Phasor-Only State Estimation

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Talk Outline

Background and terminology
State estimation
Phasor measurements
Phasor-only state estimation
  WLS: exact cancellations
  LAV: improved robustness
  Extension to 3-phase networks
Observability and error identification
Closing remarks
Background and terminology

(Static) state estimation
- Measurement type: SCADA / Synchrophasor
- Formulation: Nonlinear / Linear
- Network model: + sequence / 3-phase

Dynamic state estimation
- Wide-area, using gen/load/network variables
- Local, using gen variables, boundary measurements or estimates
Modeling

Transmission Network
[Lines + Transformers]

Gen 1

Load 1

Gen 2

Load 2

Load \( N_L \)

Gen \( N_G \)

Measurements
Modeling [ SSE ]

SSE: Estimation of the snap-shot at time “t”

Linear or NL
Pos Seq or 3-Phase
SCADA or Phasor or
Mixed SCADA/Phasor

Transmission Network
[Lines + Transformers]

- Estimated Measurements

- Load 1
- Load 2
- Load $N_L$
- Gen 1
- Gen 2
- Gen $N_G$
Modeling [ DSE ]

DSE: Single machine / zonal / wide-area
Detailed Gen and Load models
Frequency and power angle estimation

Tracking the network and dynamic gen/load state variables in real-time

- Estimated Measurements
- Zone-1
- Zone-2
- Zone $N_Z$
- Load 1
- Load 2
- Load $N_L$
- Gen 1
- Gen 2
- Gen $N_G$

Transmission Network
[Lines + Transformers]
Talk Outline

Background and terminology

**State estimation**

Phasor measurements

Phasor-only state estimation
  - WLS: exact cancellations
  - LAV: improved robustness
  - Extension to 3-phase networks

Observability and error identification

Closing remarks
Static state estimation: 
Objective and measurements

Introduced by Schweppe and his co-workers in 1969.


**Objective:** To obtain the best estimate of the state of the system based on a set of measurements.

- State variables: voltage phasors at all buses in the system
- Measurements:
  - Power injection measurements
  - Power flow measurements
  - Voltage/Current magnitude measurements
  - **Synchronized phasor measurements**
State estimation: data/info flow diagram

- Topology Processor
- Network Observability Analysis
- **WLS State Estimator**
- Bad Data Processor
- Parameter and Topology Error Processor

Analog Measurements
- \(P_i, Q_i, P_f, Q_f, V, I\)

Topology Processor

Network Observability Check

State Estimator
- \(V, \theta\)

Bad Data Processor

Load Forecasts
- Generation Schedules

Parameter and Topology Errors
- Detection, Identification, Correction

Network Parameters, Branch Status, Substation Configuration
State estimation: data/info flow diagram

- Topology Processor
- Network Observability Analysis
- Robust LAV State Estimator
- Bad Data Processor
- Parameter and Topology Error Processor

Analog Measurements
Pi, Qi, Pf, Qf, V, I

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PM +
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Phasor measurement units (PMU)

“Synchronized Phasor Measurements and Their Applications” by A.G. Phadke and J.S. Thorp

Invented by Professors Phadke and Thorp in 1988.

They calculate the real-time phasor measurements synchronized to an absolute time reference provided by the Global Positioning System.

PMUs facilitate direct measurement of phase angle differences between remote bus voltages in a power grid.
Phasor Measurement Units (PMU) Phasor Data Concentrators (PDC)

[Antenna] -> GPS CLOCK

PMU 1 <-> PT <-> CT

PMU 2 <-> PMU 3 <-> PMU 4 <-> PMU K <-> PDC

PDC <-> CORPORATE PDC <-> DATA STORAGE & APPLICATIONS

PDC <-> REGIONAL PDC <-> DATA STORAGE & APPLICATIONS

[*] IEEE PSRC Working Group C37 Report
Phasor Data Concentrator (PDC)

Taken verbatim from: IEEE C37.244-2013 PDC Guide

PDC Definition:
A function that collects phasor data, and discrete event data from PMUs and possibly from other PDCs, and transmits data to other applications.

