Coordinated Wide-Area Polytopic Control Design using Linear Matrix Inequality

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Related Publications


New Challenges in a Modern Grid

- Increased penetration of renewables
- Decommissioning of large thermal units
Overview of Damping Controllers

Flexible Alternating Current Transmission System (FACTS) [7]-[8]
- Primarily for voltage support
- Supplementary Damping Control (SDC) added

Power System Stabilizers (PSSs) [5]-[6]
- Traditional control for damping local modes
- Require careful tuning for damping inter-area modes

High-Voltage DC systems, Energy Storage Systems [9]-[11]
- SDC added for damping inter-area modes
- Controlled modulation of power flows

Methods
- Coordinated control implemented [12]-[14]
- Wide-area signals employed [15]-[19]
- Larsen & Swan
- Optimization techniques such as particle swarm optimization, genetic algorithms
- Linear Matrix Inequality (LMI) based control
Motivation and Objectives

• Most control designs:
  • Focused on only one type of controller and/or were operating point specific
  • Required changing the configurations of existing controls
  • Had higher-order complexity

Objectives

• To coordinate individual controllers such as HVDC based SDCs, Static VAr Compensators (SVCs), and PSSs
• To design a coordinated wide-area damping controller (CWADC) using LMI-based polytope having mixed $H_\infty/H_2$ control
• To develop a methodology for the selection of suitable stabilizing signals for the CWADC
• To provide flexibility in selection of feedback signals
**Mathematical Background-State Space Model**

**State-space model with $H_\infty/H_2$ * formulation**

\[
\dot{x} = Ax + B_1 \omega + B_2 u \\
z_\infty = C_1 x + D_{11} \omega + D_{12} u \\
z_2 = C_2 x + D_{22} u \\
y = C_y x + D_{y1} \omega + D_{y2} u
\]

where:
- $x$ is the system state
- $u$ is the control
- $\omega$ is a disturbance
- $z_\infty$ and $z_2$ are for $H_\infty/H_2$ optimization
- $y$ is the system output

\[
T_\infty = (C_1 + D_{12} K)(sI - A - B_2 K)^{-1}B_1 + D_{11} \\
T_2 = (C_2 + D_{22} K)(sI - A - B_2 K)^{-1}B_1
\]

* The $H_\infty$ analysis is used to evaluate how robust a system is when exposed to dynamic uncertainty. The $H_2$ design parameters are tailored more towards measuring the control effort and providing disturbance rejection.
Polytope Formation-Extension to Multiple Operating Points

**Three vertex polytope**

- **Load Increase**
- **Generation Decrease**

**Base Case** $S_1$

**Vertex of each polytope, $S_i$**

$$S_i = \begin{bmatrix} A_i & B_{i1} & B_{i2} \\ C_1 & D_{i11} & D_{i12} \\ C_{i2} & 0 & D_{i22} \\ C_{iy} & D_{iy1} & D_{iy2} \end{bmatrix}$$

**Convex combination of the systems**:

$$S\{S_1, S_2, \ldots, S_n\} = \left\{\sum_{i=1}^{n} \alpha_i S_i: \sum_{i} \alpha_i = 1, \alpha_i \geq 0 \right\}$$
Theoretical Background - Linear Matrix Inequality

- An LMI is any constraint of the form $A(p) := A_0 + p_1A_1 + p_2A_2 + ... + p_nA_n < 0$ where $p_1, p_2, ..., p_n$ are unknown vectors comprising of optimization variables; $A_0, A_1, ..., A_n$ are known symmetric matrices; $< 0$ implies “negative definite”

<table>
<thead>
<tr>
<th>Primary Advantage</th>
<th>Primary Disadvantage</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Any solution to a problem obtained using LMIs is a global optimum</td>
<td>• Inherent computational complexity of the optimization</td>
</tr>
</tbody>
</table>

\[
\begin{bmatrix}
A_i X + X A_i^T + B_{i2} Y + Y^T B_{i2}^T & B_{i1} & X C_i^T + Y^T D_i^T \\
B_{i1}^T & -I & D_{i11}^T \\
C_{i1} X + D_{i12} Y & D_{i11} & -\gamma^2 I \\
Q & C_{i2} X + D_{i22} Y & X \\
X C_{i2}^T + Y^T D_{i22}^T & X & > 0
\end{bmatrix} < 0
\]

\[
(V \otimes W) + (W \otimes (A_i X + B_{i2} Y)) + W^T \otimes (A_i X + B_{i2} Y)^T < 0
\]

