# New angles for monitoring power system area stress with synchrophasors

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We combine together phasor measurements along the border of an area to get angles across the area that measure total, internal, and external stress of area

# Some overall objectives

- Improve on measuring stress with synchrophasor voltage angle difference between 2 buses

- Distill meaningful and actionable information about a specific area of a large power system from synchrophasor monitoring.

# Angle for a single line in DC load flow

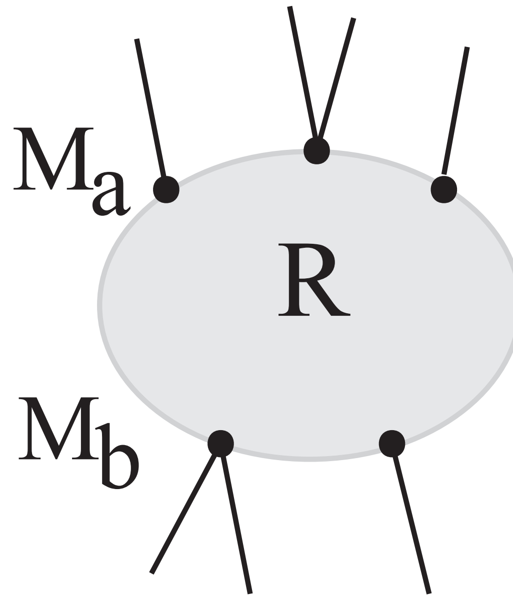

voltage phasor
angle across line $\hat{\theta}_{ab} = \theta_a - \theta_b$

line susceptance $b_{ab}$

real power through line = $b_{ab}\ \hat{\theta}_{ab}$

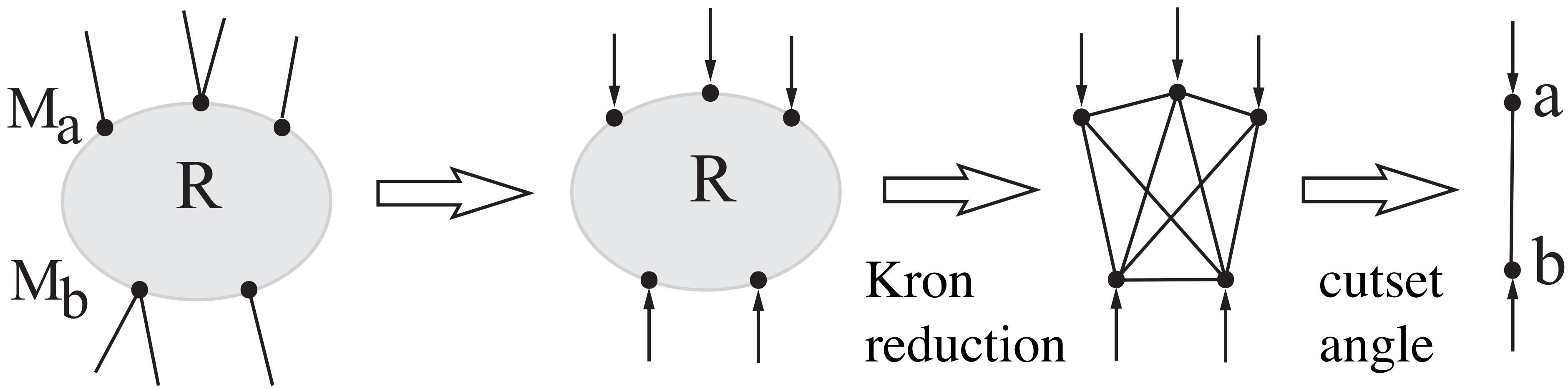
We want to get angle across an **area**
to work in the same way

# Area R with border buses




- Phasor measurements at *all* the buses that are on the border of area
- Border buses are divided into sets $M_a$ and $M_b$
- We will define angle across area from $M_a$ to $M_b$
- Use DC load flow model of area

