Security Enhancement through Direct Non-Disruptive Load Control

Final Project Report

Part I

Power Systems Engineering Research Center

A National Science Foundation
Industry/University Cooperative Research Center
since 1996

PSERC

Power Systems Engineering Research Center

Security Enhancement through Direct Non-Disruptive Load Control

Final Project Report

Part I

Report Authors

Ian Hiskens, Project Leader
Bo Gong
University of Wisconsin-Madison

PSERC Publication 06-01

January 2006

Information about this project

For information about this project contact:

Ian Hiskens
University of Wisconsin – Madison
Department of Electrical and Computer Engineering
1415 Engineering Drive
Madison, WI 53706
Phone: 608-261-1096
Fax: 608-262-5559
Email: hiskens@engr.wisc.edu

Power Systems Engineering Research Center

This is a project report from the Power Systems Engineering Research Center (PSERC). PSERC is a multi-university Center conducting research on challenges facing a restructuring electric power industry and educating the next generation of power engineers. More information about PSERC can be found at the Center's website: http://www.pserc.org.

For additional information, contact:

Power Systems Engineering Research Center
Arizona State University
Box 878606
Tempe, AZ 85287-8606
Phone: 480-965-1879

Notice Concerning Copyright Material

Acknowledgements

The Power Systems Engineering Research Center sponsored the research project entitled "Security Enhancement through Direct Non-Disruptive Load Control." The project began in 2002 and was completed in 2005. This is Part I of the final report for the project.

We express our appreciation for the support provided by PSERC's industrial members and by the National Science Foundation under grant NSF EEC-0119230 at University of Wisconsin-Madison, received under the Industry/University Cooperative Research Center program.

The authors thank all PSERC members for their technical advice on the project, especially Innocent Kamwa (IREQ), Nicholas Miller (GE Energy), and Sharma Kolluri (Entergy) who were industry advisors for the project. The authors also acknowledge Prof. Vijay Vittal of Arizona State University for technical advice, close collaboration and patience throughout the project.

Executive Summary

Dynamic security enhancement is generally associated with improvements in the response of generation and transmission systems, with network controls provided by FACTS devices and special protection schemes gaining acceptance. Load control, on the other hand, has (rightly) been viewed as disruptive to customers and, therefore, as the response of last resort. However, significant enhancements in communications, metering and computer technologies have meant that coordinated control of massive numbers of diverse loads is becoming feasible. Issues arising from such a control strategy have been explored. Our research has focused on the viability of load control for alleviating voltage collapse, and hence, for mitigating the possibility of cascading system failures.

Many customer installations include loads that can be tripped with imperceptible consequences over the short-term. Consolidation of these numerous small loads provides a non-disruptive load control capability that can be used to enhance dynamic security, for example by alleviating voltage collapse. Our research has explored a hierarchical control structure consisting of a lower-level, substation-based controller that interacts with loads, together with a higher-level, wide-area controller that formulates coordinated responses to threats of voltage instability. Load availability information is passed from the substation controller up to the higher-level controller. The resulting control signals are passed back down to the lower level for communication to the actual loads.

The higher-level, wide-area controller must be capable of responding to system events in real time. It has been found that model predictive control fulfills that requirement. Model predictive control (MPC) builds on the concept of simulation-based prediction of system behaviour. It utilizes an internal model of the system to predict the response to a disturbance. The MPC controller solves an embedded optimization problem to determine appropriate load shedding responses for restoration of voltages to acceptable secure levels. The use of trajectory sensitivities allows this optimization problem to be reduced to an approximate linear programming problem, even though the actual system may exhibit complicated nonlinear non-smooth dynamics. Errors introduced through model simplifications are corrected by subsequent repetition of the MPC prediction/optimization algorithm. Performance degradation resulting from model and measurement uncertainty was explored via a number of test cases. These investigations suggest that model predictive control is robust to significant levels of uncertainty.

Development of technology for remote monitoring and control of customer loads is well under way. These current technology developments are being driven by a desire for loads to be responsive to market signals. However, this same technology would support the monitoring and control requirements of a hierarchical security enhancement scheme.

The higher-level MPC controller requires an estimate of the current system state to determine the appropriate control response. This implies a close integration of the MPC controller with the topology processing and state estimation functions of energy management systems. For model predictive control to alleviate voltage collapse, it must respond more quickly than the

processes driving voltage degradation, which are typically transformer tap-changing and load recovery. Therefore, a response time of 30-60 seconds is generally adequate. Given normal data acquisition times of 2-4 seconds, and state estimator run times of 2-5 seconds, the required response can be easily achieved.

Based on initial investigations, non-disruptive load control appears to offer a viable option for enhancing dynamic security. Further investigations are required though to fully assess the technical and practical feasibility of the proposed control strategy. Ongoing research includes:

- Testing on a wider range of power system examples.
- Formal proof of stability and robustness properties of the MPC controller.
- Cost-benefit analysis, including appropriate incentives for customer participation in load control.

Contents

List of Figures

1 Introduction

1.1 Load control background

Dynamic security enhancement is generally associated with improvements in the response of generation and transmission systems, with network controls, such as provided by FACTS devices and special protection schemes, gaining acceptance. Load control, on the other hand, has (rightly) been viewed as disruptive to customers, and therefore the response of last resort. However recent enhancements in communications, metering and computer technologies have meant that coordinated control of massive numbers of diverse loads is becoming feasible. This project has explored issues arising from such a control strategy. Part I of the project has focused on the viability of load control for preventing voltage collapse, and hence prevention of resultant cascading system failures.

