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Impact of Wind Variability on Power System Small-Signal Reachability[1]

Alejandro D. Domínguez-García
Department of Electrical and Computer Engineering

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[1] Joint work with Christine Chen to be submitted to HICSS 2011

Outline

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The Power Grid in Context

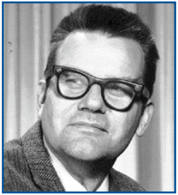


Figure: Fred C. Schweppe (1934-1988). Professor of Electrical Engineering, MIT.

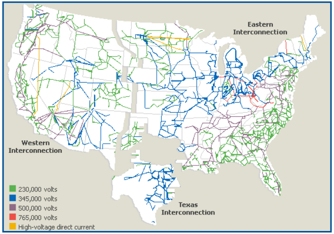


Figure: US power grid.

"I worked on aerospace problems for many years before converting to power systems, and, in my opinion at least, power problems are tougher in many respects.

...

The number of variables [in a power system] is huge, and many types of uncertainties are present.

...

Few if any aerospace problems yield such a challenging set of conditions."

– Fred. C. Schweppe, 1970

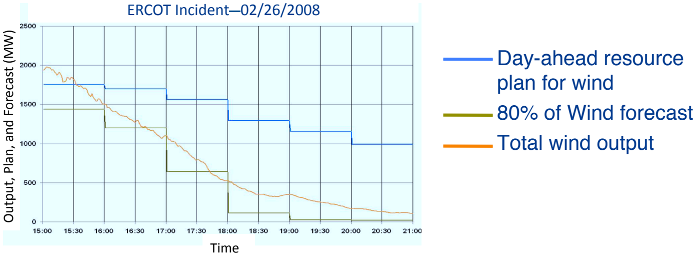
Integration of New Technologies: Blessing or Curse?



- Electric power systems worldwide are undergoing a radical transformation in structure and functionality driven by a quest to increase efficiency and responsiveness
- This added functionality provided by the integration of new technologies comes with side effects:
  - Increasing complexity in systems that are inherently complex
  - Introduction of new sources of uncertainty in systems that are inherently uncertain

Operational Uncertainty

- This uncertainty can be caused by any uncontrolled and unpredictable change (not necessarily a fault) on the demand or on the supply side of an electrical energy system
  - Generation based on intermittent renewable resources
- Operational uncertainty is not at all new to electric power systems—Demand Variation
- Extension to a significant portion of the generation is a cause of concern
  - Integration of large amounts of wind presents major challenges in operations and planning of today's power systems



Problem Statement

- As the presence of wind in the power grid increases, new tools are necessary to assess the impact of wind on the security of supply and load balancing in near real time
  - Wind variability cannot be just thought of as small disturbances (small-signal stability) or large disturbances (transient stability)
- This talk focuses on a particular aspect of the impact of wind penetration on system dynamic performance:
  - How system variables may deviate from prescribed values imposed by operational requirements due to wind variability
- We provide a method to assess whether certain variables remain within acceptable ranges while the system is subjected to uncontrolled disturbances caused by wind variability
  - Analytically tractable
  - Amenable for computer implementation

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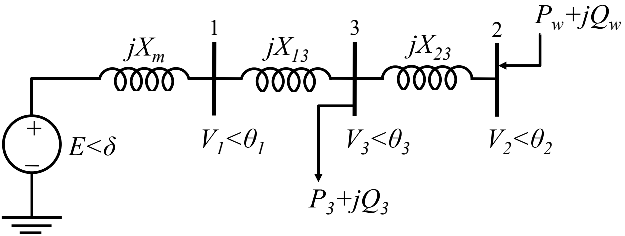
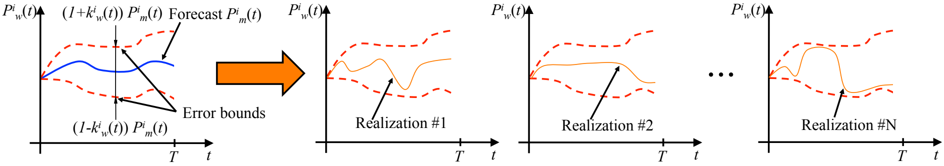
Basic Assumptions