# Reduction of area R to single line equivalent

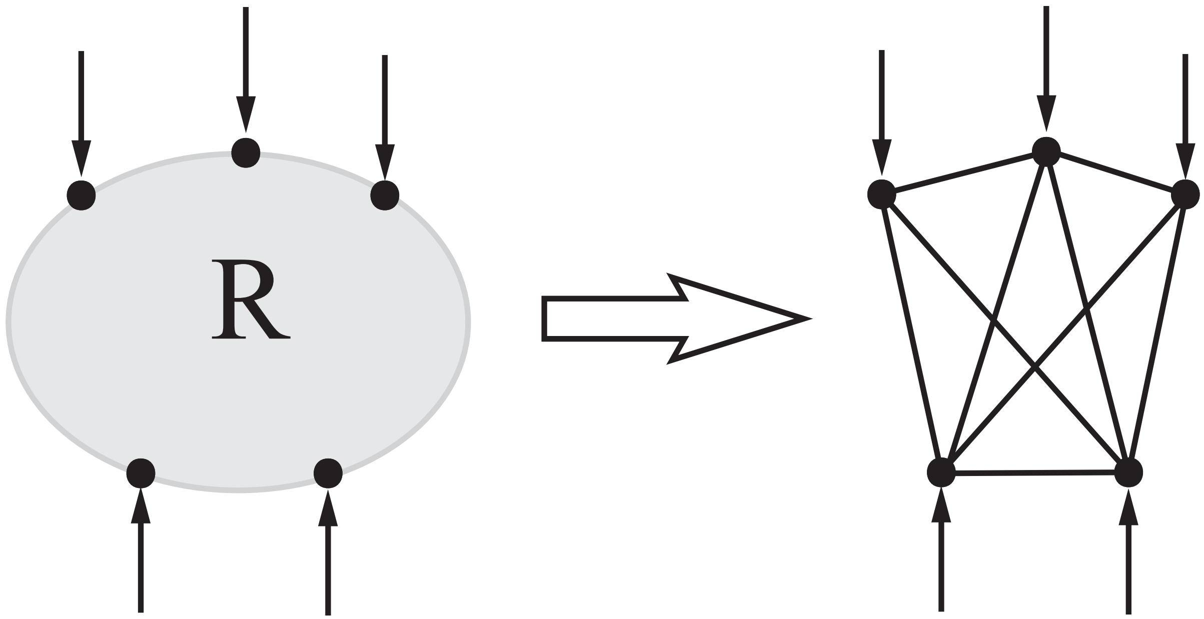



Define: angle across area $\hat{\theta}_{ab}$

area susceptance $b_{ab}$

power through area = $b_{ab}\ \hat{\theta}_{ab}$

# Kron reduction retaining border buses



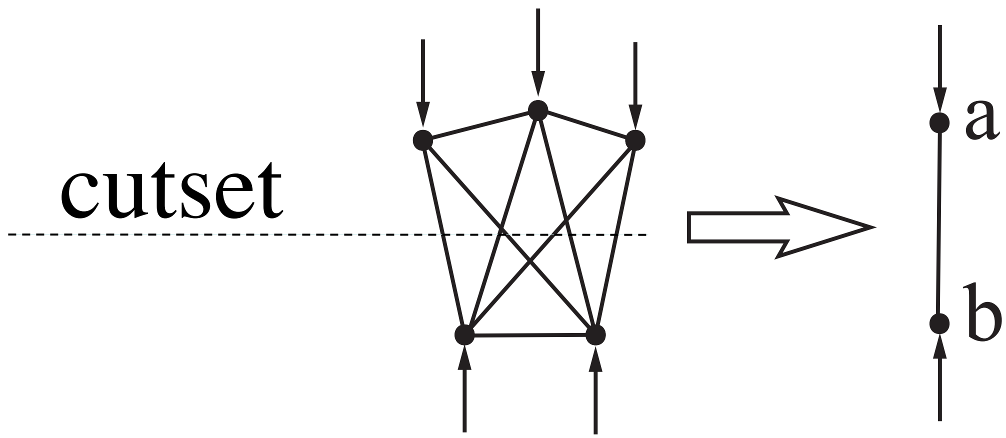

Equivalent susceptance matrix

$$= B_{mm} - B_{mn} B_{nn}^{-1} B_{nm}$$