Concepts of direct load control are not new. Underfrequency load shedding schemes have been in operation for almost as long as power systems have existed. More recently, undervoltage load shedding has become an important strategy for the prevention of voltage collapse [1]. Demand side management (DSM) schemes that primarily control water and space heating, and air conditioning, are also well established. Such schemes are designed to modify the shape of the load curve to achieve economic benefits and reliability improvements [2, 3]. They are non-disruptive, but offer limited controllability. Price-based control of loads also offers potential economic benefits [4].

Underfrequency and undervoltage load shedding are achieved by disconnecting entire distribution feeders. Implementation is simple (conceptually at least), requiring only that a trip signal be sent to the appropriate feeder circuit breaker. However such load control is clearly disruptive to consumers on the affected feeders; sensitive loads along the feeder require some form of backup. Restoration of load is normally undertaken manually, by closing the feeder circuit breaker. However many loads, in particular motor loads, draw much higher current on startup than during normal operation. This cold load pickup phenomenon must be taken into account, as it is known to cause significant restoration problems [5, 6].

Indirect forms of load control have been used previously for enhancing dynamic performance. SVCs are often equipped with a stabilizing circuit for modulating the terminal voltage, which in turn modulates local voltage-dependent loads. If tuned correctly, this load variation can damp interarea oscillations. However SVC control typically do not adapt to changing system conditions, so controller effectiveness may vary greatly between peak and light load conditions.

Load control for dynamic security enhancement is quite different to traditional load management. Security enhancement requires fast response of specific amounts of load at particular locations. DSM is too slow and too imprecise. Underfrequency load shedding works well because frequency is effectively a common, system-wide signal. Undervoltage load shedding, on the other hand, can lead to incorrect control action because voltage is a local signal that is used to infer wider system behaviour. This will be illustrated later using a simple example.

1.2 Summary of findings

Power system dynamic security can be enhanced through the coordinated control of many small, geographically distributed loads. A hierarchical control structure is suggested, with substation-based monitoring of local loads, and system-wide formulation of control responses.

Load availability information is passed up to the higher level controller. The resulting control signals are passed back down to the lower level for communication to the actual loads.

The higher-level, wide-area controller must be capable of responding to events as they occur. The project has found that model predictive control (MPC) is well suited, as this control strategy builds on the concept of simulation-based prediction of system behaviour. The project has considered performance degradation resulting from model and measurement uncertainty. It has been found that MPC is robust to significant levels of uncertainty.

The project has not addressed the economic incentives necessary to encourage consumers to participate in this form of load control. Such investigations could be based on the economic arguments underpinning traditional DSM schemes. However the benefits in this case relate solely to improvements in reliability, which can be difficult to value. Reliability-based arguments have been made to justify the cost of special protection schemes; similar arguments apply for non-disruptive load control schemes.

1.3 Outline of the report

The report is organized as follows. Concept relating to non-disruptive load control are discussed in Chapter 2. A wide-area control scheme, based on model predictive control, is presented in Chapter 3. The performance of the control strategy is explored via an example in Chapter 4. Conclusions are provided in Chapter 5.

2 Non-Disruptive Load Control

2.1 Controllable load

Many consumer installations consist of loads that are at least partially controllable [7]. Commercial loads typically involve a high proportion of air conditioning and lighting. The thermal time-constant of many commercial buildings is usually quite long. Therefore air conditioning in large multi-storied buildings can be shed with no appreciable short-term effects on building climate. Similarly, a short-term reduction in lighting load is often possible without compromising the building environment [8]. Partial load control within industrial and residential installations is also possible. In the residential case for example, one circuit within a home could be designated for interruptible supply, with a corresponding lower energy charge. That circuit could be used for lower priority loads such as dryers and/or freezers. A similar concept applies for industrial consumers. In the latter case though, it may also be possible to use backup generation to displace grid supply.

The project has not addressed the economic incentives necessary to encourage consumers to offer load for "non-disruptive" control. Such an investigation could be based on the economic arguments that underpin traditional DSM schemes. However the benefits in this case are given by reliability improvements, which can be difficult to value. Reliability-based arguments have been used to justify the cost of special protection schemes though. Similar arguments are valid for non-disruptive load control schemes.

2.2 Load consolidation

The distributed nature of non-disruptive load control implies a need for a hierarchical control structure, as suggested in Figure 1. A lower (substation) level controller is required to coordinate the many small controllable loads. In standby mode, this controller would continually poll loads to track availability of controllable load. Appropriate communications technology is described in [9]. Information retrieved from individual loads would include its real and reactive power demand, and an indicator of its load type. Using this latter information, the cold pickup behaviour of the load could be estimated. The lower-level controller would therefore build a consolidated picture of the load available to be tripped, and the likely consequences of re-energization.

Load availability information would be passed to the higher-level controller described later in Chapter 3. When a load change was required, the higher level controller would specify the amount desired at each substation. The substation-level controller would implement that request by signalling the individual loads. The anticipated and actual load responses may differ, due to the continual changes in load composition. That information would again be coordinated at the lower level and passed to the higher level in preparation for further control action.

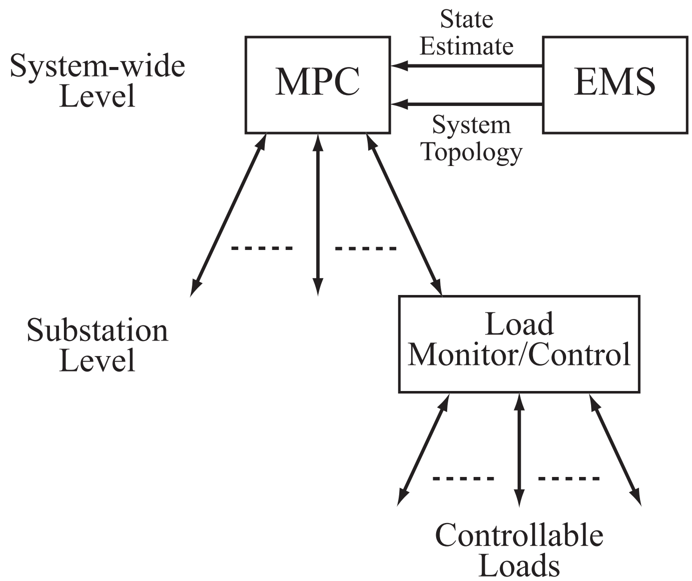


Figure 1: Hierarchical load control structure.

