Abstract—Corrective transmission switching schemes are an essential part of grid operations and are used to improve the reliability of the grid as well as the operational efficiency. Today, the transmission switching schemes are established based on the operator’s past knowledge of the system as well as other ad-hoc methods. In this paper, three topology control (corrective transmission switching) methodologies are presented along with the detailed formulation of robust corrective switching. By incorporating robust optimization into the corrective switching framework, the switching solution is guaranteed to be feasible for a range of system operating states. The robust model can be solved offline to suggest switching actions that can be used in a dynamic security assessment tool in real-time. The proposed robust topology control algorithm can also generate multiple corrective switching actions for a particular contingency. The robust topology control formulation is tested on an IEEE 118-bus test case with different uncertainty sets.

Index Terms—Mixed integer programming, power generation dispatch, power system operations, power system reliability, power transmission control, robust optimization.

NOMENCLATURE:

Indices:

- $n$, $m$: Nodes.
- $g$: Generator.
- $g(n)$: Set of generators at node $n$.
- $k$: Transmission asset (line or transformer).
- $k(n, \cdot)$: Set of lines with $n$ as the “to” node.
- $k(\cdot, n)$: Set of lines with $n$ as the “from” node.

Parameters:

- $P_{g}^{\text{max}}$: Max capacity of generator $g$.
- $P_{g}^{\text{min}}$: Min capacity of generator $g$.
- $R_{g}^{+\text{r}}$: Maximum 10-min ramp up rate for generator $g$.
- $R_{g}^{-\text{r}}$: Maximum 10-min ramp down rate for generator $g$.
- $v_{g}$: Unit commitment status of generator $g$.
- $P_{g}^{\text{e}}$: Real power supplied by generator $g$ at bus $n$ (solution obtained from unit commitment).

Variables:

- $d_{n}^{\text{fsc}}$: Forecasted system demand at bus $n$.
- $B_{k}$: Electrical susceptance of transmission line $k$.
- $P_{k}^{\text{max}}$: Max capacity of transmission line $k$.
- $P_{k}^{\text{min}}$: Min capacity of transmission line $k$.
- $N1_{k}$: Binary parameter that is 0 when $k$th transmission contingency occurred and 1 otherwise.
- $M_{k}$: Big M value for transmission line $k$.
- $M_{n}$: Big M value for load connected at bus $n$.
- $Z_{k}$: Binary variable for transmission element $k$; 0 if line is open/not in service; 1 if line is closed/in service.

I. INTRODUCTION

Even though the bulk power grid is one of the most complex systems to date, in practice, the modeling of the transmission network is simplified and limited attention is given to the flexibility in the network topology. Traditionally, transmission lines are treated as static assets, which are fixed within the network, except during times of forced outages or maintenance. This view does not describe transmission lines as assets that operators have the ability to control. Transmission switching has been studied since the 1980s and it was used as a tool to overcome various situations such as voltage violations, line overloads [1]–[4], line losses and cost reduction [5]–[7], system security [8], or a combination of these [9], [10].

Recent work has demonstrated that topology control can have significant operational as well as economic impacts on the way electrical power systems are operated today [11]–[14]. The concept of a dispatchable network is presented in [15]. Additionally, optimal transmission switching using a direct current power flow (DCOPF) formulation is presented in [13] and [16]; however, these models did not implicitly enforce N-1 reliability constraints. In [17], optimal transmission switching with an N-1 DCOPF formulation was tested on the IEEE 118-bus test case and on the RTS 96 test case. Reference [17] also indicates that substantial savings can be obtained by optimal transmission switching while satisfying N-1 reliability constraints.

There has been recent development of a different transmission switching formulation, [18], which builds on the work of
on generalized line outage distribution factors, [19]. With the use of flow canceling transactions, [18] develops a framework that is able to capture the changes in the topology and compares it to the $B - \theta$ formulation used in many preceding transmission switching papers as well as in this paper. This formulation is likely to outperform the $B - \theta$ formulation when the number of monitored lines is relatively small, something that is common practice within optimal power flow problems today.

Past literature has shown that topology control can be used to improve system operations and reliability. Such previous work has led system operators to adopt topology control as a mechanism to improve voltage profiles, transfer capacity, and even improve system reliability [20]–[22]. However, the adoption of topology control is still limited as there is a lack of systematic topology control tools. Currently, the industry adoption and implementation of topology control is based on ad-hoc methods or the operator’s past knowledge. Alternatively, transmission switching decisions can be suggested by a mathematical decision support tool. Many factors have prevented topology control from becoming a more widespread corrective action within system operations. For instance, there have been misconceptions that more transmission is always better than less, concerns over the switching actions’ effect on stability, impacts on circuit breakers, computational complexities of topology control algorithms, as well as additional concerns.

Corrective switching is one example of topology control, which is implemented today [20]. These methods are based on operators’ prior knowledge, as specified in [20, p. 107]; such actions may also be based on historical information. Ideally, corrective switching algorithms should be solved in real-time. Once the disturbance occurs, the switching algorithm is executed to suggest switching actions to alleviate any constraint violations. This approach is beneficial since the current operating status is known, which ensures the accuracy of the solution. However, the challenge of real-time mechanisms is that they must be extremely fast while also ensuring AC feasibility, voltage stability, and transient stability. Topology control models could be solved offline by estimating the operating state of the system. However, deterministic offline mechanisms also have limitations since the operating state must be predicted prior to the disturbance. The proposed offline corrective action is, thus, susceptible to its problematic reliance on perfect foresight. This paper introduces the concept of robust corrective topology control, which presents a solution to these current challenges.