PDCs may buffer data for a short time period, but do not store the data. This guide defines a PDC as a function that may exist within any given device.
Phasor voltage measurements

Reference
Reference phasor
Under the Recovery Act SGIG and SGDP programs, their numbers rapidly increased: $166 \rightarrow 1126$. 

Measurements provided by PMUs

All 3-Phases are typically measured but only positive sequence components are reported.

\[
\begin{bmatrix}
V_A \\
V_B \\
V_C
\end{bmatrix} = [T] \begin{bmatrix}
V_0 \\
V_+ \\
V_-
\end{bmatrix} \Rightarrow V_+ = \frac{1}{3}[V_A + \alpha V_B + \alpha^2 V_C]
\]

\[\alpha = e^{\frac{j2\pi}{3}}\]
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Measurement equations

**SCADA Measurements**

\[ Z = h(X) + \nu \quad \text{Non-linear Model} \]

\[ H_x : \nabla h(X) \]

**Phasor Measurements**

\[ Z = H \cdot X + \nu \quad \text{Linear Model} \]

\[ H : \text{Function of network parameters only} \]

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Phasor-only WLS state estimation

\[ Z = H \cdot X + \nu \quad \text{Linear Model} \]

WLS state estimation problem:

Minimize \[ \sum_{i} \frac{r_{i}^{2}}{\sigma_{i}^{2}} \]

Subject to \[ r = Z - H \cdot \hat{X} \quad \text{residual} \]

\[ \hat{X} = G^{-1} H^{T} R^{-1} Z \quad \text{Direct solution} \]

\[ G = H^{T} R^{-1} H \ ; \ R = E\{\nu \cdot \nu^{T}\} = \text{cov}(\nu) \]

\[ \sigma_{i}^{2} : R(i, i) \text{ error variance} \]
Consider a fully measured system:

\[ Z^m = \begin{bmatrix} V^m \\ I^m \end{bmatrix} \Rightarrow \text{Bus voltages} \]
\[ = \begin{bmatrix} U \\ Y_b \cdot A \end{bmatrix} \cdot [V] + \nu \]

- \( U \) : identity matrix
- \( Y_b \) : branch admittance matrix
- \( A \) : branch - bus incidence matrix

Note: Shunt branches are neglected initially, they will be introduced later.
Phasor-only WLS state estimation

Let

\[
\begin{bmatrix}
V^m \\
I^m
\end{bmatrix}
= \begin{bmatrix}
e^m + jf^m \\
c^m + jd^m
\end{bmatrix}
\]

\[
= \begin{bmatrix}
U \\
(g + jb) \cdot A
\end{bmatrix} \cdot [e + jf] + \nu
\]

\[
= [H_F] \cdot [e + jf] + \nu
\]
Phasor-only WLS state estimation:
Complex to real transformation

\[
\begin{bmatrix}
e^m \\
f^m \\
c^m \\
d^m
\end{bmatrix}
= \begin{bmatrix}
U & U \\
gA & -bA \\
bA & gA
\end{bmatrix}
\cdot\begin{bmatrix}
e \\
f
\end{bmatrix} + \nu
\]

\[Z = H \cdot \begin{bmatrix}
e \\
f
\end{bmatrix} + \nu\]
Phasor-only WLS state estimation: 

Exact cancellations in off-diagonals of $[G]$

$R$ is assumed to be identity matrix without loss of generality

$$G = H^T \cdot H = \begin{bmatrix} U & U \\ gA & -bA \\ bA & gA \end{bmatrix}^T \begin{bmatrix} U & U \\ gA & -bA \\ bA & gA \end{bmatrix} = \begin{bmatrix} U + A^T \left( g^T g + b^T b \right) A & 0 \\ 0 & U + A^T \left( b^T b + g^T g \right) A \end{bmatrix}$$