- In a particular system, we may have conventional and renewable-based generation
- Conventional generators are modeled using standard dynamic models
- We assume that the power injected in the system by a wind farm can be modeled as a negative load
- In any given bus, a power injection can represent:
  - An aggregated model of a single wind farm connected directly to that node
  - An aggregated model of several wind farms within the same geographical area

Wind Power Injection Model

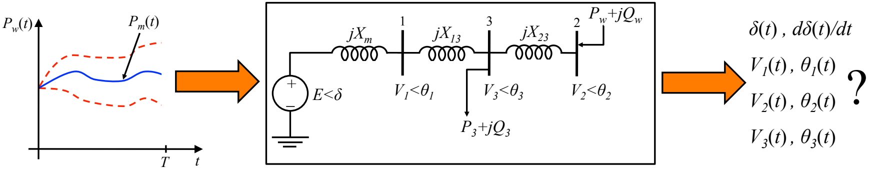


- The basic model assumes that the actual power injection can vary within some limits around a nominal (forecast) injection
- Let $P^i_w(t)$ be the power injected in node i at time $t \in [0, T]$ and let $P^i_m(t) > 0$ be the forecast wind power for $t \in [0, T]$. Then

$$P^i_w(t) \in \Omega_{P^i_w}(t) = \{|P^i_w(t) - P^i_m(t)| \leq k^i_w(t) P^i_m(t)\}, \qquad (1)$$

with $i = 1, 2, \ldots, n$, and where $k^i_w(t) \geq 0$ depends on the wind forecast error at time $t \in [0, T]$

The Reachability Problem



- Reachability. Computation of the set of possible trajectories that arise from all possible power injection scenarios
- Two possible approaches to solve the problem:
  - Extensive time-domain simulations
  - Analytical mapping of the set-theoretic power injection model into a set that contains all possible trajectories
- Methods for reachability based on ellipsoidal calculus have a long history in system theory [Schweppe ’73, Chernousko ’80, Kurzhanski and Valyi ’96, Kurzhanski and Varaiya ’02]

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Differential-Algebraic Equation Model

- The power system model is represented by a differential-algebraic equation:

$$\begin{aligned} \dot{x} &= g(x, y, u), \\ 0 &= h(x, y, w), \\ x(0) &= x_0, y(0) = y_0, \end{aligned} \tag{2}$$

  where
  - $x \in \mathbb{R}^n$ includes machine dynamic states such as angles, velocities, and the synchronous machine torque
  - $y \in \mathbb{R}^p$ includes bus voltages and angles
  - $u \in \mathbb{R}^m$ includes set points, such as voltage regulator reference and steam valve position
  - $w \in \mathbb{R}^l$ includes uncontrolled disturbances such as load demand or wind-based generated power

Ordinary Differential Equation Model

- We assume that system (3) is operating with some nominal (possibly time-varying) $u = u^*$
- We assume that the disturbance w can vary around some nominal (possibly time-varying) w^*, however $w(0) = w^*(0)$
- We assume that, for $t \in [0, T]$, the maximum variations of w around w^* are bounded and thus w belongs to a (possibly time-varying) set $\mathcal{W}$
  - This disturbance model could represent worst-case forecast error for the entries of w
- We assume that $h(\cdot, \cdot, \cdot)$ is continuously differentiable at each point of an open set $\mathcal{S}$ and $h(x^*, y^*, w^*) = 0$.
- If the Jacobian matrix $[\partial h/\partial y]_{(x^*, y^*, w^*)}$ is non-singular, then there exists function ϕ such that $y = \phi(x, w)$ locally around (x^*, y^*, w^*)
- Thus, around the system trajectory (x^*, y^*) that results from $u(t) = u^*$, $w(t) = w^*$, $x(0) = x_0$, and $y(0) = y_0$, it follows that