Robust optimization has gained a great deal of attention in recent years; for example in [23], a two-stage robust optimization technique is used for unit commitment. It deals with data uncertainty and attempts to find an optimal solution considering the worst case uncertainty realization. The solution of the robust optimization problem is guaranteed optimal for a defined uncertainty set [24], [25]. Since the optimal solution is a hedge against the worst case realization, the solution is often conservative. Robust optimization may not be preferred for many applications due to its conservative nature; however, it is in accordance with the power industry in regards to maintaining reliability.

This paper proposes the new concept of robust corrective topology control. The main idea is to use transmission switching as a control tool to mitigate constraint violations with guaranteed solution feasibility for a defined uncertainty set. The switching solution obtained from the robust corrective topology control formulation will work for all system states within the defined uncertainty set. The proposed robust corrective topology control tool is tested as a part of contingency analysis, which is conducted after solving a day-ahead unit commitment problem; however, note that the concept of robust corrective topology control is not restricted to such applications. The main contributions of this paper are summarized below.

1) Three corrective switching methodologies are identified: real-time corrective switching, deterministic planning based corrective switching, and robust corrective switching. Real-time corrective switching is the preferred process for corrective switching, but it requires extremely fast solution times. Thus, with today’s technology, the implementation of real-time corrective switching is limited. With today’s technology, deterministic planning based corrective switching can be implemented but it requires perfect foresight regarding future operating states. Therefore, implementation of deterministic planning based corrective switching is limited. To fill the technology gap between real-time corrective switching and deterministic planning based corrective switching, a robust corrective switching methodology is proposed.

2) A robust corrective topology control formulation: the robust corrective switching model is a three-stage robust optimization problem. With a pre-determined uncertainty set regarding the nodal injections (or nodal withdrawals), the robust corrective switching model will determine the corrective switching action that will be feasible for the entire uncertainty set. The robust optimization model consists of a master problem and two subproblems. The master problem will determine the corrective switching action and the subproblems will determine the worst case realization of demand within the uncertainty set (for the associated corrective switching action). The nodal injection uncertainty can be due to generation uncertainty (wind/renewables), demand uncertainty, area interchange uncertainty, as well as other causes of uncertainty. The robust corrective switching framework will work for all these different types of uncertainties. The detailed vision of the robust corrective switching framework as an end-to-end process is also presented.

3) A solution technique for solving the robust corrective switching model is presented: specifically, an iterative procedure is developed to solve the master problem and the subproblems. The master problem is a mixed integer programming (MIP) problem and the subproblems are reformulated into a single subproblem, which is a nonlinear problem. This new subproblem is converted from a nonlinear problem into a MIP problem. The proposed solution technique is tested on the IEEE 118-bus test case.

The paper is structured as follows: a detailed framework of real-time corrective switching, deterministic planning based corrective switching, and robust corrective switching are presented in Section II. The uncertainty modeling used in this paper is described in Section III. The generic deterministic
Corrective switching formulation is given in Section IV. The detailed mathematical model for robust corrective switching is given in Section V. The solution method for the corresponding problem is discussed in Section VI. The IEEE 118-bus test case is used for the robust corrective switching analysis and the results are presented in Section VII. Section VIII provides the conclusions and Section IX discusses potential future work.

II. CORRECTIVE SWITCHING METHODOLOGIES

Corrective transmission switching can be used as a control action to respond to an event. This paper proposes a robust corrective switching methodology to respond to N-1 contingencies. This section analyzes two existing methods to determine potential corrective switching actions and compares them to the proposed robust corrective switching framework. Note that corrective transmission switching actions may or may not be combined with generation re-dispatch. For the proposed robust corrective switching procedure, generation re-dispatch is taken into consideration.

A. Real-Time Topology Control

The real-time topology control model determines the corrective action(s) to take as a response to an event, e.g., a contingency. The skeleton of the real-time topology control scheme is shown in Fig. 1. When a particular contingency occurs, the corrective switching algorithm will determine the switching action in real-time based on the current system state. The resultant switching scheme will be tested to determine if the proposed topology is AC feasible and if the switching action causes instability. If the solution is feasible, it is implemented.

Ideally, it is preferred to solve for the optimal switching action in real-time because more information is known about the operating state of the grid. However, during an emergency, it is paramount that a corrective action be taken as soon as possible in order to avoid a potential blackout. Real-time corrective switching is a non-convex, nonlinear, MIP problem. Such a problem cannot be solved in real-time with available tools today. Therefore, heuristics are necessary to generate potential solutions. There are many heuristics for transmission switching that have been previously proposed in the literature [26]–[29]. These heuristics can be used to find decent solutions faster than solving a MIP. However, there is still the overarching concern that they may not be fast enough for practical large-scale applications due to the extreme importance of implementing a solution as fast as possible during an emergency. DCOPF based heuristics would still need to be checked to see if they are AC feasible and any proposed action would need to be confirmed to not cause a stability concern. Therefore, it is difficult to establish the success rate of such heuristics due to the time sensitive nature of real-time corrective actions during emergency conditions. It is also difficult to predict the solution quality of switching actions proposed by heuristics. In [10], a real-time application of topology control is proposed for an AC formulation and they have shown that this can be solved quickly but there is still the issue of transient stability of the switching action and the approach does not take into consideration generation re-dispatch.