$[G]$ matrix:

- Is block – diagonal
- Has identical diagonal blocks
- Is constant, independent of the state
Phasor-only WLS state estimation: Correction for shunt terms

\[ [Z] = (H + H_{sh}) \cdot X + \nu = H \cdot X + u \]

\[ u = H_{sh} \cdot X + \nu \]

\[ E\{u\} = H_{sh} \cdot E\{X\} \]

\[ E\{X\} = \hat{X} = G^{-1} H^T R^{-1} Z \]

\[ X^{corr} = G^{-1} H^T R^{-1} (Z - H_{sh} \cdot \hat{X}) \]

\[ = \hat{X} - G^{-1} H^T R^{-1} H_{sh} \cdot \hat{X} \]

Very sparse
Fast Decoupled WLS Implementation Results

Test Systems Used

<table>
<thead>
<tr>
<th>System Label</th>
<th>Number of Buses</th>
<th>Number of Branches</th>
<th>Number of Phasor Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>159</td>
<td>198</td>
<td>222</td>
</tr>
<tr>
<td>B</td>
<td>265</td>
<td>340</td>
<td>361</td>
</tr>
<tr>
<td>C</td>
<td>3625</td>
<td>4836</td>
<td>4982</td>
</tr>
</tbody>
</table>

Cases simulated:

Case-1: No bad measurement.
Case-2: Single bad measurement.
Case-3: Five bad measurements.
## Average MSE Values (for 100 Simulations)

<table>
<thead>
<tr>
<th>System</th>
<th>Case</th>
<th>MSE</th>
<th>Decoupled WLS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>WLS</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>0.59</td>
<td>0.59</td>
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<td>2</td>
<td>0.59</td>
<td>0.59</td>
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<tr>
<td></td>
<td>3</td>
<td>0.62</td>
<td>0.62</td>
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<tr>
<td>B</td>
<td>1</td>
<td>0.61</td>
<td>0.61</td>
</tr>
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<td>2</td>
<td>0.61</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
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<td>0.63</td>
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<tr>
<td>C</td>
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<td>0.58</td>
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</tr>
<tr>
<td></td>
<td>3</td>
<td>0.58</td>
<td>0.59</td>
</tr>
</tbody>
</table>
## Fast Decoupled WLS Implementation Results

### Mean CPU Times of 100 Simulations

<table>
<thead>
<tr>
<th>System</th>
<th>Case</th>
<th>WLS</th>
<th>Decoupled WLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>5</td>
<td>2.4</td>
</tr>
<tr>
<td>A</td>
<td>2</td>
<td>5.7</td>
<td>2.7</td>
</tr>
<tr>
<td>A</td>
<td>3</td>
<td>9.3</td>
<td>3.9</td>
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<tr>
<td>B</td>
<td>1</td>
<td>7.5</td>
<td>3.5</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>8.7</td>
<td>3.9</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
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<tr>
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<tr>
<td>C</td>
<td>2</td>
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</tr>
<tr>
<td>C</td>
<td>3</td>
<td>284.7</td>
<td>165.6</td>
</tr>
</tbody>
</table>
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L₁ (LAV) Estimator

Minimize \[ \sum_{i=1}^{m} c^T \cdot |r| \]

Subject to \[ Z = H \cdot \hat{X} + r \]

\[ c^T = [1, 1, \ldots, 1] \]

Robust but with known deficiency:

Vulnerable to leverage measurements
Motivating example: simple regression

No leverage points

Wrong data

\[ y = x \beta + w \]

\[ L_1 \text{ Estimator for } \beta \]
Motivating example: simple regression

Leverage point exists and contains gross error: \( L_1 \) estimator fails to reject bad measurement

\[
y = x \beta + w
\]

Wrong data (leverage point)
Leverage points in measurement model


– Flow measurements on the lines with impedances, which are very different from the rest of the lines.
– Using a very large weight for a specific measurement.

\[ H \ x + e = z \]

\[ R^{-1}H = \begin{bmatrix}
  x & x & x \\
  x & x & x \\
  x & x & x \\
  x & x & x \\
  Y & Y & Y \\
  x & x & x \\
\end{bmatrix} \]

Scaling both the measured value and the measurement jacobian row will eliminate leveraging effect of the measurement.