**LMI for the State Space Model**

where $X$ is the Lyapunov matrix, $Q$ is a positive definite matrix, and $\otimes$ is the Kronecker product*

Model Reduction Technique

Selective Modal Analysis

- An iterative process that simplifies the dynamic model to the oscillatory modes of interest

\[ *A = \begin{bmatrix} A_1 & A_2 \\ A_2 & 0 \end{bmatrix} \begin{bmatrix} 0 & \omega_0 I \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ A_{23} & A_{24} \end{bmatrix} \]

\[ \begin{bmatrix} A_{31} & A_{32} \\ A_{41} & A_{42} \end{bmatrix} \begin{bmatrix} A_{33} & A_{34} \\ A_{43} & A_{44} \end{bmatrix} \]

\[ A_3 \quad A_4 \]

* \(A\) is the state matrix of the original system, while \(A_1 + M_i\) is the state matrix of the reduced system, where \(M_0\) is the null matrix.
## Model Reduction Technique

**Selective Modal Analysis**

### Comparison of LMI optimization without and with SMA approach on the 4 machine – 2 area system

<table>
<thead>
<tr>
<th></th>
<th>LMI Control</th>
<th>LMI Control + Traditional SMA</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>System Size ((A/A_1))</strong></td>
<td>19 × 19</td>
<td>6 × 6</td>
</tr>
<tr>
<td><strong>Size of individual LTI system</strong></td>
<td>24 × 22</td>
<td>11 × 9</td>
</tr>
<tr>
<td><strong>Size of Polytopic System</strong></td>
<td>24 × 116</td>
<td>11 × 51</td>
</tr>
<tr>
<td><strong>Size of Closed Loop System</strong></td>
<td>24 × 111</td>
<td>11 × 46</td>
</tr>
<tr>
<td><strong>CPU Time</strong>* (seconds)</td>
<td>29.286383</td>
<td>1.209831</td>
</tr>
</tbody>
</table>

*** The computations were performed on an Intel (R) Core ™ i5 Processor having a speed of 2.40 GHz and an installed memory (RAM) of 5.86 GB
Control Effort Minimization-Partial State Feedback

- **Focus**: To control generator states which are influencing a greater number of modes by having higher participation in them.

Identify *minimum* number of states that must be controlled for improving stability of the system.

**Enhanced SMA-Proposed methodology for generator selection** [20]

- **Pre-processing**
  - a) Calculate average participation of each state, $p^\delta_{avg}$ across all modes.
  - b) Scale individual PF of each state in different modes with respect to $p^\delta_{avg}$.

- **Clustering**
  - a) k-means clustering.
  - b) Silhouette index for defining exact number of clusters.

- **Selection of set of non-critical generators, S**
Proposed Control using Partial State Feedback

1. Identify critical modes and sets of generators that participate in them
2. Compute $p_{\text{modified}}^\delta$ using k-means clustering and silhouette index
3. Define the set of non-critical generators, $\psi_{\text{non-crit}}^{\text{init}}$, based on $p_{\text{modified}}^\delta$
4. Drop a non-critical generator
   - Does SMA converge? 
   - Apply linear quadratic regulator (LQR) 
   - Is system stable? 
     - Yes: Is system stable? 
     - No: Design an LMI-based polytopic control using the critical generator set identified in previous iteration

Alternate Feedback Control Scheme

- Replace an element in the $\psi^{\text{final}}_{\text{non-crit}}$ with an element in $\psi_{\text{non-crit}}^{\text{init}} \setminus \psi^{\text{final}}_{\text{non-crit}}$
  - a) Check if SMA converges
  - b) Apply LQR control to check if the system is stable

Sequence of alternate set of critical generators identified

** $\psi^{\text{final}}_{\text{non-crit}}$ is the first set of non-critical generators dropped from control
Controller Synthesis

- **Bi-level** controller design
Flexibility of Feedback Selection

• Taking PMU measurements from alternate locations in case the primary locations encounter a problem [20]
Small Test System