Another drawback of such real-time corrective switching heuristics is that they assume the operating state will not change. State estimation would be used to estimate the system state when the algorithm is executed. However, the actual system state when the action is implemented may be different than the assumed system state due to the time it takes to run the algorithm and check for AC feasibility and system stability. While such procedures can be adjusted to reflect multiple operational states, doing so adds additional complexity to the algorithm, which further exposes the approach to the risk that it may not solve fast enough. Overall, real-time topology control mechanisms that rely on heuristics may be fast but there are still practical issues that they do not take into consideration. Thus, there is a need for topology control actions that are robust against operating states in order to increase the likelihood of obtaining a feasible solution when implemented.

B. Deterministic Planning Based Topology Control

Today, there are special protection schemes involving corrective switching that are determined based on offline analysis, [20]. The main idea of deterministic planning based corrective switching is to determine the corrective switching action offline, e.g., in a day-ahead or a week-ahead timeframe, and then feed this information into a real-time dynamic security assessment tool that can determine if the switching action is feasible. For deterministic planning based corrective switching, an assumption regarding the system state is made and switching actions will be proposed in response to selected contingencies. Then, the switching schemes will be tested for AC feasibility and system stability based on the estimated, assumed system state(s). The benefit of such a procedure is that all of the heavy computational work is done offline. The resultant switching schemes are then fed into a real-time security assessment tool that functions like a lookup table. When the particular contingency occurs, a solution from the lookup table will be selected and tested for system feasibility based on the real-time system states. If a feasible solution is found, it is implemented; if a solution is not found, the operator can resort to traditional corrective means, such as generation re-dispatch. The schematic of the deterministic planning based topology control scheme is shown in Fig. 2.

The benefit of a planning based corrective switching approach is that the real-time procedures are minimal, resulting in a fast implementation of the action. However, the drawback is that a deterministic planning based corrective switching procedure requires perfect foresight of the system states. With a small deviation from the estimated operating state, the switching action may cause a blackout instead of preventing a blackout.
However, most corrective switching schemes implemented in practice are developed offline [20]–[22]. For instance, on [21, p. 8] it states, “Open or close circuits...when previously documented studies have demonstrated that such circuit openings reliably relieve the specific condition.” As a result, corrective switching is primarily limited to unique situations where the proper corrective action is obvious or it is already a well-known action due to the operator’s prior knowledge and experience. In the literature, there are few mathematical models available that can be used to determine corrective switching schemes with guaranteed solution feasibility for a range of operating states. In order to respond to this problem, robust corrective switching is proposed.

C. Robust Corrective Topology Control

This paper proposes the robust corrective switching framework as a response to the limitations of real-time and deterministic planning based corrective switching. The proposed robust corrective switching methodology shown in Fig. 3 is a combination of real-time and planning based corrective switching methodologies. Due to robust optimization, the proposed robust corrective switching methodology is superior to deterministic policies with respect to solution reliability. The technology gap between real-time and deterministic planning based corrective switching scheme is reduced by doing most of the heavy computational work offline and the guarantee of solution feasibility for a range of operating states is achieved by developing an uncertainty set over estimated system states. The uncertainty set can be viewed as lower and upper bounds over the system parameters or a range of operating states. The topology control algorithm will find the candidate switching actions based on modeled system states (with uncertainty) and a simulated contingency. The switching solutions generated by the topology control algorithm will then be tested for AC feasibility and system stability. The resultant switching solutions will be considered as candidate switching solutions for the corresponding contingencies and will be used in connection with a real-time corrective switching algorithm. When a particular contingency occurs, the on-line dynamic security assessment tool will test the proposed robust switching actions to determine the appropriate switching action to take. This process can also be combined with previously proposed real-time corrective switching heuristics since combining these procedures together will increase the likelihood of finding a feasible corrective action fast enough.

The primary feature of robust corrective switching is that the solution is guaranteed to be feasible over a wide range of operating states. The uncertainty set may consist of variable resources, such as generation uncertainty, wind/renewable generation uncertainty, demand uncertainty, and area interchange uncertainty. It should be noted that the topology control algorithm can be used to generate multiple switching solutions for a particular contingency. This characteristic of robust corrective switching is critical as not all of the solutions generated by the topology control algorithm may be AC feasible or pass the stability check. But due to multiple potential switching actions generated by the topology control algorithm, it is more likely that at least one of them will produce a feasible operating solution.