Border bus power injections equivalent to all powers injected in R

$$P_R = P_m - B_{mn} B_{nn}^{-1} P_n$$

# further reduction with cutset angle



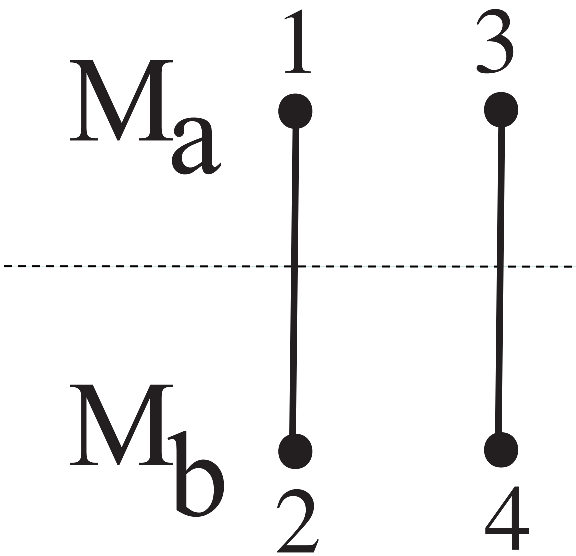

Angle $\hat{\theta}_{ab}$ across cutset is a weighted linear combination of border bus voltage angles

Cutset susceptance is sum of cutset line susceptances
Cutset power flow is sum of cutset line powers

