrix}
-\Delta \lambda \\
0 \\
0
\end{bmatrix}
\]

Separate parts,

\[-\Delta \lambda = \begin{bmatrix} I & 0 \end{bmatrix} \Delta \sigma\]

“Cost” perturbations that Result in no change in dispatch

Examine Null Space of this matrix
Dimension: buses+branches x buses+branches+constraints
Zero Dispatch Change

Simplify

\[ A^T B A \Delta \sigma + A^T B \begin{bmatrix} 0 \\ I \end{bmatrix} \Delta \xi = 0 \]

A basis for vectors \( \Delta \sigma, \Sigma \), is the given by the solution of

\[ A^T B A \Sigma = -A^T B \begin{bmatrix} 0 \\ I \end{bmatrix} \]

each vector takes the form \( ce_{st} \)
We Claim…

Each vector in a basis of the null space of the dispatch/price sensitivity matrix corresponds to a graph partitioning indicator vector. The intersection of different partitions gives rise to correlated rows in the basis.

\[
\text{Basis} = \begin{bmatrix}
+ & + & + \\
+ & + & + \\
+ & + & + \\
+ & - & + \\
+ & - & - \\
+ & - & - \\
+ & - & - \\
\end{bmatrix}
\]
Proof Outline– Graph Theory

Rewrite $A^T B A x = c e_{st}$ as

$$
\begin{bmatrix}
Q_+ & R \\
R^T & Q_- \\
\end{bmatrix}
\begin{bmatrix}
x_+ \\
x_- \\
\end{bmatrix} = \alpha
\begin{bmatrix}
e_s \\
e_t \\
\end{bmatrix}
$$

necessary because

NOTE: Elements of $R$ are non-positive. At least one element is negative since the graph is connected. $Q_+$ and $Q_-$ are both positive definite.

$$
\begin{bmatrix}
x_T^T & 0 \\
\end{bmatrix}
\begin{bmatrix}
Q_+ & R \\
R^T & Q_- \\
\end{bmatrix}
\begin{bmatrix}
x_+ \\
0 \\
\end{bmatrix} = x_T^T Q_+ x_+ = \alpha
\begin{bmatrix}
x_T^T & 0 \\
\end{bmatrix} e_s > 0
$$
Proof Outline—Graph Theory

\[
\begin{bmatrix}
Q_+ \\
R^T \\
Q_-
\end{bmatrix}
\begin{bmatrix}
x_+ \\
x_-
\end{bmatrix} = \alpha \begin{bmatrix} e_s \\
e_t \end{bmatrix}
\]

Vector \(x_+\) must correspond to a connected subgraph.

Proof by contradiction. Assume that \(x_+\) corresponds to two or more disconnected subgraphs. Then

\[
\begin{bmatrix}
x_{1+}^T \\
x_{2+}^T
\end{bmatrix}
\begin{bmatrix}
Q_+ & 0 \\
0 & Q_-
\end{bmatrix}
\begin{bmatrix}
x_{1+} \\
x_{2+}
\end{bmatrix} = \alpha \begin{bmatrix} x_{1+}^T \\
x_{2+}^T \end{bmatrix} e_s > 0
\]

Since \(e_s\) has a single nonzero element, either

\[
x_{1+}^T Q_+ x_{1+} = 0 \quad \text{or} \quad x_{2+}^T Q_+ x_{2+} = 0
\]

There is a contradiction because both \(Q_+\) and \(Q_-\) are positive definite!
Load Pocket Identification

The columns of $\Sigma$ are partition indicator vectors. A basis for the “cost” perturbation vectors

$$\text{basis for } - \Delta \lambda = [I \ 0] \Sigma$$

$$\text{Basis} = \begin{bmatrix} + & + & + \\ + & + & + \\ + & + & + \\ + & - & + \\ + & - & - \\ + & - & - \\ + & - & - \end{bmatrix}$$

- Identify suppliers that belong to non-substitutable regions.
- Specific analysis: Identify (small number of) suppliers that can change prices without changing dispatch.
Line constraints tend to both
• separate generator supply
• separate prices
118 Bus System
3 simultaneous constraints separates the generators
Perturbation tests support the sensitivity analysis
Perturbation Analysis
(just checking)

Increase Prices by $1/MWh (DC OPF)
Perturbation Analysis
(just checking)

Dispatches don’t change (DCOPF)
Perturbation Analysis
(just checking)

Increase Prices by $1/MWh In group of Three generators (AC OPF)
Perturbation Analysis
(just checking)

Dispatches
don’t change
(ACOPF)
Identify Small Groups

- Vectors of price perturbation for zero dispatch change

<table>
<thead>
<tr>
<th>-0.99</th>
<th>0.32</th>
<th>0.34</th>
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<td>-0.22</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Regroup Simultaneous Effect of All constraints by constraint

Note correlations in rows
Large System Results

PJM Heavily-Loaded Case, five binding constraints

Identifies a small number of units in the same zone that can change price without changing dispatch.
Large System Results

PJM Heavily-Loaded Case, five binding constraints

Identifies a small number of units in the same zone that can change price without changing dispatch.
Summary

• We’ve developed a practical system for identifying suppliers with market power potential.
• It uses a sensitivity-based approach that accounts for the network model.
• Demonstrated results on large realistic models.
• Presently developing visualization techniques.