$$\dot{x} = g(x, \phi(x, w), u) = f(x, u, w) \tag{3}$$

Power System Reachability Analysis

- Since $u(t) = u^*$ and $w \in \mathcal{W}$, the system can be described by

$$\dot{x} = f(x, u^*, w),$$
$$w \in \mathcal{W},\; x(0) = x_0, \tag{4}$$

- If (4) is forward complete, then the solution $x(t)$ exists for $t \in [0, T]$ and it is contained in some set $\mathcal{R}$, which is called reach set or attainability domain [Angeli and Sontag '99]
- Our goal is to compute $\mathcal{R}$
  - The computation of $\mathcal{R}$ is a difficult task and often relies on time-domain simulations for different realizations of w
  - There is also a close connection between reachability analysis and input-to-state-stability (ISS) [Sontag '89]
  - Loosely speaking, if a system is ISS, bounded inputs lead to bounded system states
  - There is some recent work on the application of ISS notions to power system reachability [Müller and D-G '10]

Dynamic Performance Requirements

- Computing the reach set allows us to determine whether or not the system violates certain performance requirements that impose bounds on the excursions of certain variables
- Constraints in the form of interval ranges on other variables of interest include voltage at certain buses
- Thus, performance requirements will constrain the excursion of the state-vector x around x_0 to some region of the state-space Φ defined by the symmetric polytope

$$\Phi = \{x : |\pi_i' (x - x^*)| \leq 1 \ \forall i = 1, 2, \ldots, p\}, \tag{5}$$

  where $\pi_i \in \mathbb{R}^n$ is a column vector
- Checking that performance requirements are met for any $w(t) \in \mathcal{W}$, with $t \in [0, T]$, is equivalent to checking that $\mathcal{R} \subseteq \Phi$

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Basic Linearization Technique

- We use optimal control notions to describe perturbations of $x(t)$ that result from perturbing w around w^* [Athans and Falb '66]
- If the variations of the disturbance w around w^* are sufficiently small, we can approximate $\mathcal{R}$ by the reach set of the system that results from linearizing (4) around x^*
- Let $x(t) = x^* + \Delta x$ and $w(t) = w^* + \Delta w$. Then $\Delta w \in \Delta\mathcal{W}$, where $\Delta\mathcal{W}$ is such that $\mathcal{W} = w^* \oplus \Delta\mathcal{W}$
- Then, small variations in the system trajectories, denoted by Δx, originating from small variation of the disturbance w, denoted by Δw, can be approximately obtained from

$$\begin{aligned} \frac{d\Delta x}{dt} &= A(t)\Delta x + B(t)\Delta w, \\ \Delta w &\in \Delta\mathcal{W}, \ \Delta x(0) = 0, \end{aligned} \tag{6}$$

  with $A(t) = \frac{\partial f(x,u,w)}{\partial x}\Big|_{x^*,u^*,w^*}$, $B(t) = \frac{\partial f(x,u,w)}{\partial w}\Big|_{x^*,u^*,w^*}$
- The reach set of (6), denoted by $\Delta\mathcal{R}$, contains all possible trajectories of Δx, for all $t \geq 0$

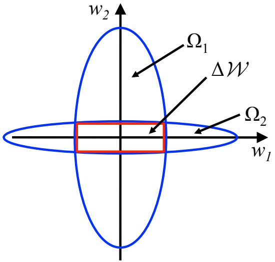
Input Space Set Approximation

- While the shape of $\Delta\mathcal{W}$ is arbitrary, it can always be bounded by an ellipsoid Ω:

$$\Delta w \in \Omega = \{\Delta w : \Delta w' Q(t)^{-1} \Delta w \leq 1\} \tag{7}$$