16 machine, 68-bus system

• System Details:
  • Represents the reduced-order equivalent model of five separate areas
  • Two types of actuators, namely, DC lines and SVCs employed for implementing the control
  • The DC lines are modeled as active power and reactive power injections
  • SVC is modeled as a reactive power injection
• Type of Analysis Performed: Modal analysis for testing the performance of CWADC
# Results-16 Machine, 68-Bus System

## Test Cases

<table>
<thead>
<tr>
<th>Number</th>
<th>Contingency Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Base Case</td>
</tr>
<tr>
<td>2</td>
<td>Flow in line 50-52 is increased from 700 to 900</td>
</tr>
<tr>
<td>3</td>
<td>Flow in line 50-52 is decreased from 700 to 455</td>
</tr>
<tr>
<td>4</td>
<td>Outage of Tie-line 1-2</td>
</tr>
<tr>
<td>5</td>
<td>10% increase in inertia of Machine at Bus 66</td>
</tr>
<tr>
<td>6</td>
<td>10% decrease in inertia of Machine at Bus 66</td>
</tr>
<tr>
<td>7</td>
<td>All loads decreased in the system to 90%</td>
</tr>
<tr>
<td>8</td>
<td>All loads increased in the system to 103%</td>
</tr>
</tbody>
</table>
Results-16 Machine, 68-Bus System

Enhanced SMA Results

SMA convergence after dropping eighth machine for Base Case
The number of machines which can be dropped varies with the test cases.

Minimum number of same six machines were chosen to be dropped, reducing the state feedback signals by 37.5%.

### Results-16 Machine, 68-Bus System

<table>
<thead>
<tr>
<th>Number</th>
<th>Sequence for dropping generators</th>
<th>Reduction in stabilizing signals(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>63, 62, 60, 61, 64, 53, 55, 57</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>63, 62, 64, 53, 55, 57</td>
<td>37.5</td>
</tr>
<tr>
<td>3</td>
<td>63, 62, 60, 64, 53, 55, 57</td>
<td>43.75</td>
</tr>
<tr>
<td>4</td>
<td>63, 62, 64, 53, 60, 55, 57</td>
<td>43.75</td>
</tr>
<tr>
<td>5</td>
<td>63, 62, 64, 53, 55, 60, 57</td>
<td>43.75</td>
</tr>
<tr>
<td>6</td>
<td>63, 62, 64, 53, 55, 57</td>
<td>37.5</td>
</tr>
<tr>
<td>7</td>
<td>63, 62, 64, 53, 55, 57</td>
<td>37.5</td>
</tr>
<tr>
<td>8</td>
<td>63, 62, 64, 53, 55, 57</td>
<td>37.5</td>
</tr>
</tbody>
</table>
Results-16 Machine, 68-Bus System

**Designed LMI Control**

Closed-loop modal analysis
Large Test System

Reduced order WECC Model

- **System Details:**
  - Two-axis model of synchronous machines with type ST1 excitation control, **PSS** models, and turbine-governor models
  - **Two wind farms** represented using Type 3 WTG generator and electric control modules
  - **HVDC lines** representing the multi-terminal Pacific DC Intertie (PDCI) and the two-terminal Intermountain Power Project (IPP); SDC is added to the multi-terminal DC line
  - One **SVC** is also present in the system
Reduced Order WECC System

Different Types of Controllers

(a) PSS

\[ \frac{sT_w}{1 + sT_w} \rightarrow \frac{1 + sT_{1PSS}}{1 + sT_{2PSS}} \rightarrow \frac{1 + sT_{3PSS}}{1 + sT_{4PSS}} \]

(b) DC-SDC

\[ \frac{sT_w}{1 + sT_w} \rightarrow \frac{1 + sT_{1DC}}{1 + sT_{2DC}} \rightarrow \frac{1 + sT_{3DC}}{1 + sT_{4DC}} \]

(c) SVC

\[ V_{bus} \rightarrow 1 + sT_{c} \rightarrow \frac{1 + sT_{s2}}{1 + sT_{s3}} \rightarrow \frac{1 + sT_{s4}}{1 + sT_{s5}} \rightarrow V_{FS} \rightarrow \frac{1}{1 + sT_{s6}} \rightarrow I_{MAX} \]

\[ \Sigma \rightarrow V_{ref} \rightarrow V_{SCS} + V_s \rightarrow V_{MIN} \rightarrow V_{MIN} \rightarrow \frac{K_{c}V_{2MIN}}{1 + sT_{s2}} \rightarrow I_{MIN} \rightarrow I_{MAX} \]
Test Cases - Reduced Order WECC System