# Defining cutset angle: 2 lines in cutset

Power through cutset = $8(\theta_1 - \theta_2) + 2(\theta_3 - \theta_4)$

$= 10\,[0.8(\theta_1 - \theta_2) + 0.2(\theta_3 - \theta_4)]$




$\hat{\theta}_{ab} = 0.8(\theta_1 - \theta_2) + 0.2(\theta_3 - \theta_4)$

Power = $10\,\hat{\theta}_{ab}$

circuit law

line 1-2 susceptance = 8

line 3-4 susceptance = 2

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cutset susceptance =10

# total angle $\hat{\theta}_{ab}$ across area

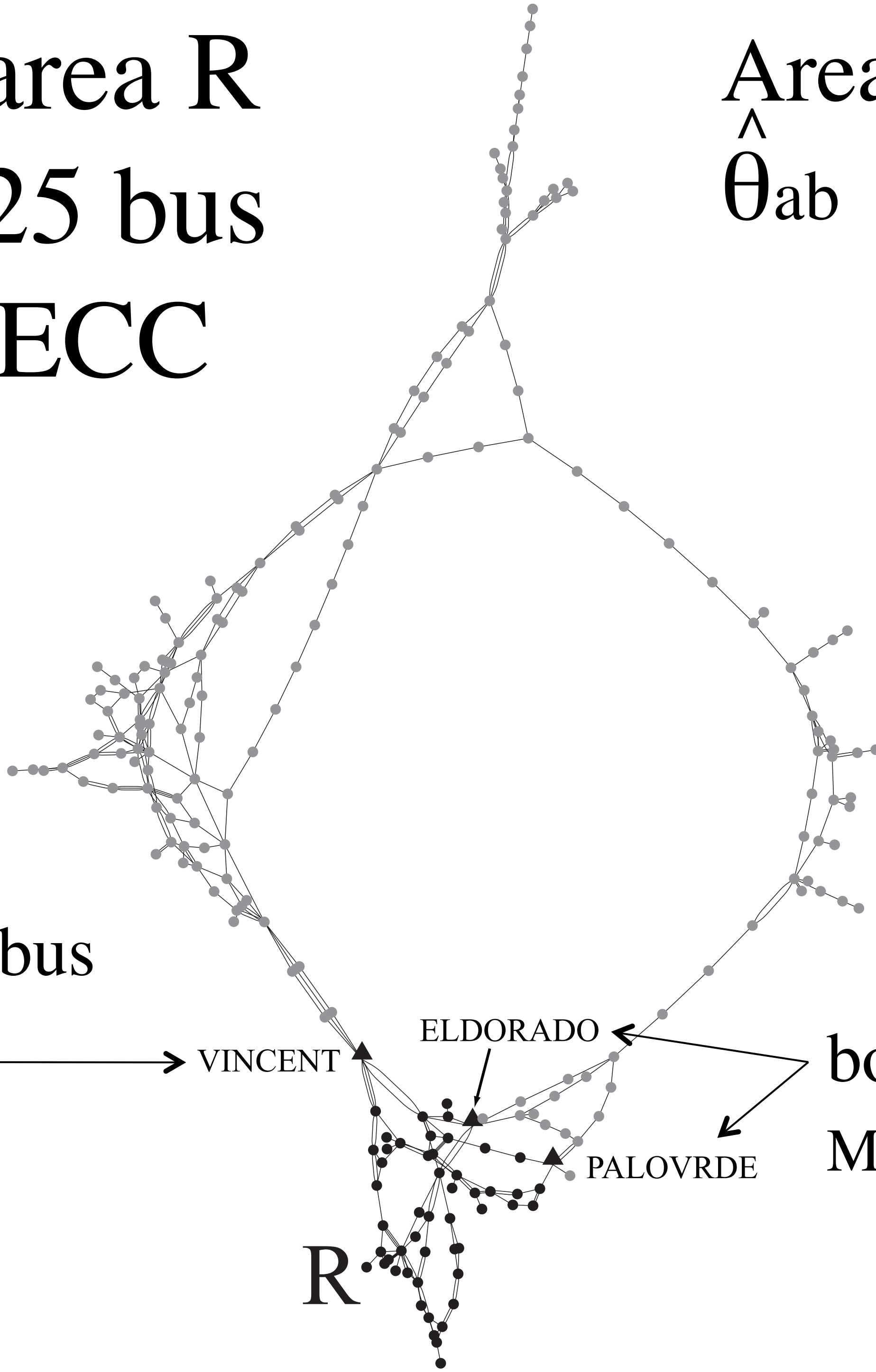
Measures stress across area

Satisfies circuit law $\quad \text{Power} = b_{ab}\ \hat{\theta}_{ab}$

… angle proportional to effective power flow through area

… angle inversely proportional to area susceptance …responds to line trips

# An area R in 225 bus WECC




border bus $M_b$

border buses $M_a$

## Area angle

$$\hat{\theta}_{ab} = 0.80\,\theta_{\mathrm{ELDORADO}} + 0.20\,\theta_{\mathrm{PALOVRDE}} - 1.00\,\theta_{\mathrm{VINCENT}}$$

