THE IMPACT OF GENERATION MIX ON PLACEMENT OF STATIC VAR COMPENSATORS

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Abstract: This paper looks at the problem of placing Static Var Compensators to provide maximum transfer capability for all possible generation mixes. The margin to low voltage limit is one of the quantities used to determine power system transfer capability. A fast method for finding the location of SVC systems that will have the greatest impact on the low voltage margin will be shown. The IEEE 24 bus system will be used to demonstrate this method over a wide range of generation patterns.

Keywords: power system control, power system reliability, power system security, power transmission reliability, power semiconductor devices, static var compensator, SVC.

I. INTRODUCTION

Electrical power systems are going through their first major change in over a hundred years. This change is brought about by the combined forces of new technologies and the restructurining of electrical utilities. These technologies (power electronics and information systems) can provide the means for the transmission system to be both flexible and competitive. Due to the marketplace forces the generation mix could change day to day. The transmission planner (and operator) will be faced with highly unpredictable generation mixes requiring new technologies to insure maximum availability and reliability. One important group of technologies is FACT devices. This paper will focus on the placement of Static Var Compensators (SVC) [1] to increase power transfer capability.

SVC uses power electronics to control its reactive power output to regulate bus voltage. Compared to mechanically switched capacitor banks, SVC reacts very fast and has high reliability. The basic applications are power oscillation damping, load compensation, bus voltage support, reduction of the total system cost, and increasing power transfer capability. There are many different factors to consider when locating an SVC. The list includes: total cost, impact on the system, and harmonic generation.

Most work in voltage support and transfer capability has been done in reactive power planning [4,5]. Current work on SVC placement [6-10] either focuses on load margin to point of collapse (PoC) boundary or oscillation damping as a measure of effectiveness. This work proposes a fast estimation method to judge SVC location based on the available transfer capability (ATC) to low voltage boundary (LVB) [3]. The main feature of this method is that it only needs a solution close to LVB to estimate the results. Without further power flow calculation and system Jacobian inversion, the method is very fast in computation, making it effective to consider many different generation mixes as well as SVC location.

II. FORMULATION OF THE POWER FLOW EQUATIONS

To consider all possible SVC locations in a power system is a formidable task. Traditionally a SVC location is chosen and the power flow equations are solved. The load is increased until a limit boundary is reached. Then, another bus is chosen for the SVC and the process is repeated. The SVC location that enables the maximum power transfer is usually selected. The inclusion of different generation and load mixes makes the problem more formidable.

To achieve this objective, the power flow equations need a particular structure. In this structure each load bus includes a shunt capacitance as a controllable parameter. The generation and load mix are defined by a unit direction vector. This allows for changes in generation and load without changing the mix. Power balance is achieved through a load scaling factor.

The generation mix is defined by a unit direction vector, \( \mathbf{P}_g \):

\[
\mathbf{P}_{\text{gen}} = \frac{1}{\sum P_o} \begin{bmatrix} P_{o1} \\ \vdots \\ P_{on} \end{bmatrix}
\]  

Total generation is defined as \( gP_{\text{gen}} \) where \( g \) is a scalar. The load is also defined by a unit direction vector:

\[
\mathbf{S}_{\text{load}} = \frac{1}{\sum \text{Real}(S_i)} \begin{bmatrix} S_{l1} \\ \vdots \\ S_{ln} \end{bmatrix}
\]

The total load is defined as \( lS_{\text{load}} \), where \( l \) is a scalar and the “slack” parameter.
The general power flow equation for an n bus system can be described in complex form as:

\[ S(\lambda, \mu) = V* = V(VV)^* \]  \hspace{1cm} (3)
\[ S(\lambda, \mu) = gP_{gen} + lS_{load} = 0 \] \hspace{1cm} (4)

where:
- \( \lambda \) system state variables containing bus voltage magnitudes \( |V| \) and angles \( \theta \)
- \( \mu \) controllable system parameter vector. (In this case \( \mu \) is a controllable shunt capacitance or SVC. The equivalent capacitance \( \omega C \), is included in \( Y \) matrix)
- \( P_{gen} \) unit generation direction vector
- \( g \) generation parameter
- \( S_{load} \) unit load direction vector
- \( l \) slack variable needed for power balance.