The timeline of the robust corrective switching scheme works as follows: after solving the day-ahead unit commitment problem, the robust corrective switching algorithm will determine the corrective switching schemes for possible contingencies. This can be seen as a form of contingency analysis, which has been modified to include robust corrective switching and it checks for a robust N-1 solution. These switching actions will be tested for AC feasibility and system stability. All of these calculations will be done offline. Once a particular contingency occurs, the real-time dynamic security assessment tool will evaluate the switching solution (if any) based on the real-time system states. If any feasible solution is obtained, it will pass the possible switching actions to the operator. Next, the operator will decide whether to implement the switching solution. The benefit of the proposed procedure is that the robust corrective switching scheme obtained from this method does not rely on ad-hoc methods, which enables corrective switching to be more widespread in order to improve operations and reliability.

The robust corrective switching scheme in this paper is based on a DCOPF framework and it guarantees the switching solution will be feasible for any operating state modeled by the uncertainty set. Since the optimal power flow (OPF) formulation is not an AC optimal power flow (ACOPF), the proposed solution must also pass an AC feasibility test. As a result, the guarantee that the solution is robust only holds for a DCOPF problem and is not guaranteed for the ACOPF problem. However, by developing a robust corrective switching formulation, we are
able to improve the chances that the proposed switching action will, indeed, be feasible as compared to deterministic corrective switching DCOPF schemes. Typically, generation re-dispatch is required to obtain an AC feasible solution, which is one of the primary reasons why corrective switching schemes may be feasible for the DCOPF but are not AC feasible. However, the proposed robust corrective switching scheme is guaranteed to be feasible (for the DCOPF) for a wide range of operating conditions; this substantially increases the chances that the chosen topology solution will have an AC feasible solution since there are many DC solutions to start with. The proposed robust corrective switching procedure can be seen as a mathematical program that is equivalent to the practice used today by operators to identify candidate switching actions based on historical studies showing the action has worked under a variety of operating conditions.

III. MODELING OF DEMAND UNCERTAINTY

Uncertainty modeling is a key part of robust optimization. In [23] and [30], polyhedral uncertainty sets are used to define demand uncertainties; they assume that each load has an upper and lower bound and that the system-wide aggregate load has an upper bound. In this paper, a simplified uncertainty model is used to represent demand uncertainty. The polyhedral uncertainty set used in this paper is presented in (1); if desired, more complex polyhedral uncertainty sets can be used instead, as in [30]:

\[
\mathcal{D} = \{ d \in \mathbb{R}^n : d_{n}^{i,\bar{x}} D_+^n \leq d_n \leq d_{n}^{i,\bar{x}} D_-^n : \forall n \}. \tag{1}
\]

In this uncertainty set, the system demand is bounded by its pre-determined lower and upper limits. The uncertainty description used in this paper is more conservative than the uncertainty sets used in [23] and [30]. The size of the uncertainty set is defined by the parameters \( D_+^n \) and \( D_-^n \). When \( D_+^n = D_-^n = 1 \), the uncertainty is zero and \( \mathcal{D} \) is a singleton, i.e., \( d_n = d_{n}^{i,\bar{x}} \). When \( D_-^n \leq 1 \) and \( D_+^n \geq 1 \), the uncertainty set is a polyhedron and its size is defined by the values of \( D_+^n \) and \( D_-^n \).

IV. DETERMINISTIC TOPOLOGY CONTROL

The parameters \( A, B, E, F, \) and \( H \) are cost vectors. The system demand in this case is the forecasted demand at each bus, \( d_{n}^{i,\bar{x}} \). Deterministic corrective switching is a MIP problem. The variable \( x \) represents the binary variable associated with the switching action, where \( x = 1 \) if the line is closed/in service or \( x = 0 \) if the line is open/out of service. The continuous variable \( y \) represents all of the OPF continuous variables, such as line currents, bus angles, and generator dispatch.

V. ROBUST CORRECTIVE TOPOLOGY CONTROL FORMULATION

In the deterministic corrective transmission switching problem, the switching action is based on a single system state. However, in the robust topology control problem, the switching action is determined based on a range of operating states. The objective of robust topology control is to find a robust switching solution in response to a contingency while not allowing any load shedding for any realizable load within the uncertainty set. It should be noted that demand response can also be used as a control mechanism in response to a contingency; however, this option is not included in this paper. Furthermore, in this paper the topology control problem is modeled as a feasibility problem; hence, vector \( c \) and \( b \) in (2) are equal to zero.

When the system demand uncertainty is zero, the topology control model presented in (2)–(6) is the same as the model given in (7)–(11). In (11), the term \( y(d) \) is used to emphasize the dependency of continuous variable \( y \) on the demand uncertainty, \( d \). The second part of the robust formulation is further divided into two parts and results into a three-stage optimization problem as shown in (12). The objective of a three stage robust problem is to find a feasible topology under the worst case demand. The first stage will determine the topology or switching actions, whereas stages two and three will determine the feasibility of the switching action for the entire uncertainty set:

\[
\min_{x \in \mathcal{L}} \left( c^T x + \max_{d \in \mathcal{D}} \min_{y \in \Omega(x, d)} b^T y \right) \tag{12}
\]

\[
\text{s.t. } Fx \leq f, \quad H y \leq h, \quad Ax + By \leq g, \quad E y = d, \quad x \in \{0, 1\}. \tag{13}
\]