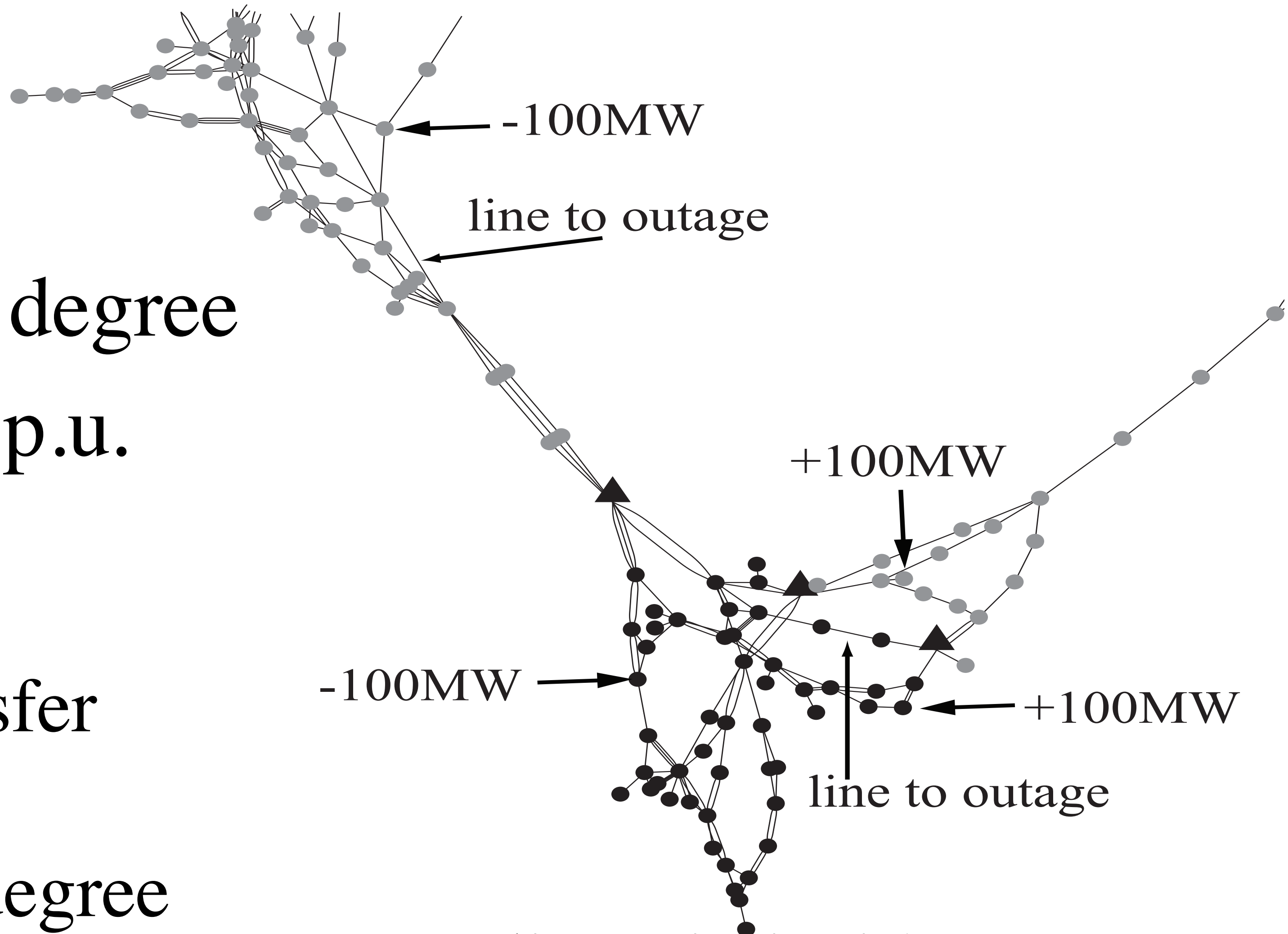
= weighted combination of border bus angles

thanks to C-C Liu for 225 bus model

# Area angle base case

$\hat{\theta}_{ab} = 10.9$ degree

$b_{ab} = 91.6$ p.u.




100 MW transfer inside area gives + 0.25 degree

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100 MW transfer through area gives + 0.60 degree