**III. SENSITIVITY RELATIONSHIPS**

Basic sensitivity concepts can be illustrated in Fig. 1. This figure plots the voltage on a bus \( i \) against the power transfer of the system (also called PV curve). In this example it is assumed that a solution is known for voltage at \( g_0 \), as indicated by the point A. Using this solution, linear techniques provide an estimate of the additional power transfer \( \Delta g_i \). This is defined as the point where the bus voltage equals its low voltage limit at B (at LVB). In this case \( \Delta g_i \) is:

\[ \Delta g_i = \frac{V_{LVB} - V_i}{dV/dg} \] \hspace{1cm} (5)

A more systematic approach starts with representing the power flow equations (3) and (4) as a set of algebraic equations:

\[ f(x, \mu, g) = 0 \] \hspace{1cm} (6)

The vector \( x \) denotes the states or dependent variables of the system. It contains the slack variable \( l \) and bus voltage magnitudes and angles. The vector \( \mu \) denotes the controllable parameters, in this case a shunt capacitance at each PQ load bus. The scalar \( g \) denotes the total generation.

If it is assumed that \( x_0, \mu_0, g_0 \) are solutions to the power load flow without SVCs, then:

\[ f(x_0, \mu_0, g_0) = 0 \] \hspace{1cm} (7)

where \( \mu_0 = 0 \). It is now possible to determine changes in the systems states, \( x \), due to small changes in \( \mu \) and \( g \). Expanding about the base solution using Taylor-series it follows from (7):

\[ f_x \Delta x + f_\mu \Delta \mu + f_g \Delta g = 0 \]

or

\[ \Delta x = -f_x^{-1} f_\mu \Delta \mu - f_x^{-1} f_g \Delta g \] \hspace{1cm} (8)

where the Jacobian matrices \( f_x \), \( f_\mu \), and \( f_g \) are evaluated at \( x_0, \mu_0, g_0 \). The inverse of \( f_x \) is the same Jacobian inverse used in the Newton’s method and needs to be calculated once for each generation direction. For convenience of notation, equation (8) can be rewritten in terms of sensitivity matrices \( M \) and \( G \) as:

\[ \Delta x = M \Delta \mu + G \Delta g \] \hspace{1cm} (9)

with

\[ M = -f_x^{-1} f_\mu \] \hspace{1cm} (10)
\[ G = -f_x^{-1} f_g \] \hspace{1cm} (11)

**IV. ESTIMATING AVAILABLE TRANSFER CAPABILITY (ATC) TO THE LOW VOLTAGE BOUNDARY (LVB) AFTER A SVC IS INSTALLED**

Starting from the initial load flow solution at \( x_0, \mu_0, g_0 \), sensitivity relationships discussed above can be used to estimate the increase in generation margin when a single SVC is added to the system. To approximate the new ATC to LVB, \( \Delta g \), estimates of the change in bus voltage \( V_i \) and the tangent \([11,12]\) \( dV_i/dg \) need to be calculated, (see Fig. 1.).

Using (9) the change in the voltage at the \( i^{th} \) and \( k^{th} \) buses due to controlling voltage at the \( k^{th} \) bus using the parameter \( \mu_k \) are:

\[ \Delta V_i = M_{ik} \Delta \mu_k \] \hspace{1cm} (12)
\[ \Delta V_k = M_{kk} \Delta \mu_k \] \hspace{1cm} (13)

where generation is fixed at \( g_0 \). The components \( M_{ik} \) and \( M_{kk} \) of the sensitivity matrix \( M \) are the terms that couple the voltages on bus \( i \) and \( k \) to the control parameter \( \mu_k \). From these relationships the sensitivity of the \( i^{th} \) bus voltage to change in the \( k^{th} \) bus voltage can be expressed as:

\[ \Delta V_i = M_{ik} (M_{kk})^{-1} \Delta V_k \] \hspace{1cm} (14)
V. SVC RATINGS

In the last section methods were presented to estimate the increase in power transfer using SVC(s). In these approximations it was assumed that the SVC(s) could hold the bus voltage to 1 pu at the maximum transfer level. This assumption implies a minimum SVC rating. In some case this value will be too large to be a practical alternative.

