Line Limit Preserving Power System Equivalent

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Abstract—A reduced order equivalent of a power system is a simplified model of the original system with the ability to preserve some key characteristics and to provide adequate fidelity in simulation studies with considerably reduced computational requirements. In this paper, an algorithm to construct an equivalent system with the ability to retain thermal line limits is presented. The application of Kron’s reduction and power transfer distribution factor is used in the creation of the reduced system. The criteria for this method and its step-by-step procedure with the implementation to a small power system are described. Also, the result of its application to the IEEE 118 bus system is provided.

Index Terms—Line limit, Kron’s reduction, power transfer distribution factor (PTDF), power system equivalent

I. INTRODUCTION

An equivalent power system is a model system with a reduced number of nodes and branches than the corresponding full system. The purpose of making an equivalent system is basically to have higher efficiency in computer simulation without excessively sacrificing the fidelity of the simulation results of the original system. Traditional methods such as Ward equivalent have been used to build power system equivalents [1]-[6]. The Ward equivalent fully retains the internal system which is in proximity, but equivalences the external system which is in distant. Thus, this does not work well in terms of retaining desired characteristics across the entire system after significant reduction of elements from the original system. Which characteristics of the original system are preserved in the reduced model depends on the kind of analysis being performed. This paper focuses on creation of an equivalent system that preserve the thermal line limits of the original system. To have more accurate attributes of the original system, the equivalent system has internal system that is spread throughout the system [7], [8]. Therefore, the limits of the reduced portion of the system become important. This is contrast with the Ward equivalent in which the equivalent is completely external area to the retained system.

Line limits are a fundamental part of the formulation for Power Flow problems, Optimal Power Flow (OPF) problems and Security-Constrained Optimal Power Flow (SCOPF) problems for electric power systems. While solving such problems for large-scale networks, it is common practice to use equivalent networks derived from detailed models so as to reduce the computational burden. Kron’s reduction is a widely used method for power system network reduction, which is based on successive node (or bus) elimination [9]. Although Kron’s reduction is capable of calculating the line admittance values for equivalent lines, the information about line limits is lost in the equivalent network (with fewer nodes).

It is inevitable to consider power injections at buses when determine line limits in power systems since bus injections from generation or loads might have impact on them. Without considering bus injections, total transfer capability (TTC) is direction independent. With bus injections, however, TTC is reduced to available transfer capability (ATC) to meet existing transmission commitments and becomes direction dependent. This can be illustrated in Fig. 1 in which the bus 2 is to be equivalenced. When bus 2 has no bus injection and TTC can be easily obtained by choosing the lower value of two transmission line limits for both directions between buses 1 and 3. This lower line limit would be assigned as the equivalent line limit when bus 2 is equivalenced. In this case, ATC is identical to the TTC. When bus 2 has the bus injection of 50 MW load, the ATC becomes 90-50=40 MW in the right to left direction and 100-50=50 MW in the opposite direction.

Even though bus injections play a role in finding line limits in the reduced system, it is not easy to solve the complicated problem. It is known that for large systems it would not be possible to assign limits to equivalent lines that exactly match those of the original system for all operating points. Therefore,
this paper focuses on calculating equivalent line limits without considering bus injections by unloading systems as a precursor of the comprehensive study in the future.

The Limit Preserving Equivalent (LPE) algorithm, presented in this paper, is based on Power Transfer Distribution Factor (PTDF) and capable of translating limits of physical lines in a full-scale model to limits of equivalent lines in reduced-node model. This will make equivalent networks an attractive option for carrying out studies about the full-scale network, such as system capacity assessments, total transfer capability calculations, operational reliability (security) analyses [10], economic analyses [11], and load-pocket identification [12], [13].

This paper is organized as follows. Section II discusses features of network reduction and criteria of the proposed algorithm. Special cases of obtaining line limits are introduced in Section III. Section IV explains the proposed LPE algorithm in detail with a small system example. In Section V, application of the algorithm to a network example is presented. Finally, computational aspects of the algorithm and conclusion are drawn in Section VI and VII, respectively.