3 Wide-area Control

3.1 Local undervoltage load control

Load control provides an effective means of alleviating voltage collapse. For example the cascading failure of the North American power system in August 2003 could have been avoided by tripping a relatively small amount of load in the Cleveland area [10]. The most effective load shedding strategies are not always so obvious though. Low voltages often provide a good indication of locations where load shedding would assist in relieving system stress [1]. However counter-examples are easy to generate. The simple system of Figure 2 provides an illustration.

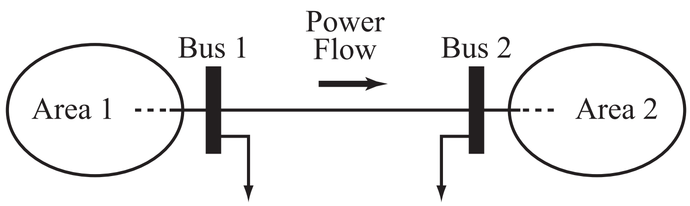


Figure 2: Illustration of inappropriate undervoltage load shedding.

Consider the situation where the power being exported from Area 1 to Area 2 overloads the corridor between buses 1 and 2. (This may be a consequence of line tripping between these buses.) As a result of the overload, lines forming the corridor will demand higher levels of reactive power, causing voltages at both end buses to fall. Undervoltage load shedding at bus 1, without a matching reduction in Area 1 generation, would actually lead to an increase in the power flow over the troublesome corridor, exacerbating the line-overload situation.

This is illustrated in Figure 3, where an initial line trip between buses 1 and 2 reduces the voltages. In this first case, undervoltage load shedding is set to trip 10% of the load at bus 1 whenever that bus voltage remains below 0.95 pu for 10 sec. Notice that each time a load block trips, the voltages at buses 1 and 2 are driven lower, and the system becomes more stressed. On the other hand, undervoltage load shedding at bus 2 achieves its desired goal, as shown in Figure 4.

Clearly situations arise where a coordinated approach to load shedding is required. A range of such load shedding schemes have been proposed and/or implemented, see for example [11, 12, 13]. This paper suggests an approach based on model predictive control.

3.2 Model predictive control

Model predictive control (MPC) is a discrete-time form of control, with commands issued at periodic intervals [14, 15]. Figure 5 illustrates the MPC process. Each control decision is obtained by first estimating the system state. This provides the initial condition for prediction (simulation) of subsequent dynamic behaviour. The prediction stage is traditionally formulated as an open-loop optimal control problem over a finite horizon. The solution of this optimal control problem provides an open-loop control sequence. MPC applies the initial control value from that sequence. The process is repeated periodically, with the state estimator giving a new initial condition for a new prediction (optimal control) problem.

The optimization problem underlying MPC involves open-loop prediction of system behaviour. Actual behaviour invariably deviates from that predicted response though. However

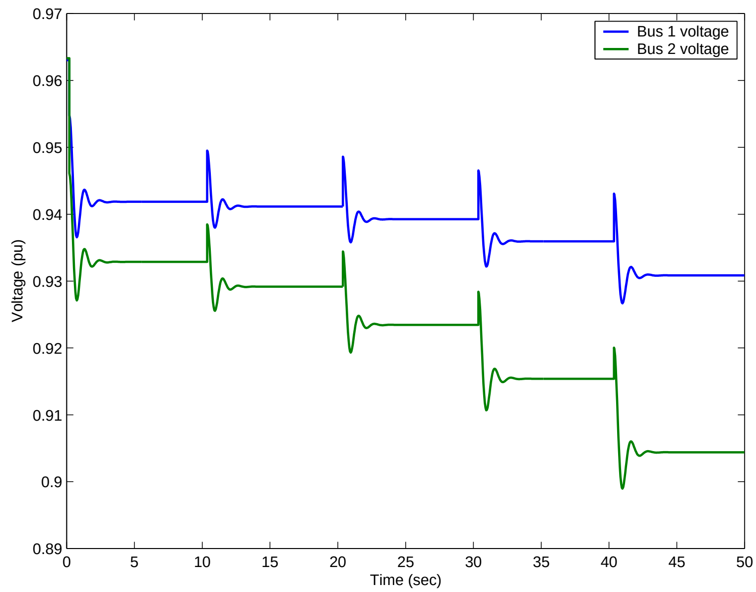


Figure 3: Undervoltage load shedding at bus 1.