Focus: To create change in power transfers in and around WECC

*Intertie Path #26*

<table>
<thead>
<tr>
<th>Number</th>
<th>Generation</th>
<th>Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>No Change</td>
<td>No Change</td>
</tr>
<tr>
<td>2</td>
<td>#91-Increase by 250 MW</td>
<td>#71-Increase by 250 MW</td>
</tr>
<tr>
<td>3</td>
<td>#91-Decrease by 250 MW</td>
<td>#71-Decrease by 500 MW</td>
</tr>
<tr>
<td>4</td>
<td>#8881-Increase by 250 MW</td>
<td>#69-Increase by 250 MW</td>
</tr>
<tr>
<td>5</td>
<td>#8881-Decrease by 500 MW</td>
<td>#69-Decrease by 500 MW</td>
</tr>
<tr>
<td>6</td>
<td>#8881-Increase by 250 MW</td>
<td>#78-Increase by 400 MW</td>
</tr>
<tr>
<td></td>
<td>#91-Increase by 250 MW</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>#8881-Decrease by 250 MW</td>
<td>#78-Decrease by 500 MW</td>
</tr>
<tr>
<td></td>
<td>#91-Decrease by 250 MW</td>
<td></td>
</tr>
</tbody>
</table>
# Results - Open-loop Eigenvalue Analysis

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Damping of critical modes(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mode 1</td>
</tr>
<tr>
<td>v₁</td>
<td>3.71% @1.737Hz</td>
</tr>
<tr>
<td>v₂</td>
<td>0.27% @0.397Hz</td>
</tr>
<tr>
<td>v₃</td>
<td>3.74% @1.736Hz</td>
</tr>
<tr>
<td>v₄</td>
<td>3.63% @1.742Hz</td>
</tr>
<tr>
<td>v₅</td>
<td>3.71% @1.737Hz</td>
</tr>
<tr>
<td>v₆</td>
<td><strong>-0.36% @0.399Hz</strong></td>
</tr>
<tr>
<td>v₇</td>
<td>3.23% @0.430Hz</td>
</tr>
</tbody>
</table>

*Open-loop eigenvalues*
## Results - Controller Interactions

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Critical Mode from Table I</th>
<th>Damping(%)</th>
<th>PSS + SVC</th>
<th>DC-SDC + PSS + SVC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_1$</td>
<td>Model1</td>
<td>3.51</td>
<td>3.71</td>
<td></td>
</tr>
<tr>
<td>$v_2$</td>
<td>Model1</td>
<td>3.47</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>$v_3$</td>
<td>Model1</td>
<td>3.54</td>
<td>3.74</td>
<td></td>
</tr>
<tr>
<td>$v_4$</td>
<td>Model1</td>
<td>3.42</td>
<td>3.63</td>
<td></td>
</tr>
<tr>
<td>$v_5$</td>
<td>Model1</td>
<td>3.51</td>
<td>3.71</td>
<td></td>
</tr>
<tr>
<td>$v_6$</td>
<td>Model1</td>
<td>3.46</td>
<td>-0.36</td>
<td></td>
</tr>
<tr>
<td>$v_7$</td>
<td>Model1</td>
<td>3.49</td>
<td>3.23</td>
<td></td>
</tr>
</tbody>
</table>
Results-System Reduction

Enhanced SMA Results using Primary Control Set

SMA convergence for Base Case
Results-System Reduction

Enhanced SMA Results using Primary Control Set

Black Dots: Eigenvalues of $A$
Green Circles: Eigenvalues of $A_1 + M_{100}$

SMA convergence for Case 2
Results-Modal Analysis

**Designed LMI Control using Primary Control Set**

Closed-loop modal analysis

5.5% damping
Small Disturbance-Time-Domain Analysis

Disturbance-Simultaneous increase of load at bus #78 by 200 MW and a decrease in load at bus #71 by 400 MW at time, \( t = 0.5 \text{ sec} \)
Small Disturbance-Controller Outputs*