Leave zero injections since they are error free by design. Incorporate equality constraints in the formulation.
Properties of $L_1$ estimator

- Efficient Linear Programming (LP) code exists to solve it for large scale systems.
- Use of simple scaling eliminates leverage points. This is possible due to the type of phasor measurements (either voltages or branch currents).
- $L_1$ estimator automatically rejects bad data given sufficient local redundancy, hence bad data processing is built-in.
Conversion to Equivalent LP Problem

\[ \begin{align*}
\min \ c^T |r| & \quad \rightarrow \quad \min \ c^T y \\
\text{s.t.} \quad Hx + r = z & \quad \rightarrow \quad \text{s.t.} \quad My = z \\
\quad y \geq 0
\end{align*} \]

\[ c^T = \begin{bmatrix} 0_n & 0_n & c_m & c_m \end{bmatrix} \]

\[ y = \begin{bmatrix} X_a^T & X_b^T & U^T & V^T \end{bmatrix}^T \quad x = X_a - X_b \]

\[ M = \begin{bmatrix} H & -H & I & -I \end{bmatrix} \quad r = U - V \]
Bad data processing

WLS: Post estimation bad data processing / re-estimation

\[ \Omega = R - HG^{-1}H^T \]

Normalized Residuals Test (NRT):

\[ G = H^T R^{-1}H \]

\[ r_i^N = \frac{|r_i|}{\sqrt{\Omega_{ii}}} \quad z_{i}^{new} = z_{i}^{bad} - \frac{R_{ii}}{\Omega_{ii}} r_i^{bad} \]

Update the estimates

NRT is repeated as many times as the number of bad data.

\[ L_1: \text{Linear Programming Solution} \]

LP problem is solved once. Choice of initial basis impacts solution time/iterations.
IEEE 30-bus system

Line 1-2 parameter is changed to transform $I_{1-2}$ into a leverage measurement

Case 1: Base case true solution

Case 2: Bad leverage measurement without scaling

Case 3: Same as Case 2, but using scaling
# Small system example

<table>
<thead>
<tr>
<th>Case</th>
<th>V (pu)</th>
<th>θ (deg)</th>
<th>V (pu)</th>
<th>θ (deg)</th>
<th>V (pu)</th>
<th>θ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Case-1</strong></td>
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<tr>
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<td>1</td>
<td>5.2</td>
</tr>
<tr>
<td>2</td>
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<td>-3.74</td>
<td>0.9999</td>
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<td>-3.74</td>
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<tr>
<td>3</td>
<td>0.9952</td>
<td>0.13</td>
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<tr>
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<td>0.9851</td>
<td>-7.41</td>
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Large test case: 3625 bus + 4836 branch utility system

Case a: No bad measurement.
Case b: Single bad measurement.
Case c: Five bad measurements.

<table>
<thead>
<tr>
<th></th>
<th>Case a</th>
<th>Case b</th>
<th>Case c</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LAV</strong></td>
<td>3.33 s.</td>
<td>3.36 s.</td>
<td>3.57 s.</td>
</tr>
<tr>
<td><strong>WLS</strong></td>
<td>2.32 s.</td>
<td>9.38 s.</td>
<td>50.2 s.</td>
</tr>
</tbody>
</table>
Phasor-only state estimation: WLS versus $L_1$ (LAV)

WLS:
- Linear solution
- Requires bad-data analysis
  - Normalized residuals test
  - Re-weighting (not applicable)
- No deficiency in the presence of leverage measurements, with *scaling*.
- Exact cancellations in $[G]$

$L_1$ (LAV):
- Linear programming *(single solution)*
  
  *computationally competitive in s-s*
- Does not require bad-data analysis
- No deficiency in the presence of leverage measurements, with *scaling*.