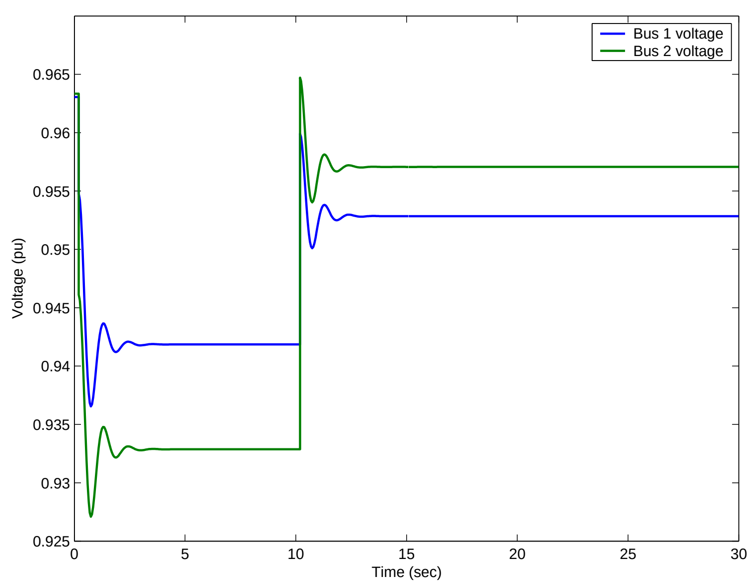


Figure 4: Undervoltage load shedding at bus 2.

feedback is effectively achieved through the correction applied when the next MPC control signal is issued. This is illustrated in Figure 5.

Power system dynamic behaviour often involves interactions between continuous dynamics and discrete events, particularly during voltage collapse when many discrete devices switch. Formulation of optimal control problems for such hybrid behaviour is fraught with technical difficulties. However it is shown in Appendix A that this problem may be approximated, through the use of trajectory sensitivities, as a linear (time-varying) discrete-time optimal control problem. Such formulations are explored thoroughly in [16].

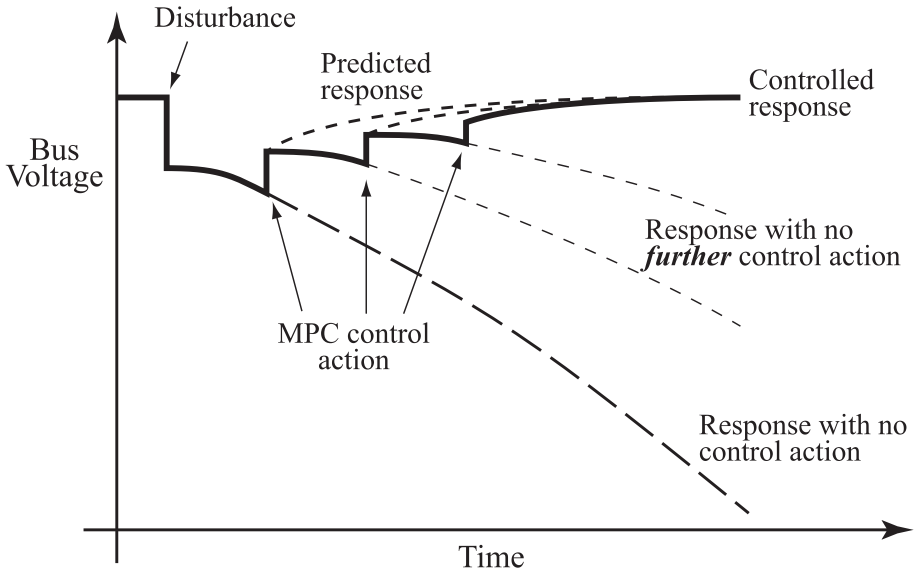


Figure 5: MPC response.

3.3 MPC implementation

3.3.1 Prediction of nominal system behaviour

For MPC to determine a control response at time t_k, prediction of future behaviour is necessary. The first step in this prediction process is the acquisition of an estimate of the current system state x_k. Power system state estimators are now capable of providing an estimate of power flow states (voltage magnitudes and angles) in under 30 sec. The internal states of dynamic components, such as generators and their controllers, can then be estimated from those power flow states. This process is similar to the initialization phase of standard simulation packages. The outcome is a complete, consistent, initial state estimate x_k.

Prediction also requires knowledge of the nominal control $\tilde{u}_k$ at time t_k. That nominal control is composed of two parts,

$$\tilde{u}_k = u_{k-1} + \Delta\tilde{u}_k \tag{1}$$

where u_{k-1} describes the actual load shed prior to t_k, which is provided by feedback from the substation-based controllers, and $\Delta\tilde{u}_k$ gives the initial guess for the change in the load shedding requirements at t_k. The aim of MPC is to optimally correct that initial guess. Accuracy of the initial guess is therefore not particularly crucial, with heuristics such as simple undervoltage strategies sufficing. In fact, it has been found that setting $\Delta\tilde{u}_k = 0$ is often quite acceptable.

Given x_k and $\tilde{u}_k$, simulation then generates the nominal system behaviour. Referring to Appendix A, behaviour can be expressed as the flow $x(t) = \phi(x_k, \tilde{u}_k, t)$, for $t \geq t_k$. Linearization around this nominal trajectory provides the time-varying linear model (22)-(23), from which the (open loop) optimal control updates $\Delta u_{k+i}, i = 0, ..., N-1$ are computed. These updates describe perturbations from the initial control $\tilde{u}_k$. MPC implements the new control signal $u_k = \tilde{u}_k + \Delta u_k$, or in other words adjusts the control by $\Delta\tilde{u}_k + \Delta u_k$ from the previous control u_{k-1}. Notice that even though the full sequence $\Delta u_{k+i}, i = 0, ..., N-1$ is computed by MPC, the control adjustment utilizes only the initial sample Δu_k from that optimal sequence.

The general model of Appendix A must be tailored to the specific requirements of non-disruptive load control. Those details are outlined in the following sections.

3.3.2 Load model

MPC implementation is not limited to any particular load model. It is important though that the effect of load control action is incorporated into the model. For example voltage dependent load could be modelled as

$$P(V, u) = (1 - u)P_0 \left(\frac{V}{V_0}\right)^{\alpha} \tag{2}$$

and similarly for reactive power $Q(V, u)$. No load shedding (full load) corresponds to $u = 0$, while complete load shedding is given by $u = 1$.[1] Emergency control requires periodic adjustment of u.