II. PROBLEM STATEMENT

A. Network Reduction

Consider a power system with \( N \) buses indexed by \( \{1, \ldots, N\} \). Let \( \mathcal{E} \) denote the set of all lines and \( \mathbf{Y} \) its \( N \times N \) bus admittance matrix. To eliminate a given subset of buses, Kron’s reduction has been adopted in this paper to build the equivalent bus admittance matrix for the reduced power system. Although a closed-form solution is available through matrix computation, for efficient computations, practical implementations of partial factorization of \( \mathbf{Y} \) proceed by sequentially eliminating one bus at a time. Specifically, to eliminate a given bus \( k \), the admittance between any two buses \( i \) and \( j \) can be updated as

\[
Y'_{ij} = Y_{ij} - \frac{Y_{ik} Y_{kj}}{Y_{kk}} = Y_{ij} + \tilde{Y}_{ij}
\]

For the original system, the bus admittance \( Y_{ij} \) takes value 0 if there is no direct line between \( i \) and \( j \). Otherwise, it is related to the corresponding line admittance quantity and is prescribed with a given line flow limit denoted by \( F_{ij} \). The essential goal of the present paper is to update the line limit \( F'_{ij} \) for the reduced system.

Even though the update in (1) holds generally for any two chosen buses \( i \) and \( j \), the bus admittance only changes if and only if buses \( i \) and \( j \) are both neighbors of bus \( k \), i.e., \((i,k), (j,k) \in \mathcal{E}\) and \((j,k) \in \mathcal{E}\). Clearly, if either \( Y_{ik} \) or \( Y_{kj} \) is 0, then the fractional term \( \tilde{Y}_{ij} \) in (1) becomes also 0, in which case the corresponding line limit stays unchanged as \( F_{ij} = F'_{ij} \). Otherwise, if \( Y_{ij} \) has indeed changed as given by (1), it is equivalent to adding another parallel line to connect buses \( i \) and \( j \), as illustrated in Fig. 2.

However, this equivalent line has no associated line limit, which is exactly the challenge of our proposed line limit persevering equivalent approach. Hence, the proposed LPE algorithm intends to provide the line limit for any pair of neighboring buses of the eliminated bus.

B. Criteria for Line Limit Preservation

To tackle this, it is desirable to match the total transfer capability (TTC) between the first neighbor buses in the reduced system and those in the full system. Here, it should be noted that only the eliminated lines are considered in calculating TTC of the original system. The TTC is defined as the amount of power that can be transferred over the transmission network in a reliable manner while meeting all of a specific set of defined pre- and post-contingency system conditions [15]. In this paper, however, since only bus-to-bus transfers are concerned, contingencies are not considered in the calculation of the TTC. As will be detailed soon, the TTC attributes are closely related to the line limits of a power system, which speaks for their importance as the criterion of the LPE algorithm. Before that, notice that the TTC is independent of direction as bus injection is not considered in the paper. This may lead to different TTC values depending on the direction of power flows. However, such differences are assumed negligible in this paper.

To this end, suppose a bus \( k \) is to be eliminated, which converts the original \( N \)-bus system to the reduced (\( N-1 \))-bus one. As mentioned earlier, it is of interest to assign the equivalent line limits corresponding to \( k \)’s neighbor buses, as denoted by \( \mathbf{S} = \{s_i|l| i \in \{1, S\} \} \) of cardinality \( S \). By eliminating bus \( k \), all the \( S \) lines in \( \mathcal{L} = \{l_i|l \in \{1, S\} \} \) will also be eliminated, notice that this assumes possible parallel lines have been combined to a single line. Similarly, let \( \mathbf{H} = \{h_r|r \in \{1, H\} \} \) of cardinality \( H \) denote the set of lines in the original system directly connecting any two buses in \( \mathbf{S} \) with admittance \( Y_{ij} \) in (1). Also, let \( \mathcal{L} = \{l_i|l\in \{1, L\} \} \) the set of equivalent lines added after the elimination. Clearly, \( \mathcal{L} \) has \( L = (\frac{S}{2}) \) number of lines whose limits are to be determined. Finally, the set of power transaction bus-pairs is denoted as \( \mathbf{W} = \{w_p = \{s_i, s_j\}|l, j \in \{1, S\}, i \neq j \} \) of cardinality \( W = (\frac{S(S-1)}{2}) \), while each \( w_p \) is independent of direction.