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Observability and error identification
Closing remarks
3-phase operation: motivation and observations

• State estimators implicitly assume balanced operating conditions and use positive sequence network model and measurements. This assumption is sometimes questionable.

• Power measurements are non-linear functions of state variables.

• Phasor measurements are linearly related to the states.
Further motivation

• Develop a state estimator capable of solving state estimation for three-phase unbalanced operating conditions,
  yet:
  • Utilize existing, well developed and tested software as much as possible.
Proposed approach: Modal decomposition revisited

- Decompose phasor measurements into their symmetrical components
- Estimate individual symmetrical components of states separately (in parallel if such hardware is available)
- Transform the estimates into phase domain to obtain estimated phase voltages and flows.
Modal decomposition of measurement equations

\[ Z = HV + e \]
\[ Z^T = [Z_v^T Z_i^T] \]

Phasor domain measurement representation
H: 3mx3n
V: 3nx1
Z: 30x1

Modal domain vectors
T: 3x3
\( V_S = TV_P \) and \( I_S = TI_P \)

\[ T_Z = \begin{bmatrix} T & 0 & \cdots & 0 \\ 0 & T & \cdots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & T \end{bmatrix} \]

T: 3mx3m

\[ T_Z Z = T_Z HV + T_Z e \]
Modal decomposition of measurement equations

\[ V = T_v V_M \]

\[ T_v = \begin{bmatrix} T^{-1} & 0 & \cdots & 0 \\ 0 & T^{-1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & T^{-1} \end{bmatrix} \]

\[ Z_M = H_M V_M + e_M \]

\[ Z_M = T_Z Z \]

\[ H_M = T_Z H T_V \]

\[ e_M = T_Z e \]

Zero sequence

\[ Z_0 = H_0 V_0 + e_0 \]

\[ V_0 = G_0^{-1} H_0^T R_0^{-1} Z_0 \]

\[ G_0 = H_0^T R_0^{-1} H_0 \]

Positive/ Negative sequence

\[ Z_r = H_r V_r + e_r \]

\[ V_r = G_r^{-1} H_r^T R_r^{-1} Z_r \]

\[ G_r = H_r^T R_r^{-1} H_r \]

\[ Z_0, Z_r : mx1 \]

\[ V_0, V_r : nx1 \]

\[ H_0, H_r : mxn \]

- Fully decoupled three relations
- Smaller size (Jacobian) matrices
Large scale 3-phase system example: 3625-buses and 4836-branches

Case a: No bad measurement.  
Case b: Single bad measurement.  
Case c: Five bad measurements.

<table>
<thead>
<tr>
<th></th>
<th>Case a</th>
<th></th>
<th>Case b</th>
<th></th>
<th>Case c</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LAV</td>
<td>WLS</td>
<td>LAV</td>
<td>WLS</td>
<td>LAV</td>
<td>WLS</td>
</tr>
<tr>
<td>Zero Seq.</td>
<td>3.52 s.</td>
<td>2.32 s.</td>
<td>3.65 s.</td>
<td>9.28 s.</td>
<td>3.78 s.</td>
<td>25.51 s.</td>
</tr>
<tr>
<td>Positive Seq.</td>
<td>3.61 s.</td>
<td>2.62 s.</td>
<td>3.63 s.</td>
<td>9.41 s.</td>
<td>3.66 s.</td>
<td>26.01 s.</td>
</tr>
<tr>
<td>Negative Seq.</td>
<td>3.21 s.</td>
<td>2.22 s.</td>
<td>3.45 s.</td>
<td>9.01 s.</td>
<td>3.62 s.</td>
<td>24.82 s.</td>
</tr>
</tbody>
</table>
Talk Outline

Background and terminology
State estimation
Phasor measurements
Phasor-only state estimation
  WLS: exact cancellations
  LAV: improved robustness
  Extension to 3-phase networks