The MPC algorithm must take account of the limits on the amount of load that is available for control. Let the maximum amount of load that can be shed at a particular location j be u^j_{max}. Also, it will be assumed that once load is shed, it cannot be returned to service over the prediction horizon. Then at any interval over that horizon,

$$0 \leq \tilde{u}^j_k + \Delta u^j_k \leq \tilde{u}^j_k + \Delta u^j_{k+1} \leq ... \leq \tilde{u}^j_k + \Delta u^j_{k+N-1} \leq u^j_{max} \tag{3}$$

where $\tilde{u}^j_k$ is the nominal control, i.e., the initial guess for the desired load shedding, and Δu^j_{k+i} is the actual adjustment at time t_{k+i}. Note that this still allows previously shed load to be restored by a subsequent MPC operation. If that is not reasonable, i.e., there is a latching mechanism, then (1) implies replacement of (3) by

$$-\Delta\tilde{u}^j_k \leq \Delta u^j_k \leq \Delta u^j_{k+1} \leq ... \leq \Delta u^j_{k+N-1} \leq u^j_{max} - \tilde{u}^j_k. \tag{4}$$

It may also be appropriate to limit the load change at any interval, according to

$$\Delta u_{minchange} \leq \Delta u \leq \Delta u_{maxchange}. \tag{5}$$

For example, such limits may be necessary to avoid excessive voltage steps. All these limits can be combined together to give

$$\Delta u^j_{min} \leq \Delta u^j_k \leq \Delta u^j_{k+1} \leq ... \leq \Delta u^j_{k+N-1} \leq \Delta u^j_{max} \tag{6}$$

where Δu^j_{min} and Δu^j_{max} are the most stringent minimum and maximum limits respectively.

3.3.3 Voltage constraints

The aim of the MPC process is to shed just enough load that bus voltages recover to within acceptable voltage bounds $[V_l \; V_u]$. Driving voltages to within these bounds terminates the

[1]In fact, if the load were partially served by local distributed generation, it is (theoretically) possible for the bus to become a net exporter of energy, corresponding to $u > 1$. This will not be explored further though.

voltage collapse process. This requirement is implemented in the MPC optimization formulation by placing constraints on the voltages at the prediction horizon,

$$V_l \le V(t_{k+N}) \le V_u \tag{7}$$

where voltages V are a subset of the algebraic states y, and t_{k+N} is the time at the end of the prediction horizon.

3.3.4 MPC optimization

The MPC optimization process seeks to determine minimal load changes Δu^j that ensure voltage constraints (7) are satisfied. This results in a nonlinear, constrained, dynamic embedded optimization problem. An iterative process is required to solve such problems, with each iteration involving simulation over the prediction horizon. However by using the (approximate) model of Appendix A, the solution process can be substantially simplified. The voltages at the prediction horizon, required to ensure (7) is satisfied, become simply

$$V(t_{k+N}) \approx V_{nom}(t_{k+N}) + \Delta V_{k+N} \tag{8}$$

where V_{nom} describes the voltages predicted by simulating the nominal trajectory, and from (23),

$$\Delta V_{k+N} = C_{k+N} \Delta x_{k+N}. \tag{9}$$

The errors in this approximation will result in (slightly) sub-optimal load controls Δu being applied by MPC. However the effects of that sub-optimality only persist until the subsequent MPC cycle. At that time, the whole optimization process is repeated.

The objective of shedding the minimal amount of load can be expressed in the form $\min_{\Delta u} \sum_j \left| \tilde{u}_k^j + \Delta u_{k+N-1}^j \right|$. By observing (3) or (4), it is clear that the amount of load shed is always positive, so the objective can be restated

$$\min_{\Delta u} \sum_j (\tilde{u}_k^j + \Delta u_{k+N-1}^j). \tag{10}$$

The minimization is unaffected by $\sum_j \tilde{u}_k^j$, so the objective function can be further simplified. Collecting together the objective and constraints gives the linear programming (LP) problem

$$\min_{\Delta u} \sum_j \Delta u_{k+N-1}^j \tag{11}$$

subject to the linear model (22),(23) and inequality constraints

$$\Delta u_{min} \le \Delta u_k \le \Delta u_{k+1} \le \dots \le \Delta u_{k+N-1} \le \Delta u_{max} \tag{12}$$
$$V_l \le V_{nom}(t_{k+N}) + \Delta V_{k+N} \le V_u. \tag{13}$$

Such problems can be solved efficiently, even for very large sets of equations.

4 Example

4.1 System description

The small system of Figure 6 is well established as a benchmark for exploring voltage stability issues [1, 17, 18]. An outage of any one of the feeders between buses 5 and 7 results in voltage collapse behaviour. This is illustrated in Figure 7 for a line outage at 10 seconds. In response to the line trip, voltages across the right-hand network dropped. This caused load tap changers (LTCs) to respond in an attempt to restore load bus voltages. However tap changing actually drove voltages lower, resulting in voltage collapse.

Two situations were considered, 1) no over-excitation limiter (OXL) on generator 3 (solid red curve), and 2) inclusion of an OXL on generator 3 (dashed blue line.) Both exhibit undesirable voltage behaviour, though the OXL clearly induced a more onerous response. The reactive support provided by generator 3, for the two cases, is shown in Figure 8. The OXL ensures that reactive demand does not rise to a damaging level.

The studies presented subsequently explore the MPC model detail required to achieve adequate control. To enable this comparison, the system was modelled precisely. A sixth order model (two axes, with two windings on each axis) [19] was used for each generator, and IEEE standard models `AC4A` and `PSS1A` for all AVRs and PSSs respectively. The OXL model was taken from [18]. A standard induction motor model [18] was used for the industrial load at bus 8, and a static voltage dependent representation for the bus 9 load. The AVR of transformer LTC3 was represented by a model that captured switching events associated with deadbands and timers [20].