Sensitivity methods can again be used to approximate the size of the SVC(s). From (9) the sensitivity of the kth bus voltage to changes in g and m can be expressed as:

$$\Delta V_k = M_{ik} \Delta \mu_k + G_k \Delta g$$

(24)

The rated pu (per unit) reactive power required to hold bus k at 1.0 pu voltage can be approximated from (24):

$$\Delta \mu_k \approx M_{ik} \left[ \Delta V_k - G_k \Delta g \right]$$

(25)

where the change in voltage is:

$$\Delta V_k = 1.0 - V_k$$

(26)

and $\Delta g = \Delta g_k$ as evaluated in (20). Similar relationships can be derived for two or more SVCs.

VI. TEST RESULTS

A. Results consider only one direction

A modified IEEE 24 bus system [13] as shown in Fig. 2 is
used to test the estimation method against traditional results. The modified 24 bus system has 10 generators with the synchronous var generator (bus 14) removed. The 24 bus system uses 100MVA base. The normal loading totals 28.5 per unit. As load increases, buses 3, 4, 5, 6, 8, 9, 10 and 24 will have large voltage drops due to some generators reaching their MVAR limits.

Fixed generation and load directions are selected, as shown in (1) and (2). The output ratio between generators is fixed as generation increases. Every bus load ratio is also fixed as total load increases. Starting from zero generation and loading, continuation power flow method [2] is used to increase generation until LVB is reached. Here 0.9 pu is used as the low voltage limit for all buses. At LVB, the lowest bus voltage is 0.9 pu.

TABLE 1  
KEY LOW VOLTAGE LOAD BUSSES AT LVB

<table>
<thead>
<tr>
<th>V(3) (pu)</th>
<th>V(4) (pu)</th>
<th>V(5) (pu)</th>
<th>V(6) (pu)</th>
<th>V(8) (pu)</th>
<th>V(9) (pu)</th>
<th>V(10) (pu)</th>
<th>V(24) (pu)</th>
<th>Generator at MVAR limit</th>
<th>Total generation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.900</td>
<td>0.922</td>
<td>0.947</td>
<td>0.918</td>
<td>0.918</td>
<td>0.944</td>
<td>0.938</td>
<td>0.916</td>
<td>15.16</td>
<td>3977MW</td>
</tr>
</tbody>
</table>

Along a selected direction, at LVB, the ATC for generators are 398MW (bus 1), 398(2), 531(7), 608(13), 183(15), 109(16), 497(18), 497(21), 331(22) and 426(23) respectively, totaling 3977 MW, with generators 15 and 16 at MVAR limit. Key low voltage buses are shown in Table 1. Bus 3 reached its low voltage limit first.

At this calculated ATC to LVB point, the estimation method is performed first for single SVC placement, using (19), (20) and (25). To compare, traditional method is also used to obtain the exact solution, using continuation power flow and holding the SVC bus at 1 per unit. Results are shown in Table 2. Column 2 and 3 show the percentage transfer increases over the original level of 3977MW, after a load bus is hold at 1.0 per unit by SVC. Column 4 and 5 show the rating of SVC that is required. Result show that a SVC at bus 9 can increase power transfer by as much as 29% (796MW), with SVC rating of 351MVAr. Note that the bus that reaches its low voltage limit (bus 3) may not be the best place to install a SVC (bus 9).

At the same original ATC to LVB point as in Table 1, two SVC placement was also estimated as in (21-23). All possible pairs of load buses are estimated with SVCs holding two load bus voltages at 1.0 per unit. Three best pairs are shown in Table 3. Of all the combinations, the optimal is bus 6 and bus 9. They together will increase ATC to LVB by over 37% (1471MW), with total SVC rating of 5.37 pu (537MVAr).