The set \( \Omega(x, d) \) is a set of feasible solutions for a fixed topology and demand \( d \), which is represented by \( \Omega(x, d) = \{ y : H y \leq h, \ Ax + By \leq g, \ E y = d \} \). In (12), the \( \max \min \) part of the problem determines the worst case cost or demand associated with the switching solution (determined in the first stage) and can be combined
The resultant problem is shown in (14)–(16):
\[
\begin{align*}
\max_{d, \varphi, \lambda, \eta} & \quad \lambda^T (Ax - g) - \varphi^T h + \eta^T d \\
\text{s.t.} & \quad \lambda^T H - \varphi^T H + \eta^T E = b^T, \\
& \quad d \in D, \lambda \geq 0, \varphi \geq 0, \eta \text{ free}.
\end{align*}
\]
(14a)
(14b)
(14c)
(15)
(16)

\(\varphi, \lambda, \text{and } \eta\) are dual variables of constraints (4)–(6), respectively. In (14), the term \(\eta^T d\) is nonlinear. In [23], an outer approximation technique is used to solve this bilinear problem. However, this approach assumes that the problem is feasible and it guarantees only local optimality. The corrective switching problem is a feasibility problem and thus, it requires a global solution. Therefore, the outer approximation technique is not suitable for the robust corrective switching problem.

Since the DCOPF problem is a convex problem, the new subproblem formulation presented by (14)–(16) can be reformulated into a MIP problem. By classifying all extreme points of the polyhedron representing the uncertainty set, we can guarantee a robust solution due to the convexity of the DCOPF problem, i.e., we can guarantee that all interior points are feasible if the robust solution is feasible for all extreme points of the polyhedron. This reformulation allows us to solve the nonlinear problem (14)–(16) by mixed integer programming while still being able to guarantee a global optimal solution. This reformulation procedure is also used in [30]. The MIP reformulation for the polyhedron representing the demand uncertainty is shown by (40)–(43).

The master problem is a MIP problem and represented by (17)–(18) and the subproblem is represented by (14)–(16):
\[
\begin{align*}
\min_{x \in X} & \quad c^T x \\
\text{s.t.} & \quad Fx < f, \ x \in \{0, 1\}.
\end{align*}
\]
(17)
(18)

The robust corrective switching formulation used in this paper is presented in (20)–(32), with an objective presented by (19). The formulation includes generator limit constraints (20)–(21), generator contingency ramp up and ramp down constraints (22)–(23), line limit constraints (24)–(25), transmission switching constraints (26)–(27), the node balance constraint (28), and demand uncertainty (29)–(30). The maximum number of line switchings per solution are limited by parameter \(M\) in (31):
\[
\begin{align*}
\min_{x_k \in X} & \quad \left(0 + \max_{d \in D} \min_{P_g, \theta_n, \theta_m, \epsilon \in \{\{Z\}, d\}} \sum_{\forall g} \right) \\
\text{s.t.} & \quad P_g \geq P_{g_{\text{min}}} u_g, \forall g, \\
P_g & \geq (-R_{g_{\epsilon}} - P_{g_{\text{pc}}}), \forall g, \\
P_g & \leq (-R_{g_{\epsilon}} + P_{g_{\text{pc}}}), \forall g, \\
P_k & \geq -P_{k_{\text{max}}} Z_k N_1 k, \forall k, \\
P_k & \geq -P_{k_{\text{max}}} Z_k N_1 k, \forall k, \\
P_k - B_k (\theta_n - \theta_m) + (1 - Z_k N_1 k) M_k \geq 0, \forall k, \\
P_k - B_k (\theta_n - \theta_m) - (1 - Z_k N_1 k) M_k \leq 0, \forall k, \\
\sum_{\forall k(n \neq \epsilon(n))} P_k - \sum_{\forall k(n)} P_k + \sum_{\forall g(n)} P_g = d_n, \forall n.
\end{align*}
\]
(19)
(20)
(21)
(22)
(23)
(24)
(25)
(26)
(27)
(28)

The complete robust corrective switching problem is split into two parts: a master problem, and a subproblem. The master problem is \(\min \ 0\) with constraints represented by (31)–(32), which determine the topology. The subproblem is a two part optimization problem, which determines the worst case demand for a particular topology. The first part of the subproblem is represented by an objective \(\max\) with constraints (29)–(30), which determines the worst case system demand within the uncertainty set. The second part of the subproblem is represented by the objective \(\min\) with constraints (20)–(28).

This second part of the subproblem is a DCOPF formulation that evaluates the feasibility of the system demand, which is selected in the first part of the subproblem.

The objective of the third stage’s dual is given in (33), where \(\alpha_g^+, \alpha_g^-, \Omega_k^+, \Omega_k^-, F_k^+, F_k^-, S_k^+, S_k^-, L_n\) are dual variables associated with constraints (20)–(28), respectively. When the second stage and the third stage of the subproblem are combined together, the term \(d_n L_n\) in (33) makes the objective nonlinear. The nonlinearity of the dual objective is removed by restructuring the nonlinear problem into a MIP problem. The resultant subproblem is given in (34)–(36), where the dual formulation of the third stage subproblem is combined with the demand uncertainty:
\[
\begin{align*}
\max & \quad -\sum_{\forall g} P_{g_{\text{max}}} u_g \alpha_g^+ + \sum_{\forall g} P_{g_{\text{min}}} u_g \alpha_g^- \\
& \quad + \sum_{\forall g} (-R_{g_{\epsilon}} - P_{g_{\text{pc}}}) \Omega_g^+ + \sum_{\forall g} (-R_{g_{\epsilon}} + P_{g_{\text{pc}}}) \Omega_g^- \\
& \quad - \sum_{\forall k} P_{k_{\text{max}}} Z_k N_1 k (F_k^+ + F_k^-) + \sum_{\forall n} d_n L_n \\
& \quad - \sum_{\forall k} (1 - Z_k N_1 k) M_k (S_k^+ + S_k^-).
\end{align*}
\]
(33)