In all cases MPC was set to run every $T = 50$ seconds, with an horizon time of $2T = 100$ seconds. The control objective was to restore the voltages of buses 6 and 8 above 0.98pu by shedding minimum load at buses 8 and 9. (These two sets of buses were chosen to avoid symmetry between load-shed buses and voltage-regulated buses.) This objective was achieved by solving the LP optimization problem (11)-(13) for the corresponding values of Δu at each MPC step.

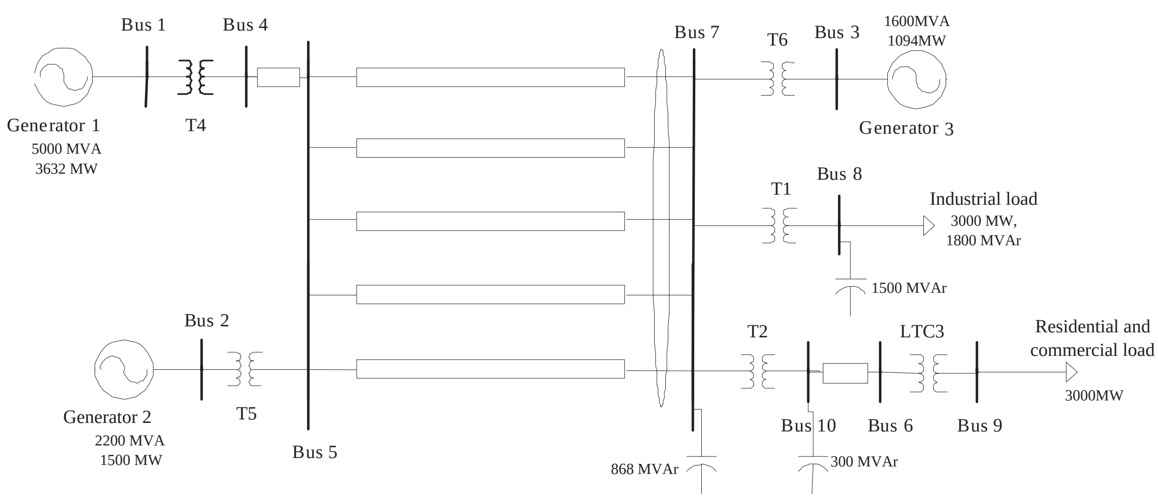


Figure 6: Voltage collapse test system.

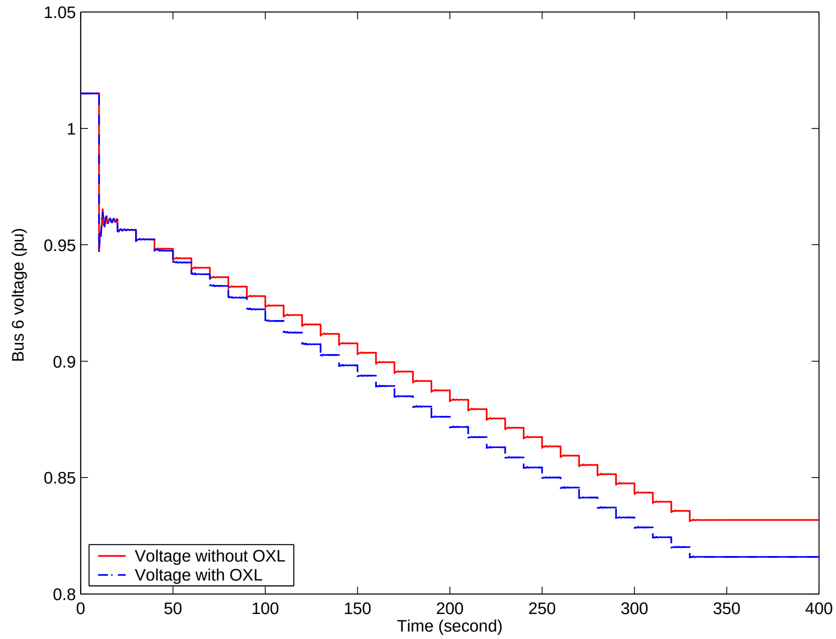


Figure 7: Voltage behaviour without MPC.

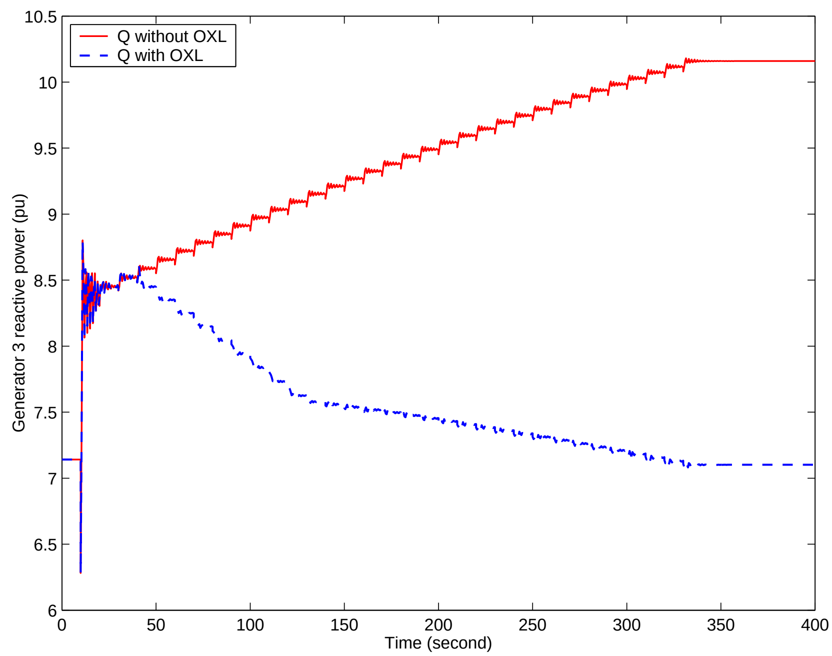


Figure 8: Reactive support provided by generator 3, without MPC.