- $\Delta\mathcal{W}$ is usually a symmetrical polytope, i.e., each entry of w lies within an interval around the corresponding entry of w^*
- A symmetrical polytope can always be approximated to a high degree of precision by the intersection of a family of ellipsoids

$$\Delta\mathcal{W} = \bigcap_i \Omega_i \tag{8}$$



Linearized Model Reach Set Computation

- The reach set $\Delta\mathcal{R}$ can be computed as

$$\Delta\mathcal{R} = \bigcap_i \Delta\mathcal{R}_i, \tag{9}$$

where $\Delta\mathcal{R}_i$ is the reach set that results from the Ω_i ellipsoid bounding the set $\Delta\mathcal{W}$

- Each $\Delta\mathcal{R}_i$ can be computed as the intersection of a family of ellipsoids

$$\Delta\mathcal{R}_i = \bigcap_\eta \mathcal{X}_{i,\eta}, \ \forall\eta \in \mathbb{R}^n \text{ such that } \eta'\eta = 1, \tag{10}$$

with $\mathcal{X}_{i,\eta} = \{x : x'\Psi_{i,\eta}(t)^{-1}x \leq 1\}$, where for each η, $\Psi_{i,\eta}$ is obtained by solving

$$\frac{d\Psi_{i,\eta}}{dt} = A\Psi_{i,\eta} + \Psi_{i,\eta}A' + \beta_{i,\eta}\Psi_{i,\eta} + \frac{1}{\beta_{i,\eta}}BQ_iB', \tag{11}$$

where $\beta_{i,\eta} = \sqrt{\frac{\eta' e^{At} Q_i e^{A't}\eta}{\eta'\Psi_{i,\eta}\eta}}$ [Kurzhanski and Varaiya '02]

Outline

Introduction

The Basic Setup

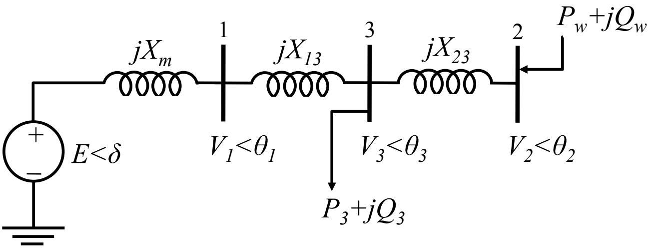
Power System Dynamic Model

Variational Approach to Power System Reachability

Tool Development and Case Studies

Concluding Remarks

Three-Bus System Example



- A synchronous machine is connected to bus 1
- An augmented classical model describes machine dynamics:
  - There is an additional equation to describe the governor model
  - The mechanical torque T_m becomes an additional state variable
  - The valve position P_c becomes an external reference
- Wind power is injected at bus 2
- There is a load connected to bus 3 served by both the synchronous generator and the wind-based power generation

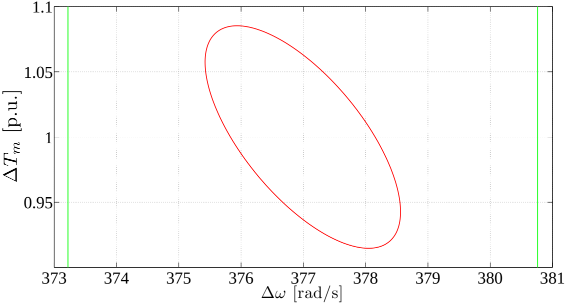
Numerical Results

Table: Model Parameter Values (all in per unit unless otherwise specified).