line trip inside area gives + 2.51 degree

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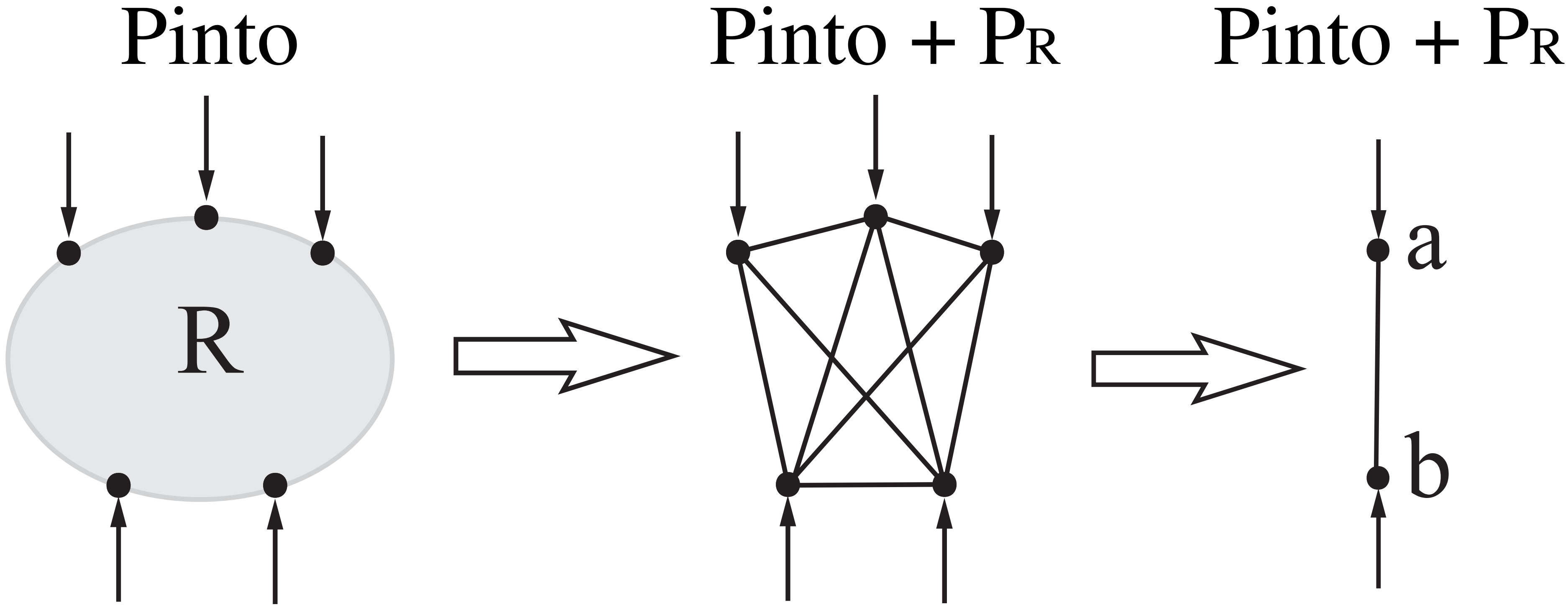
line trip outside area gives + 0.38 degree

# Kron reduction adds power injections $P_R$




Pinto is external power flowing into area

$P_R$ is equivalent to all powers injected in area R

# total stress = external + internal stress

total power through area = $P_{into} + P_R$

Divide by area susceptance $b_{ab}$ to get

total angle across area $\hat{\theta}_{ab} = \hat{\theta}_{into} + \hat{\theta}_R$

$\hat{\theta}_{into}$ = external stress angle

$\hat{\theta}_R$ = internal stress angle

Monitor $\hat{\theta}_{ab}$ and $P_{into}$ with synchrophasors.