B. Results consider the generation space

So far, only one direction of generation is considered. Due to the fast speed nature of the estimation method, full generation space can be estimated as well, assuming the load direction is still fixed as before. To visualize the results, generators are divided into three groups. Each generator inside the group has fixed ratio of output to each other. The outputs between groups, however, are selected at all possible combinations. Therefore the direction of three groups will scan a 3-D space. The points of ATC to LVB of all directions will form a surface in 3-D space, as shown in Fig. 4-5. We call it the original ATC to LVB surface since no SVC is added. The three groups are buses (1,2,7), (13,15,16,23) and (18,21,22). Group (13,15,16,23) reaches MW generation limit in some directions. The generation direction is chosen as:

$$P_{gen} = a \times (0.3_{1st} + 0.3_2 + 0.4_1) + b \times (0.46_{1st} + 0.14_{1st} + 0.08_{1st} + 0.32_7) + c \times (0.38_{18} + 0.38_{21} + 0.25_{22})$$

where \(a, b, c\) are parameters that scan the whole 3-D space.

When \(a = b = c = \frac{1}{3}\), the generation direction is the same as the first example in Table 1-3. To better illustrate the concept, Fig. 3 draws the first example in 3-D space. In Fig. 3, the partial surface is the original ATC to LVB surface. The one direction of the first example forms a single point on the surface. The line pointing out of the surface shows the surface change due to a SVC at bus 3. Shown also are the
Table 4

FOR 10 EVENLY SELECTED GENERATION COMBINATION DIRECTIONS, THE LIST OF SOME LOW VOLTAGE BUSES AND THEIR VOLTAGES AT LVB

<table>
<thead>
<tr>
<th>Load bus voltages (per unit)</th>
<th>Total generation (100 MW)</th>
<th>Generators at MVAR limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>V(3) V(4) V(5) V(6) V(8) V(9) V(10) V(24)</td>
<td>21.15</td>
<td>16*</td>
</tr>
<tr>
<td>0.960 0.960 0.983 0.960 0.959 0.958 0.988 0.978</td>
<td>31.72</td>
<td>16.15*</td>
</tr>
<tr>
<td>0.916 0.941 0.967 0.950 0.938 0.936 0.967 0.964</td>
<td>20.52</td>
<td>16.15</td>
</tr>
<tr>
<td>0.900 0.926 0.939 0.909 0.915 0.940 0.960 0.963</td>
<td>13.10</td>
<td>none</td>
</tr>
<tr>
<td>0.903 0.924 0.951 0.939 0.900 0.900 0.937 0.964</td>
<td>20.98</td>
<td>16.15</td>
</tr>
<tr>
<td>0.916 0.923 0.948 0.952 0.930 0.910 0.940 0.969</td>
<td>39.77</td>
<td>16.15</td>
</tr>
<tr>
<td>0.900 0.922 0.947 0.918 0.913 0.918 0.944 0.937</td>
<td>26.20</td>
<td>16.15</td>
</tr>
<tr>
<td>0.900 0.903 0.928 0.919 0.915 0.908 0.933 0.927</td>
<td>24.18</td>
<td>16.15</td>
</tr>
<tr>
<td>0.900 0.935 0.964 0.958 0.941 0.928 0.968 0.902</td>
<td>20.75</td>
<td>16.15</td>
</tr>
<tr>
<td>0.900 0.934 0.961 0.944 0.936 0.925 0.960 0.900</td>
<td>30.75</td>
<td>16.15</td>
</tr>
<tr>
<td>0.916 0.929 0.990 0.991 0.965 0.949 0.990 0.900</td>
<td>18.95</td>
<td>16.15</td>
</tr>
<tr>
<td>(* Generators reached power limit before LVB is reached)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5