To calculate the TTC, PTDF is introduced to linearly approximate the impact of power flowing on any line with respect to the power transfer of a transaction \( w_p \). Specifically, the PTDF of any line \( l_i \in \mathcal{L} \) for a transaction \( w_p \in \mathbf{W} \) is denoted by \( \varphi_{l_i}(w_p) \in (0,1) \). The power transfer for the
transaction \( w_p \) that obeys the limit of line \( l_i \) is upper-bounded by \( F_{l_i} / \phi_{l_i}(w_p) \). Hence, the TTC of transaction \( w_p \) before the reduction is determined by the minimum of those upper bounds, as follows:

\[
P^{(w_p)} = \min_{l_i \in E} \left\{ \frac{F_{l_i}}{\phi_{l_i}(w_p)} \right\}
\]

(2)

Similarly, once the post-reduction PTDF value \( \phi_{l_i}(w_p) \) is available, the corresponding TTC in the reduced system can be calculated as:

\[
\tilde{P}^{(w_p)} = \min_{l_i \in \tilde{E}} \left\{ \frac{\tilde{F}_{l_i}}{\phi_{l_i}(w_p)} \right\}
\]

(3)

where \( \tilde{F}_{l_i} \) is the limit for the equivalent lines which is the objective values to calculate in the paper. Even though line limits use MVA for its unit, MW is used for power transfers as well as TTCs since reactive power is operating point oriented, hence it is not very significant to consider. To preserve the line limits, the proposed LPE algorithm intends to match the TTC quantities as given by (2) and (3) for any transaction \( w_p \).

Interestingly, we only need to account for the eliminated lines in \( E \) and the newly added equivalent lines in \( \tilde{E} \) since the lines in \( \mathcal{H} \) keep their line parameters after equivalencing.

There are two common approaches to calculate PTDFs [15, 16]; the linearized ac method that includes loss in the calculation, and the lossless dc method that does not. In this paper, the lossless dc method is used to calculate PTDFs because of its simplicity and computational advantages. One of the characteristics of PTDF to note is that PTDFs on the retained lines in the reduced system are not affected by equivalencing. This is because PTDFs are based on the line parameters, and the ratio of parameters of a retained line to the rest of the system does not change by equivalencing according to the Kron’s reduction.

Fig. 3 shows a 4-bus system on the left and bus 1 is being eliminated resulting in the 3-bus system on the right. Lines have limits in MVA and reactances in p.u. The PTDFs are in % with bus 2 as a source and bus 3 a sink for both systems in Fig. 3. By removing bus 1, three equivalent lines are added between the first neighbor buses, but their line limits are unknown. As mentioned above, the PTDFs on the retained lines from the original system remains unchanged in the equivalent system. However, the limits for the newly created equivalent lines are yet to be determined.

TTCs on the lines being eliminated for the transaction between the first neighbor buses and bus 1 in the original system can be calculated with (2) and the results are shown in Table I. The TTCs between the same buses in the reduced system should match the values in Table I and they will be compared later in Section IV.

In each step of equivalencing, generation and load would be moved to the retained buses. However, relocating eliminated generation/load to neighbor buses in the process of equivalencing is not covered as the system is unloaded in this paper.

### III. Special Cases

This section will focus on how to combine series and parallel lines. These two cases not only give a simple illustration of the general algorithm, but also become useful for preprocessing of the algorithm.

#### A. Series Calculation

Consider a set of \( m \) lines in series \( \mathcal{L}_S = \{l_1, l_2, ..., l_m\} \) and each line has its own admittance and line limit as shown in Fig. 4. The series lines in \( \mathcal{L}_S \) are being equivalenced to one single line with the admittance \( y'_l \) and the new line limit \( \tilde{F}_l \).