Then

$$\hat{\theta}_R = \hat{\theta}_{ab} - \frac{P_{into}}{b_{ab}}$$

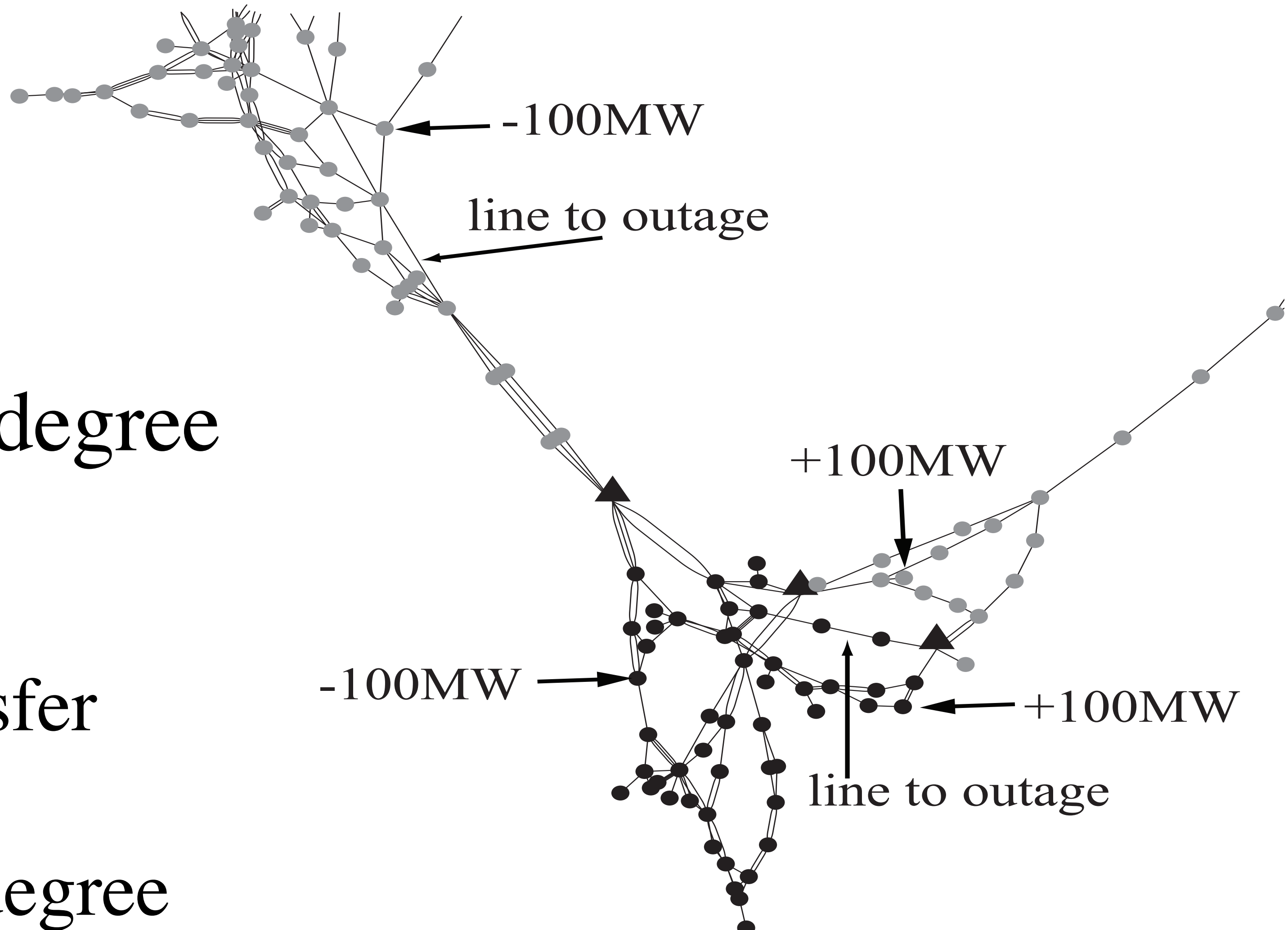
# Properties of internal area angle $\hat{\theta}_R$

- Reacts to changes in injections inside area. Insensitive to injections or redispatch outside area.
- Reacts to line trips inside the area. Insensitive to line trips outside area.

Internal area angle monitors specific area

# Internal area angle base case

$\hat{\theta}_R = 1.07$ degree




100 MW transfer inside area gives + 0.24 degree

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100 MW transfer through area gives + 0 degree

line trip inside area gives + 2.63 degree

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line trip outside area gives + 0 degree

# Conclusions

- angle across area is a weighted combination of phasor measurements at boundary of area;
  easy to compute but needs DC load flow model to compute weights
- derived from Kron reduction and cutset angle
- area angle obeys circuit laws and monitors stress in specific area
- area angle = internal angle due to power injections inside area + angle due to power flows into area
- internal angle only responds to changes in area

# Future directions

- Move towards practical application: choice of areas and PMU locations, model detail needed
- Multiple areas and model reduction
- Monitor line trips, generation changes in area
- AC load flow; complex voltages and currents
- Supply area-specific quantities for data-mining

For details, google
Ian Dobson papers
and download synchrophasor conference papers
at 2010 HICSS, PESGM, and IREP