Observability and error identification

Closing remarks
No reference bus or reference PMU is needed or should be used

- Eliminate the reference phase angle from the SE formulation.
- Bad data in SCADA as well as phasor measurements can be detected and identified with sufficiently redundant measurement sets.
Numerical Example

: Power Injection  : Power Flow  : Voltage Magnitude

: PMU
**Numerical Example**

**Error in bus 1 phase angle**

<table>
<thead>
<tr>
<th>Test A</th>
<th>Test B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Measurement</strong></td>
<td><strong>Normalized residual</strong></td>
</tr>
<tr>
<td>$\theta_8$</td>
<td>7.5</td>
</tr>
<tr>
<td>$p_{4-7}$</td>
<td>5.73</td>
</tr>
<tr>
<td>$p_{5-6}$</td>
<td>5.6</td>
</tr>
<tr>
<td>$\theta_{12}$</td>
<td>5.2</td>
</tr>
<tr>
<td>$V_1$</td>
<td>4.53</td>
</tr>
</tbody>
</table>
Merging Observable Islands

- PMUs can be placed at any bus in the observable island.
- SCADA pseudo-measurements can merge observable islands only if they are incident to the boundary buses.
Robust Metering

• Bad data appearing in “critical measurements” can NOT be detected.

• Adding new measurements at strategic locations will transform them, allowing detection of bad data which would otherwise have been missed.

Note:
When a “critical measurement” is removed from the measurement set, the network will no longer be OBSERVABLE.
IEEE 57-bus system

<table>
<thead>
<tr>
<th>Number of Critical Meas.</th>
<th>Number of PMU needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>2</td>
</tr>
</tbody>
</table>

Critical Meas. | Type
---|---
1 | F41-43
2 | F36-35
3 | F42-41
4 | F40-56
5 | I-11
6 | I-24
7 | I-39
8 | I-37
9 | I-46
10 | I-48
11 | I-56
12 | I-57
13 | I-34
IEEE 118-bus system

<table>
<thead>
<tr>
<th>Number of Critical Meas.</th>
<th>Number of PMU needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>13</td>
</tr>
</tbody>
</table>
Impact of PMUs on Parameter Error Identification

- Are there cases where errors in certain parameters can not be identified without synchronized phasor measurements?

Cases where more than one set of parameters satisfies all SCADA measurements.
Illustrative Example

\[ J(x, p_1, p_2) = J(x', p'_1, p'_2) \]

\[ \frac{x'_{kl}}{x_{kl}} = \frac{\theta_k - \theta'_l}{\theta_k - \theta_l} \quad \frac{x'_{lm}}{x_{lm}} = \frac{\theta_l - \theta_m}{\theta_l - \theta_m} \]

\[ P_l = P_{lm} - P_{kl} \]
Illustrative Example

\[ P_{kl} = \frac{\theta_k - \theta_l}{x_{kl}} \]
\[ P_{lm} = \frac{\theta_l - \theta_m}{x_{lm}} \]

\[ J(x, p_1, p_2) = J(x', p_1', p_2') \]
\[ x'_{kl} = \frac{\theta_k - \theta_l'}{\theta_k - \theta_l} \]
\[ x'_{lm} = \frac{\theta_l' - \theta_m}{\theta_l - \theta_m} \]

\[ P_l = P_{lm} - P_{kl} \]
Remarks and Conclusions

• LAV estimator will be a computationally viable and effective alternative to WLS estimator when the measurement set consists of only PMUs. Scaling can be a simple yet effective tool in the presence of leverage measurements.

• Use of phasor measurements for SE allows modal decomposition of measurement equations.

• WLS estimator implementation has built-in simplifications due to exact cancellations in the gain [G] matrix.

• Given sufficiently redundant phasor measurements, fast and robust state estimation is possible even for very large scale power grids.


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  Murat Gol, Ph.D.  2014
Thank You

Any Questions?