AVERAGE RESULTS OF SINGLE SVC TO INCREASE ATC TO LVB OVER MANY GENERATION DIRECTIONS

<table>
<thead>
<tr>
<th>SVC bus</th>
<th>Average on 10 directions of 10 generators in 3 groups</th>
<th>Average on 220 directions of 10 generators in 10 groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average ATC to LVB increase (100MW)</td>
<td>Average SVC rating (100MVAR)</td>
<td>Average ATC to LVB increase (100MW)</td>
</tr>
<tr>
<td>3</td>
<td>2.33 2.09 1.81 1.59</td>
<td>1.34 1.34 1.34 1.34</td>
</tr>
<tr>
<td>4</td>
<td>1.09 1.15 1.45 1.14</td>
<td>1.34 1.34 1.34 1.34</td>
</tr>
<tr>
<td>5</td>
<td>0.33 0.86 1.21 0.87</td>
<td>1.34 1.34 1.34 1.34</td>
</tr>
<tr>
<td>6</td>
<td>0.67 0.91 1.02 0.82</td>
<td>1.34 1.34 1.34 1.34</td>
</tr>
<tr>
<td>7</td>
<td>1.52 2.13 3.44 2.85</td>
<td>1.34 1.34 1.34 1.34</td>
</tr>
<tr>
<td>8</td>
<td>3.43 3.19 2.44 2.60</td>
<td>1.34 1.34 1.34 1.34</td>
</tr>
<tr>
<td>9</td>
<td>1.26 1.59 1.38 1.34</td>
<td>1.34 1.34 1.34 1.34</td>
</tr>
<tr>
<td>10</td>
<td>1.59 2.41 1.11 1.81</td>
<td>1.34 1.34 1.34 1.34</td>
</tr>
<tr>
<td>11</td>
<td>1.00 1.61 0.77 1.30</td>
<td>1.34 1.34 1.34 1.34</td>
</tr>
<tr>
<td>12</td>
<td>1.40 2.13 0.81 1.49</td>
<td>1.34 1.34 1.34 1.34</td>
</tr>
<tr>
<td>13</td>
<td>0.56 1.04 0.36 0.76</td>
<td>1.34 1.34 1.34 1.34</td>
</tr>
<tr>
<td>14</td>
<td>0.66 1.62 0.34 1.09</td>
<td>1.34 1.34 1.34 1.34</td>
</tr>
<tr>
<td>15</td>
<td>0.19 1.28 0.10 0.90</td>
<td>1.34 1.34 1.34 1.34</td>
</tr>
<tr>
<td>24</td>
<td>2.16 1.85 1.21 1.34</td>
<td>1.34 1.34 1.34 1.34</td>
</tr>
</tbody>
</table>

The power transfer is more than bus 9 for some generation directions. Generation direction is also an important factor for choosing SVC locations.

To determine the best place for single SVC placement over all directions of the three groups of generators, average of the results is listed in Table 5 for the above evenly chosen 10 directions. Results show that bus 9 is the best place to place a SVC to increase ATC to LVB, followed by bus 3 and 24.

When the fixed relations among generators in each group are relaxed, the full generator space is 10-dimensional. In the 10-D space, averages of results are calculated over 220 evenly selected directions as listed in Table 5. It shows the best location is bus 8 followed by bus 9 and 3.

Since the selection of direction set will affect the results very much, system planner should determine the set of future generation directions before selection a best place for SVC placement.
The fast estimation makes it easy to consider SVC locations for a wide range of generation mixes.

Also rating is an issue. An optimization method could be used with this estimation to reduce the cost. The cost function could include rating, SVC bus voltage, required margin and generation and load directions.

VIII. REFERENCES


IX. BIOGRAPHIES

Robert H. Lasseter (F'92) received a Ph.D. in physics from the University of Pennsylvania, Philadelphia, in 1971. He was a Consultant Engineer with the General Electric Company until 1980 when he joined the University of Wisconsin-Madison. His main interest is the application of power electronics to utility systems including hardware, methods of analysis and simulation.

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