4.2 Perfect MPC model

This initial investigation considered the ideal (though unrealistic) situation where the internal MPC model exactly matched the real system. The voltages at the regulated buses are shown in Figure 9. It is apparent that in response to the initial MPC load control command, both

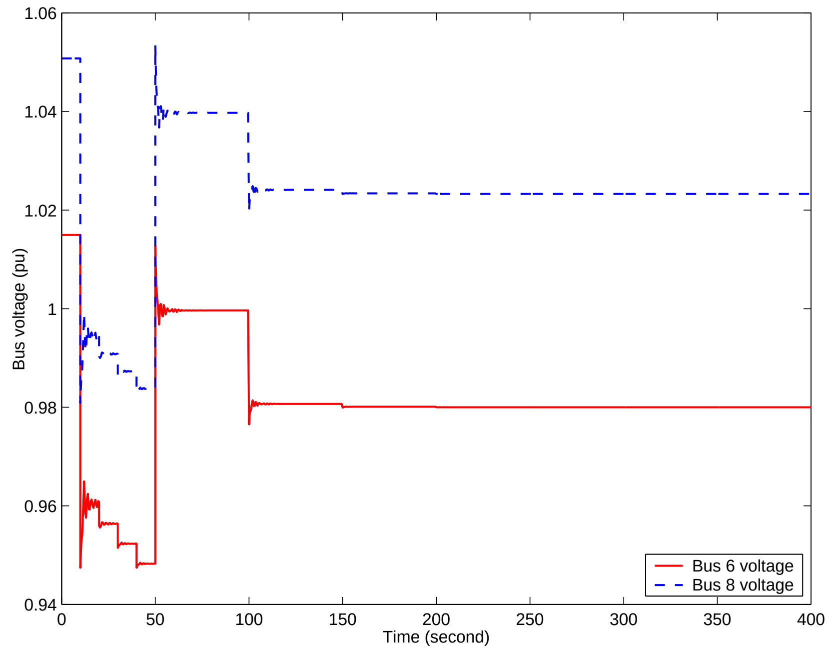
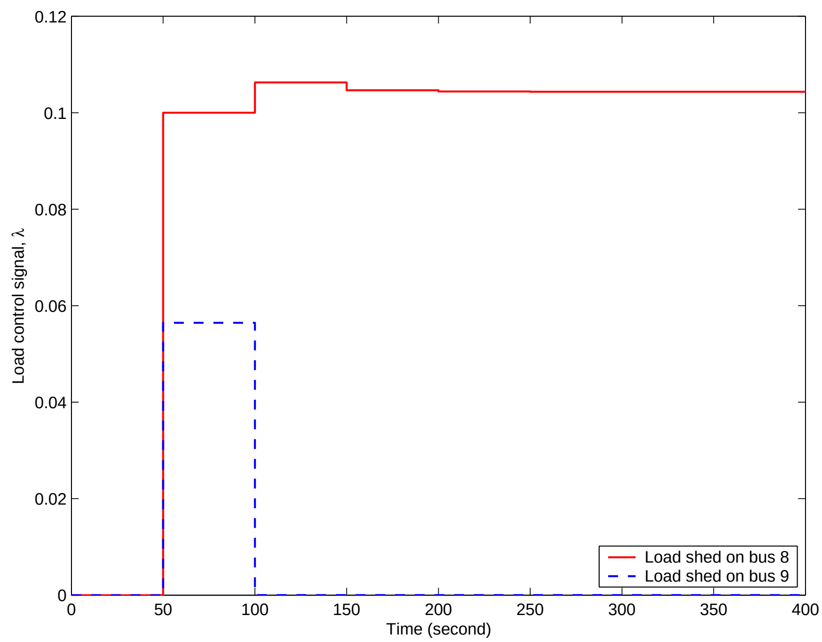


Figure 9: Voltage behaviour, perfect MPC model.



x

Figure 10: Load control signals, perfect MPC model.

voltages rose above their specified minimum values. The initial MPC command therefore over-compensated for the collapsing voltages by shedding too much load. This was a consequence of approximating perturbed trajectories in (13) using trajectory sensitivities. The voltage overshoot was corrected with the second MPC control command though, with the bus 6

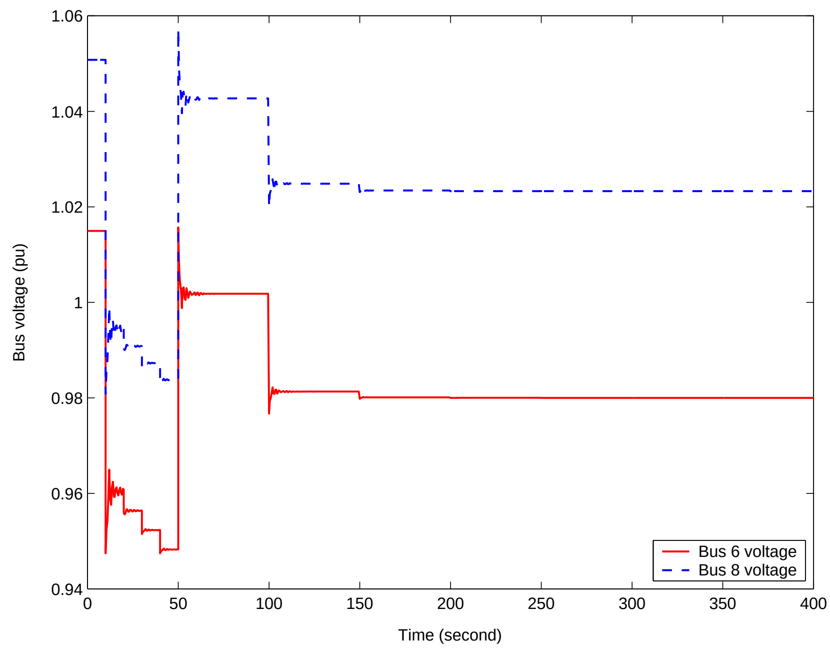


Figure 11: Voltage behaviour, imprecise load response.