![Fig. 4. Series line equivalencing](image)

The limit of the equivalent line for multiple series lines must be the minimum of the series line limits. This is because the equivalent line limit should represent the maximum power that can flow on the original series lines without violating the limits. Therefore, the new equivalent line limit for series lines can be obtained as follows:

\[
\tilde{F}_l = \min_{l_i \in \mathcal{L}_S} \{F_{l_i}\}
\]

(4)
B. Parallel Calculation

Consider a set of \( m \) lines in parallel \( L_{p} = \{ l_{1}, l_{2}, ..., l_{m} \} \) and each line has its own admittance and line limit as shown in Fig. 5. The parallel lines in \( L_{p} \) are being equivalenced to a single line with the admittance \( y_{i}' \) and the new line limit \( F_{i} \).

![Parallel line equivalencing](image)

The new limit of the combined parallel lines is calculated by determining which line in the parallel bundle is binding.

\[
\bar{F}_{i} = \min_{l \in L_{p}} \left\{ F_{i} \times \frac{y_{i}'}{y_{i}} \right\}
\]

(5)

The algorithm may create a lot of parallel equivalent lines as a consequence. The combination of parallel lines and the calculation of its parameter and limit is a crucial building block in the series of node eliminations. During any stage of elimination, this will allow one to consider parallel configurations of lines between nodes as one single line. Therefore, for illustrating the algorithm for a general step of single node elimination, it is sufficient to consider a single line between any two nodes. This can be thought of as a pre-processing procedure, and results in a neater algorithm.

IV. LINE LIMIT PRESERVING EQUIVALENT ALGORITHM

The goal to achieve in preserving line limits is to match the TTCs between any of first neighbor buses in the reduced system match the TTCs between the same buses in the original system. In the process, buses are eliminated one at a time for the computational advantage. The overview of the algorithm is described below. As each bus is being equivalenced,

1) Combine limits of parallel lines to be eliminated
2) Calculate PTDFs between the first neighbor buses of the bus that is being eliminated
3) Calculate TTCs between the first neighbor buses, only considering the lines that are being eliminated
4) Determine limits for equivalent lines so the TTCs in the reduced system match those in the original system
5) In case of non-exact limits for equivalent lines, determine lower and upper limits

Selection of a set of buses to be eliminated could vary according to the application of the algorithm. However, the order of bus elimination might have a great impact on the results of the algorithm since the computational expense depends on the number of first neighbor buses. The number of first neighbor bus lines can be greatly reduced by applying Tinney Scheme 2 [17]. This scheme calculates the number of first neighbor buses for all the candidates at each iteration and choose the next bus with the fewest first neighbors to be eliminated.

To explain the algorithm in detail, consider the \( N \)-bus system from Section II with the same notation. In the process of equivalencing bus \( k \), assume \( f \) lines are eliminated from the \( N \)-bus system and \( e \) equivalent lines are introduced in the reduced (\( N-1 \))-bus system. The objective is to find the limits for the \( e \) equivalent lines so that TTCs for every pair of the first neighbor buses in the equivalent system equal those in the original system, but only considering the eliminated \( f \) lines when calculating the TTCs of the original system.

First of all, combine parallel lines between buses in \( S \) to a single line with a combined line limit using (5). After that, calculate PTDFs between the first neighbor buses in \( S \) on the lines in \( L \). With the PTDFs and the given line limits of the original system, TTCs for each transaction in the original system can be obtained with (2). The TTC for each transaction \( w_{p} \) in the original system should be less than or equal to the power transfers on equivalent lines for the same transaction from the reduced system as shown in Table II. Note that there are \( e \) transactions since \( L = W = (\mathbb{J}) = e \), hence Table II is always square. By multiplying each PTDF term for both sides of the equation in each entry, inequality constraints for the limits of the equivalent lines can be obtained as a product of the TTC for a transaction from the original system and the PTDF for the same transaction from the reduced system as shown in Table III. Those entries give the minimum limit needed to allow for the original TTCs. All the entries in each row have to be smaller than or equal to the corresponding equivalent line limit.