P_3	Q_3	P_m	Q_w	X_{13}	X_{23}	X_m	M	D [s/rad]	T_m	ω_s [rad/s]	k_w
1	0.5	0.4	0	0.1	0.2	0.15	$\frac{1}{15\pi}$	0.04	1	120π	0.3

- Reachability analysis was conducted using the parameter values in the Table above
- The nominal value of the wind-based power generation, $P_m = 0.4$ p.u. represents 40% of the total demand at bus 2
- The equilibrium voltage magnitudes and angles needed in the linearized model are:
  - $E_{1o} = 1.13$ p.u.
  - $V_{1o} = 1$ p.u.
  - $V_{2o} = 0.94$ p.u.
  - $V_{3o} = 0.94$ p.u.
  - $\theta'_{1o} = -6.12°$
  - $\theta'_{13o} = 3.65°$
  - $\theta'_{23o} = 3.89°$

Numerical Results



- The reach set depicted above is the result of 30% variation in P_w around the nominal value
- The reach set is entirely contained within the region defined by the solid vertical traces
  - The acceptable frequency range of the WECC system

Tool Development Effort

- We are modifying RPI's MATLAB-based Power Systems Toolbox (PST) small-signal stability analysis capability to our needs
- We are developing our own custom-made MATLAB-based code to compute reachable sets
- PST has the following capabilities which are relevant to our analysis
  - PST performs linearization around a steady-state operating point to produce the following form

    $$\dot{x} = Ax + Bu,$$

    where x are machine states and u are inputs.
  - PST allows the user to choose from a variety of input variables, such as mechanical power, exciter reference voltage, and real power modulation at a load bus

Analysis of New England System

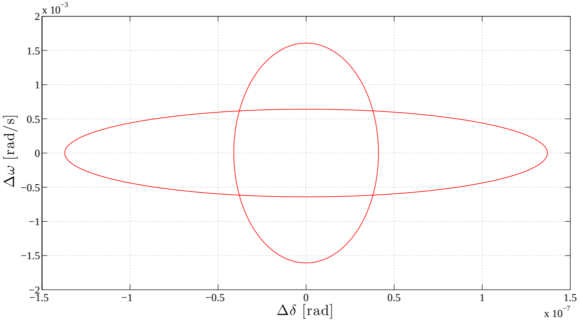
- Simplified model:
  - 10 machines, 39 buses, and 66 states
- We model the wind farm power injection as a negative load.
- To illustrate the method, we replace 4 machines in the NE system with negative loads.
- The loads are perturbed around the operating point and represent inputs.
- PST produces the following set of differential equations:

$$\begin{bmatrix} \dot{x} \\ \tau_1 \dot{P}_{w_1} \\ \tau_2 \dot{P}_{w_2} \\ \tau_3 \dot{P}_{w_3} \\ \tau_4 \dot{P}_{w_4} \end{bmatrix} = \begin{bmatrix} A & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ P_{w_1} \\ P_{w_2} \\ P_{w_3} \\ P_{w_4} \end{bmatrix} + \begin{bmatrix} b_1 & b_2 & b_3 & b_4 \\ K_1 & 0 & 0 & 0 \\ 0 & K_2 & 0 & 0 \\ 0 & 0 & K_3 & 0 \\ 0 & 0 & 0 & K_4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

- We assume the time constants for P_w are much faster than those of the machines and apply singular-perturbation method to remove the latter 4 differential equations.

NE System Analysis Numerical Results

- The reach set is a multi-dimensional ellipsoid, and we project the ellipsoid onto two relevant states $\Delta\delta$ and $\Delta\omega$ of one machine in the system.



- The reach set depicted above is the result of 30% variation in P_w's around their nominal values.

Outline

Some Challenges and Ongoing Work

- Reachability analysis is computationally expensive
- Scalability of ellipsoidal techniques to analyze large system remains to be proven
- Limits of the small-signal approximation for a given disturbance-size needs to be investigated
- Current tool development:
  - We are modifying RPI's Power Systems Toolbox (PST) small-signal stability analysis capability to our needs
  - We are developing our own custom-made code to compute reachable sets
- We will investigate the adaptability of existing commercial tools to support the proposed method