A big-M formulation is used to represent the extreme points of the polyhedron representing the uncertainty set. The drawback of such an approach is that it causes a poor relaxation. To overcome this problem, CPLEX’s indicator constraint modeling approach is used to model (40)–(43):
\[
\begin{align*}
\max & \quad -\sum_{\forall g} P_{g_{\text{max}}} u_g \alpha_g^+ + \sum_{\forall g} P_{g_{\text{min}}} u_g \alpha_g^- \\
& \quad + \sum_{\forall g} (-R_{g_{\epsilon}} - P_{g_{\text{pc}}}) \Omega_g^+ + \sum_{\forall g} (-R_{g_{\epsilon}} + P_{g_{\text{pc}}}) \Omega_g^- \\
& \quad - \sum_{\forall k} P_{k_{\text{max}}} Z_k N_1 k (F_k^+ + F_k^-) + \sum_{\forall n} d_n L_n \\
& \quad - \sum_{\forall k} (1 - Z_k N_1 k) M_k (S_k^+ + S_k^-).
\end{align*}
\]
(34)

\[
\begin{align*}
\text{s.t.} & \quad -\alpha_g^+ + \alpha_g^- - \Omega_k^+ - \Omega_k^- + L_n = 0, \forall g, \\
& \quad F_k^+ + F_k^- + S_k^+ - S_k^- + L_n - L_m = 0, \forall k.
\end{align*}
\]
(35)
(36)
VI. SOLUTION METHOD FOR ROBUST CORRECTIVE TOPOLOGY CONTROL

The robust topology control problem is a three-stage problem with a master problem and two subproblems. However, it is reformulated into a two-stage problem with a master problem and a subproblem. The solution method proposed in this paper is an iterative process between the master problem and the subproblem. The master problem is a MIP, which determines the system topology. The subproblem is a nonlinear problem, which is converted into a MIP and it searches for the worst case demand for the particular topology. For the proposed solution method, it is assumed that the unit commitment problem is solved prior to solving the robust corrective switching problem.

A. Initialization

The unit commitment problem is first solved with the fixed, initial topology. The solution of this unit commitment problem, the unit commitment status, the generators’ scheduled dispatch, and the acquired reserves, are fed into the robust topology control framework. The first step of solution method is to solve the dual problem given by (44), where $Z_k$ represents the initial topology. The model presented in (44) is the dual of the DCOPF problem. The dual variables of constraints (35)–(37) are $P^+, P^-$, respectively. If the problem is infeasible, then the proposed unit commitment solution is not N-1 reliable and a cut must be added to the master problem in the form of (46). The proposed approach will then search for a robust corrective switching action that enables the solution to be N-1 compliant, if such a solution exists:

\[
- \sum_{k \in \mathcal{N}} B_k S_k^+ + \sum_{k \in \mathcal{N}} B_k S_k^- + \sum_{k \in \mathcal{N}} B_k S_k^+ - \sum_{k \in \mathcal{N}} B_k S_k^- = 0, \forall n
\]  
\( (37) \)

\[
\alpha^+_g, \alpha^-_g, \Omega^+_g, \Omega^-_g \geq 0, \forall g
\]  
\( (38) \)

\[
F_k^+, F_k^-, S_k^+, S_k^- \geq 0, \forall k
\]  
\( (39) \)

\[
\eta_n - L_n d_{n,ns}^+ D_n^+ + (1 - D_n) M_n \geq 0, \forall n
\]  
\( (40) \)

\[
\eta_n - L_n d_{n,ns}^- D_n^- - (1 - D_n) M_n \leq 0, \forall n
\]  
\( (41) \)

\[
\eta_n - L_n d_{n,ns}^- D_n^- + D_n M_n \geq 0, \forall n
\]  
\( (42) \)

\[
\eta_n - L_n d_{n,ns}^+ D_n^- - D_n M_n \leq 0, \forall n
\]  
\( (43) \)

B. Master Problem: Topology Selection

The master problem is a MIP problem and its objective is to determine the system topology. The master problem contains a topology selection formulation and combinatorial cuts. The master problem is represented by (45)–(47). For iteration $j \geq 1$

\[
\min \gamma
\]  
\( (45) \)

\[
s.t. \; 1 \leq \sum_{k \in \mathcal{N}} Z_k + \sum_{k \in \mathcal{N}} (1 - Z_k); \forall l \leq j
\]  
\( (46) \)

\[
\sum_{k \in \mathcal{N}} (1 - Z_k) \leq M
\]  
\( (47) \)

At each iteration, the master problem finds a feasible solution and then passes $Z_k$ to the subproblem as an input parameter. The solution $Z_k$ will be evaluated for the worst case scenario in the subproblem. If the master problem is infeasible, this states that all of the possible topologies are infeasible and there is no feasible switching action for the defined uncertainty set, as shown in stage 1 of Fig. 4.

