A Zero-Reflection Controller for Electromechanical Disturbances in Power Networks

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Approach

• Use a continuum model to gain insight for the design of generator controls.

• Focus on traveling waves and impedance-matching controllers.
Background - general

• Power System Models - ODEs, DAEs

• Analysis – simulation, modal analysis, energy methods.

• Control – modal approach.
Background – continuum model

Distributed model, PDE description, electromechanical wave behavior.

- Semlyen (1974) Introduced model, traveling waves, standing waves, parallels to EM.
- Thorp et al. (1998) Develop model to study traveling waves observed in phasor measurements.
A Power System Model

Distributed parameters:
- $z$: impedance per unit length
- $h$: inertia per unit length
- $d$: damping per unit length

Denote $z^{-1} = g - j b$
A Power System Model

\[
\frac{2h\Delta}{\omega_s} \frac{d^2 \delta}{dt^2} = p_g \Delta - d \Delta \frac{d \delta}{dt} \\
- \frac{E^2 b}{\Delta} \left( \sin(\delta(x) - \delta(x - \Delta)) + \sin(\delta(x) - \delta(x + \Delta)) \right) \\
- \frac{E^2 g}{\Delta} \left( 2 \cos(\delta(x) - \delta(x - \Delta)) - \cos(\delta(x) - \delta(x + \Delta)) \right)
\]

(Swing Equation Model)

Neglect \( g \), \( d \), and examine the limit as \( \Delta \to 0 \)

\[
\frac{2h}{\omega_s} \frac{\partial \omega}{\partial t} = p_g - \frac{\partial P}{\partial x}
\]
Continuum Model

\[ \frac{2h}{\omega_s} \frac{\partial \omega}{\partial t} = p_g - \frac{\partial P}{\partial x} \]

where

\[ P = -E^2 b \frac{\partial \delta}{\partial x} \]

differentiate w.r.t. time

Telegrapher’s Equation (we’ll use later)
Continuum Model

\[
\frac{2h}{\omega_s} \frac{\partial \omega}{\partial t} = p_g - \frac{\partial P}{\partial x}
\]

where

\[
P = -E^2 b \frac{\partial \delta}{\partial x}
\]

combine

\[
\frac{2h}{\omega_s} \frac{\partial^2 \delta}{\partial t^2} = p_g - E^2 b \frac{\partial^2 \delta}{\partial x^2}
\]

linear undamped wave equation

(a more detailed nonlinear model is found in Thorp et al.)
Continuum Model - Higher Dimensions

\[
\frac{2h}{\omega_s} \frac{\partial^2 \delta}{\partial t^2} = p_g - E^2 b \nabla^2 \delta
\]

Same equation, parameters have different units.

*Does the original discrete model exhibit wave-like behavior?*
A Power System Model

\[ \frac{d\delta_k}{dt} = \omega_k \quad \forall k \]

\[ 2H \frac{d\omega_k}{\omega_s dt} = p_k - 0.01\omega_k \]

\[ -6 \left( \sin(\delta_k - \delta_{k-1}) + \sin(\delta_k - \delta_{k+1}) \right) - \left( 2 - \cos(\delta_k - \delta_{k-1}) - \cos(\delta_k - \delta_{k+1}) \right) \quad 2 \leq k \leq 63 \]

\[ 2H \frac{d\omega_1}{\omega_s dt} = p_1 - 0.01\omega_1 \]

\[ -6 \sin(\delta_1 - \delta_2) - (1 - \cos(\delta_1 - \delta_2)) \quad k = 1 \]

\[ 2H \frac{d\omega_{64}}{\omega_s dt} = p_{64} - 0.01\omega_{64} \]

\[ -6 \sin(\delta_{64} - \delta_{63}) - (1 - \cos(\delta_{64} - \delta_{63})) \quad k = 64 \]
Initial Conditions and Disturbance

• Initial conditions: angle varies by $2\pi$ over the string of generators.

• Gaussian pulse perturbation: $\tilde{\delta}_k = \frac{1}{2} e^{-0.1(k-15.5)^2}$
Continuum Model - Characteristic Impedance

Forward traveling wave

\[
\frac{\partial P}{\partial t} = -E^2 b \frac{\partial \omega}{\partial x}
\]

\[
\frac{\partial P^+}{\partial y} = \frac{E^2 b}{v} \frac{\partial \omega^+}{\partial y}
\]

\[
P^+ = P\left(t - \frac{x}{v}\right) = P(y)
\]

\[
\omega^+ = \omega\left(t - \frac{x}{v}\right) = \omega(y)
\]

\[
P^+ = \frac{E^2 b}{v} \omega^+
\]

\[P^+ \text{ and } \omega^+ \text{ are in constant proportion in the traveling wave.}\]
Continuum Model - Characteristic Impedance

Forward traveling wave

\[ P^+ = \frac{E^2 b}{v} \omega^+ \]

Characteristic Impedance

\[ C_o = \frac{\omega^+}{P^+} = \frac{v}{E^2 b} = \sqrt{\left(\frac{\omega_s}{2h}\right)\frac{1}{E^2 b}} \]
Reflection Coefficients

Terminating the string of generators such that frequency and power are in constant proportion, $C=\frac{\omega}{P}$, leads to the Following Reflection Coefficient:

$$R = \frac{C - C_0}{C + C_0}$$

$R = 1$ positive reflection (in frequency)

$C \rightarrow \infty$, open ended, negative reflection in power

$R = -1$ negative reflection (in frequency)

$C = 0$, infinite bus, angle and frequency are fixed.

$R = 0$ no reflection
A Zero Reflection Controller

Terminate the string of generators with the characteristic impedance.

Imposed Generator Control Objective: \( \tilde{P}_{\text{end}} = \frac{\omega}{C_0} \)

deviations from nominal values

Generator Model

\[
\frac{d\delta}{dt} = \omega \\
2H \frac{d\omega}{\omega_s} dt = p_g + P_{\text{end}} - D \omega
\]

Using \( p_g \) as the input, the control objective cannot be achieved… at least not exactly…
A Zero Reflection Controller

Generator Model

\[ \frac{d\delta}{dt} = \omega \]
\[ \frac{2H}{\omega_s} \frac{d\omega}{dt} = P_g + P_{end} - D\omega \]

Try the following feedback control

\[ \tilde{P}_g = K \left( \tilde{P}_{end} - \frac{\omega}{C_o} \right) - \tilde{P}_{end} + D\omega \]

control objective
Results - 1D Case

No Control

Zero-Reflection Controller at Generator 64
2D Grid / no control

T = 0 s
T = 2.4 s
T = 6 s
T = 10.8 s
T = 16.8 s
T = 24 s
2D Grid / Zero Reflection Controller
2D Grid / Zero Reflection Controller
Perturbed Parameters ±90%
L-Shaped Grid / No Control
L-Shaped Grid / Zero Reflection Controller
Perturbed Parameters ±90%
Conclusions

• Conceptual Contributions:
  • Traveling waves $\rightarrow$ eliminate reflections.
  • Place controllers where reflections are expected.
  • Controller uses deviations in both power and frequency as inputs.

• Future Work: lots!

more realistic models, realistic assessment, irregular topologies, other control opportunities, and more.