* The rating of the SVC is 100 MVAR. The modulation limit for DC-SDC is set to ±125 MW.
Results-Alternate Signal Control Set

*Designed LMI Control using an Alternate Control Set*

Closed-loop modal analysis
Small Disturbance-Alternate Signal Control Set
Small Disturbance-Incorporation of Delays

Random delay between 25 ms-100 ms added to control signals
Extensions to Multi-Polytopic Design

- Deviation of an operating point post-contingency is reflected in the measurements.
- Trajectory that the current point follows to reach its destination is a measure of the domain-of-attraction of the different equilibrium points.
- By employing this trajectory as a guideline, the most suitable equilibrium point for different disturbances can be identified.
Multi-Polytopic Design using CART [21]

- CART used to identify the polytope inside which the operating point lay

Test cases for 16 machine, 68-bus system

<table>
<thead>
<tr>
<th>Number</th>
<th>Contingency Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Base case</td>
</tr>
<tr>
<td>2</td>
<td>One of double circuit line 1-2 is outage</td>
</tr>
<tr>
<td>3</td>
<td>One of double circuit line 8-9 is outage</td>
</tr>
<tr>
<td>4</td>
<td>Load of group 4 increased by 20%</td>
</tr>
<tr>
<td>5</td>
<td>Load of group 5 increased by 10%</td>
</tr>
<tr>
<td>6</td>
<td>Generation of group 4 increased by 20%</td>
</tr>
<tr>
<td>7</td>
<td>Generation of group 5 increased by 10%</td>
</tr>
<tr>
<td>8</td>
<td>10% constant power + 10% constant current + 80% constant impedance</td>
</tr>
<tr>
<td>9</td>
<td>20% constant power + 40% constant current + 40% constant impedance</td>
</tr>
<tr>
<td>10</td>
<td>Load of group 4 decreased by 20%</td>
</tr>
<tr>
<td>11</td>
<td>Generation of group 4 decreased by 10%</td>
</tr>
<tr>
<td>12</td>
<td>Generation of group 5 decreased by 10%</td>
</tr>
<tr>
<td>13</td>
<td>Load of group 5 decreased by 10%</td>
</tr>
</tbody>
</table>
Results-16 Machine, 68-Bus System [21]

Dynamic response of closed-loop system for different polytopic controllers

Output of CART

<table>
<thead>
<tr>
<th>Samples</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pol. No.</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Samples</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>Pol. No.</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Samples</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
<td>29</td>
<td>30</td>
</tr>
<tr>
<td>Pol. No.</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
Test Cases

<table>
<thead>
<tr>
<th>Vertex No.</th>
<th>Parameters</th>
<th>P1 (Load 1-10)</th>
<th>P2 (Outages of line 118-124)</th>
<th>P3 (Generation 1-3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>v1</td>
<td>100%</td>
<td>No outages</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td>v2</td>
<td>95%</td>
<td>One (out of the four) line is out</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td>v3</td>
<td>105%</td>
<td>Three (out of the four) lines are out</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td>v4</td>
<td>100%</td>
<td>One (out of the four) line is out</td>
<td>95%</td>
<td></td>
</tr>
<tr>
<td>v5</td>
<td>100%</td>
<td>One (out of the four) line is out</td>
<td>105%</td>
<td></td>
</tr>
</tbody>
</table>

Dynamic responses of closed-loop system for unknown scenario with no control, fixed control, and adaptive control

Polytopes in parameter space for the Reduced Order WECC System
Conclusions

• Designed coordinated controls for mitigating inter-area oscillations using an enhanced SMA and LMI-based polytopic design
• Coordinated operations of different controllers already present in the system
• Differentiated non-critical set of generators from the critical set, and created a partial state feedback for the complete system, with the aid of only the critical set
• Identified alternate feedback sources in case of loss of primary feedback signals
• Effectively damped critical oscillatory modes under changing operating conditions
Ongoing Work

• Solve the control problem using a machine learning (ML)-based polytopic control design:
  • Take advantage of large amounts of historical data being collected
  • Combine LMI-based control with load and renewable generation forecasts to better account for systemic uncertainty

• An ML-based approach would:
  • **Relieve** the computational burden of LMIs
  • Offer **scalable** performance
References


Thank You!

Questions??

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