One entry for each column has to have an equality constraint so that the TTC of that transaction can be determined. Also, one entry for each row has to have an equality constraint so that the limit of that equivalent line can be determined. Often, the solutions are trivial such that just choosing the largest value in each row could be the exact solutions. If each row and each column has one solution, then those set of solutions are the exact equivalent line limits since they satisfy all the inequality and equality constraints.

<table>
<thead>
<tr>
<th>Table II</th>
<th>INEQUALITY CONSTRAINTS FOR POWER TRANSFER CAPACITIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{F_{11}}{Q_{11}} \geq p(w_{1}) )</td>
<td>( \frac{F_{11}}{Q_{11}} \geq p(w_{2}) )</td>
</tr>
<tr>
<td>( \frac{F_{12}}{Q_{12}} \geq p(w_{1}) )</td>
<td>( \frac{F_{12}}{Q_{12}} \geq p(w_{2}) )</td>
</tr>
<tr>
<td>( \frac{F_{1e}}{Q_{1e}} \geq p(w_{1}) )</td>
<td>( \frac{F_{1e}}{Q_{1e}} \geq p(w_{2}) )</td>
</tr>
<tr>
<td>( \frac{F_{21}}{Q_{21}} \geq p(w_{1}) )</td>
<td>( \frac{F_{22}}{Q_{22}} \geq p(w_{2}) )</td>
</tr>
<tr>
<td>( \frac{F_{2e}}{Q_{2e}} \geq p(w_{1}) )</td>
<td>( \frac{F_{2e}}{Q_{2e}} \geq p(w_{2}) )</td>
</tr>
</tbody>
</table>

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TABLE III
INEQUALITY CONSTRAINTS FOR EQUIVALENT LINE LIMITS

<table>
<thead>
<tr>
<th>( \bar{F}_{i} \geq )</th>
<th>( w_1 )</th>
<th>( w_2 )</th>
<th>...</th>
<th>( w_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p^{(w_i)} \times \bar{q}_{i1}^{(w_i)} )</td>
<td>( p^{(w_2)} \times \bar{q}_{i1}^{(w_2)} )</td>
<td>...</td>
<td>( p^{(w_e)} \times \bar{q}_{i1}^{(w_e)} )</td>
<td></td>
</tr>
<tr>
<td>( p^{(w_1)} \times \bar{q}_{i2}^{(w_1)} )</td>
<td>( p^{(w_2)} \times \bar{q}_{i2}^{(w_2)} )</td>
<td>...</td>
<td>( p^{(w_e)} \times \bar{q}_{i2}^{(w_e)} )</td>
<td></td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>...</td>
<td>( \vdots )</td>
<td></td>
</tr>
<tr>
<td>( p^{(w_1)} \times \bar{q}_{ie}^{(w_1)} )</td>
<td>( p^{(w_2)} \times \bar{q}_{ie}^{(w_2)} )</td>
<td>...</td>
<td>( p^{(w_e)} \times \bar{q}_{ie}^{(w_e)} )</td>
<td></td>
</tr>
</tbody>
</table>

A. Example of Exact Solution Case

Consider the 4-bus system in Fig. 3 for example. The inequality constraints for equivalent line limits in the reduced 3-bus system are shown in Table IV. In this case, selecting the largest value in each row satisfies all of the inequality constraints for each line limit and also enforces all the transactions, hence producing the exact solutions. Fig. 6 shows the reduced 3-bus system with the new limits for the equivalent lines and the TTC of 217 MW for the transaction between bus 2 and 3. With the new limits, the TTCs between the first neighbor buses in the equivalent system can be calculated using (3) and they are identical with the original system as shown in Table V.