C. Subproblem: Worst Case Evaluation

The objective of the subproblem is to determine the worst case demand associated with the topology (determined in the master problem). The subproblem is a MIP and presented in (34)–(43). If the subproblem is feasible and the objective is equal to zero, then it proves that, for a given topology, there is no system demand within the uncertainty set that will produce an infeasible OPF solution. In other words, the corresponding topology is feasible for the entire uncertainty set; hence, a robust solution is obtained. On the other hand, if the subproblem’s objective is non-zero, then the corresponding topology is infeasible for a particular demand within the uncertainty set. Hence, that topology is discarded and a feasibility and/or combinatorial cut is applied to the master problem in form of (46). Equation (46) is known as a combinatorial cut, which prevents the master problem from choosing any prior binary $Z_k$ solution that is known to be infeasible. The master problem is solved again and the process continues till the robust solution is found or all possible topologies are confirmed to be infeasible. The solution method for the robust topology control problem is summarized in Fig. 4.
VII. RESULTS

The computational study for robust corrective switching is performed on the IEEE 118-bus test case. The test case consists of 54 generators, 118 buses, and 186 transmission lines. The IEEE 118-bus test case given in [31] does not have generator information. Therefore, generator information from the Reliability Test System-1996 [31] is used. The fuel costs given in [12] are used to calculate generator operating costs. The basic unit commitment model presented in [11] is adopted. A 24-h unit commitment problem is solved. The reserve requirements for the unit commitment problem is the sum of 5% of demand supplied by hydro generators and 7% of demand supplied by non-hydro units or the single largest contingency, whichever is greater. It is assumed that at least 50% of total required reserves will be supplied by spinning reserves and the rest will be supplied by non-spinning reserves. This assumption is in line with CAISO’s guidelines for spinning reserve and non-spinning reserve [32]. The hour 16 solution of the unit commitment problem is used for deterministic as well as robust corrective switching analysis. The IEEE 118-bus test case in [31] does not have emergency transmission rating. Therefore, it is assumed that the emergency thermal rating for the transmission elements is 125% of the steady state operating limits.

A. Deterministic Corrective Switching

In the deterministic corrective switching analysis, the demand uncertainty is assumed to be zero. The switching action is determined with the static demand levels used in the unit commitment problem. It is observed that 10 transmission contingencies (out of 186) can only be alleviated if transmission switching is combined with generation re-dispatch, i.e., generation re-dispatch on its own cannot satisfy these 10 transmission contingencies. The generation re-dispatch allows each unit to change within 10 min of its ramping capability. This result is important because, traditionally, such contingencies are mitigated by expensive generation re-dispatch. Moreover, these 10 transmission contingencies have multiple corrective switching actions. The ability of the corrective switching algorithm to generate multiple solutions for a single contingency is critical from a system operations point of view. The corrective switching formulation is based on a DC framework. Therefore, the solution needs to be tested for AC feasibility and system stability requirements. Hence, the probability of having at least one AC feasible and stable corrective switching solution is higher if the corrective switching algorithm generates multiple corrective solutions.

It is also observed that the solution for corrective transmission switching will not always be “to open the congested line”, but frequently it will be “to open a lightly loaded line”. This demonstrates that the commonly held assumption that congested lines are the top candidate lines for switching is not always correct. Furthermore, such examples demonstrate the need for systematic tools for topology control.

B. Robust Corrective Switching Analysis

For robust corrective switching analysis, ±14.3%, i.e., ±324.5 MW, demand uncertainty is assumed. For computational simplicity, the demand uncertainty is assumed only on 50% of the system MW demand involving roughly half of the load buses. It is also assumed that all of the system reserves are available within 10 min and the generators are allowed to change their outputs within its 10-min ramp rate. Of the 186 transmission contingencies, 159 can be alleviated by dispatching reserves alone. While corrective switching is not required for these 159 contingencies, topology control can still be useful in response to these contingencies because it can reduce the need for a costly system re-dispatch; furthermore, the topology control algorithm provides multiple feasible switching solutions for these 159 transmission contingencies. The 7 transmission contingencies listed in Table I require corrective transmission switching actions in order to avoid load shedding, i.e., generation re-dispatch alone was not sufficient to respond to the contingencies. Note that these robust corrective switching solutions involve both corrective switching and generation re-dispatch.

The first column of Table I represents the transmission contingency and the second column represents the corresponding corrective switching actions. All 7 of these transmission switching contingencies can only be alleviated if corrective transmission switching is employed. For instance, a contingency on line 111 can only be mitigated by switching line 108 or 109 combined with generation re-dispatch. No feasible solution is available with generation re-dispatch alone due to network congestion. The switching solutions for the other 6 transmission contingencies are documented in Table I.

The contingencies 111, 115, 148, and 154 have multiple robust corrective switching actions. Table I shows that there can be multiple switching solutions for a single contingency. Similarly, one switching action may alleviate multiple contingencies. For instance, the robust switching solution to open line 141 mitigated 3 transmission contingencies. This result shows the potential of robust corrective switching to generate multiple candidate switching solutions for a real-time dynamic security assessment tool to evaluate switching actions for real-time operations.

In the last column of Table I, the number of deterministic corrective switching solutions, for a particular contingency, is presented. It shows that the number of possible deterministic corrective switching solutions is much more as compared to the number of robust solutions. However, the robust solutions guarantee solution feasibility over a wide range of operating states whereas the deterministic solutions do not guarantee solution
feasibility if there is any change in the operating state. Therefore, the possibility of having a successful corrective action with the deterministic corrective switching solutions is far less than the potential success rates for the robust corrective switching solutions.

For a contingency on line 63, with the initial topology no feasible solution is obtained with a fixed demand. Hence, the unit commitment solution is not N-1 compliant. However, with the robust corrective switching framework, an N-1 feasible solution exists; furthermore, the robust corrective switching framework is able to produce an N-1 feasible solution that is robust against the demand uncertainty. This result is extremely important and powerful as we have proven that topology control can take a solution that is N-1 infeasible for a deterministic fixed demand and make it N-1 feasible even with a high level of demand uncertainty. Indeed, the assumption that transmission switching must degrade system reliability is false.

The computational time for ±1.3% uncertainty set is about 10 min per contingency with a 2.93 GHz, Intel i-7 processor with 8 GB of RAM. It is also observed that the computational time increases with small increases in the uncertainty set. For instance, a 1% decrease in uncertainty causes a 13% drop in computational time.

VIII. CONCLUSION

In this paper, three different corrective switching methodologies are presented: real-time, deterministic planning based, and robust corrective switching. Real-time corrective switching is very difficult to implement with today’s technology due to a lack of computational power and the practical barriers of needing to ensure AC feasibility, voltage stability, and transient stability. Deterministic planning based corrective switching can be solved offline, but such an approach relies on predicting the operating state. Furthermore, the deterministic planning based methods cannot guarantee solution feasibility over a wide range of system states. The proposed method of robust corrective switching fills the technology gap between the real-time and the deterministic planning based corrective switching methodologies. The offline mechanism of robust corrective switching generates multiple solutions and can be implemented in real-time with the help of a real-time dynamic security assessment tool. As a result, the proposed robust corrective switching model provides a mathematical decision support tool that integrates topology control into every day operations by being able to guarantee robust solutions.

While deterministic corrective switching frameworks may suggest many potential switching solutions, the empirical results presented in this paper show that many of these solutions will be infeasible for minor changes in the operating state. In contrast, the robust corrective switching scheme presented in this paper guarantees solution feasibility for a wide range of system states, given a DCOPF formulation. In addition, the robust corrective switching formulation demonstrates the ability of generating multiple corrective switching actions for a particular contingency. Moreover, a single resulting corrective switching solution is capable of mitigating multiple contingencies.

Day-ahead unit commitment problems with proxy reserve requirements do not guarantee N-1 feasibility. Contingency analysis is used to determine whether there are contingencies that cannot be satisfied by the unit commitment solution. The results have shown that robust corrective topology control can be used to reduce the occurrence of contingencies that are not satisfied by the re-dispatch capabilities of the unit commitment solution alone. Furthermore, the numerical results prove that topology control does not necessarily degrade system reliability; on the contrary, it can help the system to achieve N-1 feasibility even with uncertainty.

While transmission switching exists today, it is used to a limited extent; historical information or the operators’ prior knowledge are the primary mechanisms to establish and implement corrective switching as opposed to using a mathematical framework to identify corrective switching actions. The electric grid is one of the most complex engineered systems to date. Relying on only prior observations to determine potential corrective switching actions limits our capability to harness the existing flexibility in the transmission network. Systematic procedures that are capable of capturing such complexities should be preferred over such limited methods. Furthermore, the hardware requirements to implement topology control (circuit breakers) already exist, leaving only the need to develop the appropriate decision support tools, which are low in cost, to obtain such benefits.

IX. FUTURE WORK

Future research will involve AC feasibility and stability tests. Future work should also examine the integration of this technique into a dynamic security assessment tool as well as work to improve the computational performance.

REFERENCES


Akshay S. Korad (S’11) received the B.E. degree in electrical engineering from Pune University, India, in 2005 and the M.S. degree in control systems from Arizona State University, Tempe, AZ, USA, in 2010. Currently, he is pursuing the Ph.D. degree at Arizona State University in the School of Electrical, Computer, and Energy Engineering.

Kory W. Hedman (S’05–M’10) received the B.S. degree in electrical engineering and the B.S. degree in economics from the University of Washington, Seattle, WA, USA, in 2004 and the M.S. degree in economics and the M.S. degree in electrical engineering from Iowa State University, Ames, IA, USA, in 2006 and 2007, respectively. He received the M.S. and Ph.D. degrees in industrial engineering and operations research from the University of California, Berkeley, CA, USA, in 2007 and 2010, respectively. Currently, he is an Assistant Professor in the School of Electrical, Computer, and Energy Engineering at Arizona State University, Tempe, AZ, USA. He previously worked for the California ISO (CAISO), Folsom, CA, USA, on transmission planning and he has worked with the Federal Energy Regulatory Commission (FERC), Washington, DC, USA, on transmission switching.