![Figure 6](image)

Fig. 6. Reduced 3-bus system with equivalent line limits for the transaction between bus 2 and 3

TABLE IV
INEQUALITY CONSTRAINTS FOR EQUIVALENT LINE LIMITS IN THE 4-BUS SYSTEM EXAMPLE

<table>
<thead>
<tr>
<th>( w_{23} )</th>
<th>( w_{24} )</th>
<th>( w_{34} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{F}_{i23} \geq )</td>
<td>( 50.8 )</td>
<td>( 4.8 )</td>
</tr>
<tr>
<td>( \bar{F}_{i24} \geq )</td>
<td>( 5.2 )</td>
<td>( 41.4 )</td>
</tr>
<tr>
<td>( \bar{F}_{i34} \geq )</td>
<td>( 19.2 )</td>
<td>( 18.6 )</td>
</tr>
</tbody>
</table>

B. Example of Non-Exact Solution Case

Exact solutions may not exist in some cases since it would not be always possible to assign limits to equivalent lines that fit all operating points. In this case, solutions can be bound with lower and upper limits. Consider that the limit on line 1-4 in the original 4-bus system is reduced to 20 MVA in order to have non-exact equivalent line limits. The reduced line limit results in reduced TTCs as shown in Table VI and inequality constraints for equivalent lines are calculated accordingly in Table VII.

![Table VI](image)

TABLE VI
TTCs OF 4-BUS SYSTEM WITH REduced LINE LIMIT

<table>
<thead>
<tr>
<th>( w_{p} )</th>
<th>Binding line</th>
<th>( P^{(w_p)} ) (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-3</td>
<td>1-3</td>
<td>217.0</td>
</tr>
<tr>
<td>2-4</td>
<td>1-4</td>
<td>57.2</td>
</tr>
<tr>
<td>3-4</td>
<td>1-4</td>
<td>48.3</td>
</tr>
</tbody>
</table>

![Table VII](image)

TABLE VII
INEQUALITY CONSTRAINTS FOR EQUIVALENT LINE LIMITS FOR NON-EXACT SOLUTION CASE

<table>
<thead>
<tr>
<th>( w_{23} )</th>
<th>( w_{24} )</th>
<th>( w_{34} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{F}_{i23} \geq )</td>
<td>( 50.8 )</td>
<td>( 1.6 )</td>
</tr>
<tr>
<td>( \bar{F}_{i24} \geq )</td>
<td>( 5.2 )</td>
<td>( 13.8 )</td>
</tr>
<tr>
<td>( \bar{F}_{i34} \geq )</td>
<td>( 19.1 )</td>
<td>( 6.2 )</td>
</tr>
</tbody>
</table>

In this case, taking the largest value in each row does not satisfy equality constraints for \( w_{34} \). The allowable power flow in transaction \( w_{34} \) is overestimated since none of the entries in its column is enforced. This sets upper bound for the equivalent line limits.

Furthermore, it is possible to have lower bound by insuring that the flow in every transaction is equal or less than the TTCs in the original system. However, some of the inequality constraints would be in violation, these limits underestimate the TTC in some transactions. In this paper, lower limits are motivated by defining a “limit violation cost” (LVC) for each entry in the matrix, which is the sum of violations for all entries in the row as follows:

\[
V_i^{(w_r)} = \sum_{w_r \neq w_p} \max_{w_r \neq w_p} \left( \bar{q}_i^{(w_r)} \times \bar{q}_i^{(w_r)} - \bar{q}_i^{(w_r)} \times \bar{q}_i^{(w_r)} \right)
\]

(6)

The LVC matrix for the inequality constraints can be obtained using (6) as shown in Table VIII. To illustrate this in detail, for the first row of the table, the entry for \( w_{23} \) is 0 because it involves no limit violations as it is the largest in the row. However, the entry for \( w_{24} \) is related with the other two transactions in the row, hence its LVC is calculated as \((50.7 - 1.6) + (9.9 - 1.6) = 57.4\) and the third entry becomes \(50.7 - 9.9 = 40.8\). Therefore, each entry is the sum of the differences between itself and larger entries in the row.
Now, this is a resource allocation problem that one entry from each row and each column has to be chosen so that it minimizes the sum of the limit violation costs. The Hungarian method (also known as Munkres assignment algorithm) is one of the algorithms that solve this kind of minimum matching problem [18]. According to the Hungarian method, if 0.0 were chosen for both the first and second row, and 9.6 instead of 0.0 in the third row, it would provide minimum sum of limit violation costs in Table VIII. For the lower bound solution, therefore, the new limits would be 50.7 MW for line 2-3, 13.8 MW for line 2-4 and 9.5 MW for line 3-4.

Table IX compares the TTCs on the eliminated lines in the original system and those on the equivalent lines in the equivalent system. When the largest value in each row is chosen for the equivalent lines as the upper limits, the TTC in transaction $w_{24}$ is overestimated about 123%. On the contrary, when the power flow for transaction $w_{24}$ is enforced by applying the Hungarian method to the limit violation cost matrix, the TTC in transaction $w_{23}$ is underestimated about 50%.

If a line with two limits is involved in the next equivalencing, this upper limit and lower limit becomes input for the next process of limit calculation. Both of them will generate another upper and lower limit and the bigger value of the upper limit and the smaller value of the lower limit should be chosen for the equivalent line limits.

V. NETWORK EXAMPLE

The IEEE 118-bus system is used to apply the proposed LPE algorithm. A set of buses to be eliminated are selected based on the amount of power flow going through the bus less than 60 MW. Thus, total 56 buses are chosen to be eliminated and the resultant system has 62 buses with 42 retained lines. As aforementioned, Tinney Scheme 2 is applied to reduce the number of neighbor buses as it reduces the number of fill-ins. Fig. 7 shows the results of the simulation.

The left axis shows the normalized average TTC which is the post-elimination average TTC on the eliminated lines divided by the pre-elimination average TTC on the equivalent lines. Therefore, 54 steps with exact line limits produce 1 and the rest two steps with non-exact line limits show their upper and lower boundaries in the figure. The number of fill-ins introduced during the equivalencing process gradually increase according to Tinney Scheme 2. Generally, lines with high impedance after a series of equivalencing steps can be ignored, but in this case, just one equivalent line has more than 3 p.u. impedance with its exact limit of 11.03 MW. Simulation results with a random order of bus elimination show that many more lines have upper and lower boundaries and also it takes much more time to run the simulation as the efficiency of the algorithm significantly depends on the number of the first neighbor buses.

VI. COMPUTATIONAL ASPECTS

Consider an N-bus system, in which M buses are being equivalenced in a sequential manner. Let $s_i$ be the number of first neighbor buses for bus $i$. For each elimination step, $(\frac{n}{2})$ of equivalent line admittances are calculated with Kron’s reduction. Also, the number of calculations of PTDFs in pre- and post-elimination stages is $(\frac{n}{2}) (\frac{n}{2} + s_i)$. For non-exact solution cases, minimum matching algorithm is also needed. The brute-force approach for assignment problems has to consider $n!$ combinations where $n$ is the dimension of the matrix, which is the number of equivalent lines, $(\frac{n}{2})$ in the paper. However, the Hungarian algorithm reduces the complexity to $O((\frac{n}{2})^3)$. Therefore, the complexity of the proposed algorithm can be rapidly increased with the number of first neighbor buses. However, Tinney Scheme 2 reduces the number of fill-ins by ordering the buses during the partial factorization and consequently the computational burden. In addition, the impedances of certain equivalent lines become very high after a series of equivalencing steps and hence they can be ignored in the process to reduce the computation further.

VII. CONCLUSION

In this paper, the procedure to calculate equivalent line limits when building an equivalent power system is presented. The proposed algorithm assumes that the system is unloaded so the bus injections from generation or loads do not affect on line
limits. For computational advantage, the Kron’s reduction is applied with partial factorization in order to obtain the line parameters of the equivalent system as buses are sequentially equivalenced. The next bus to be eliminated is chosen based on the bus valence to reduce the computation by reordering buses during the partial factorization. TTCs are calculated using line limits and PTDFs of pre- and post-elimination stages. The preserved attribute by assigning limits to the equivalent lines was that TTCs between the first neighbor buses of in the original system match with those for the same buses in the equivalent system. The algorithm can determine exact limits of equivalent lines in the reduced system when it is possible. In case where exact limits do not exit, the solutions could be bounded with upper and lower limits. There are a couple of areas that this algorithm has to deal with in the future research. The impact of power injections on line limits needs further investigation. In addition, how to handle the range of line limits in case of non-exact solution cases will be important issue as those ranges of limits have to be single values for the reduced system to be used in any applications.

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