voltage falling to its lower limit of 0.98 pu. At this step all of the bus 9 load was actually restored; see Figure 10 for the load shedding commands. Note that negligible MPC action is required beyond the second control interval.

4.3 Imprecise load response

The nature of non-disruptive load control means there will always be some uncertainty in the amount of load that is actually available for control. To investigate this situation, the load control signals generated by MPC were randomly perturbed by up to $\pm 10\%$. Figure 11 shows that performance was only slightly degraded.

4.4 Realistic implementation

It is unrealistic to expect that the MPC controller could maintain a complete, accurate system representation. To investigate this case, the MPC internal model was altered to make use of a simplified generator representation. Also the OXL was removed from the MPC model. Furthermore, load uncertainty was incorporated, as in Section 4.3. Voltage response and load control signals are shown in Figures 12 and 13 respectively. It is apparent that model approximation did not adversely affect the quality of MPC regulation.

These results are encouraging, though certainly not definitive. The degree to which MPC can tolerate model inaccuracy is core to practical power system implementation. This is the focus of on-going research.

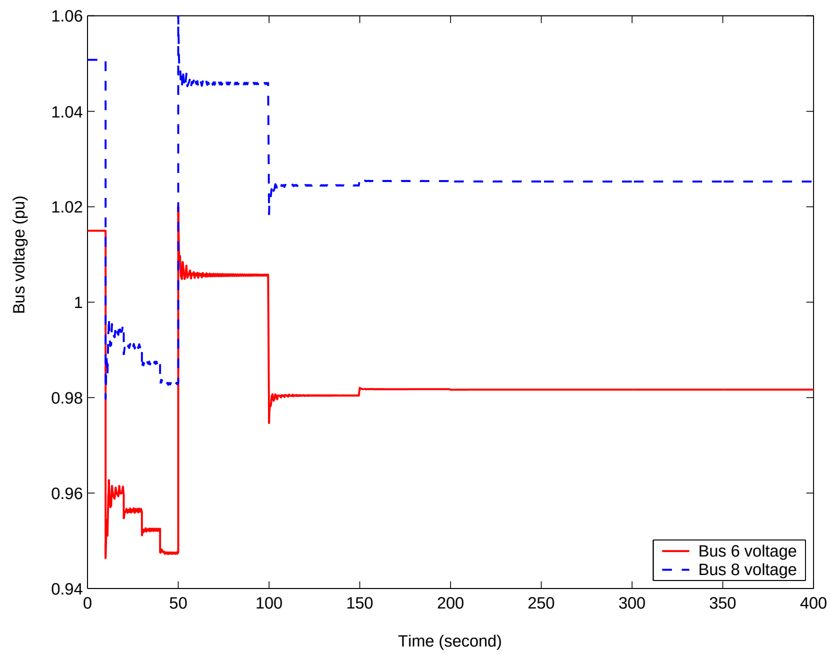


Figure 12: Voltage behaviour, approximate MPC model.

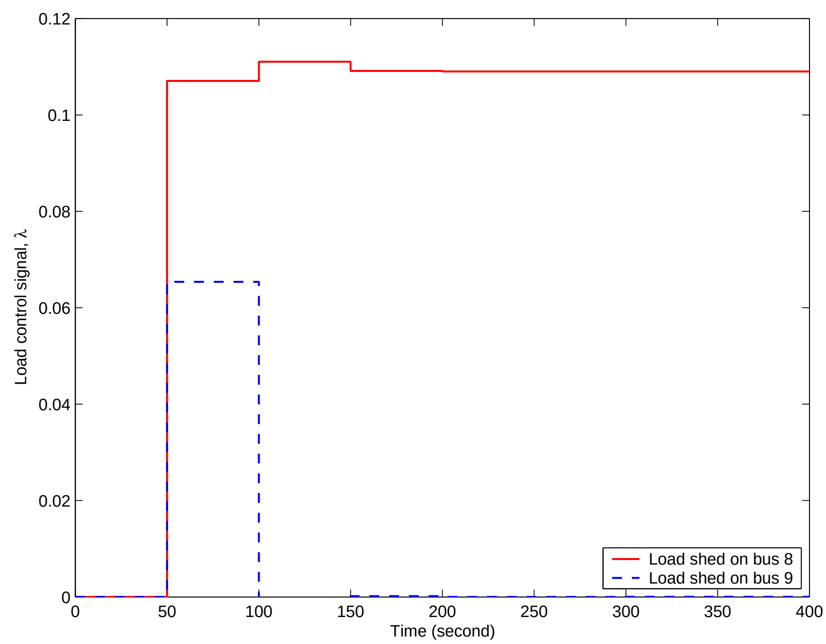


Figure 13: Load control signals, approximate MPC model.

5 Conclusions

Many consumer installations include load components that can be tripped with imperceptible effects over the short-term. Consolidation of such load fragments provides a non-disruptive load control capability that can be used to alleviate voltage collapse. The project has explored a hierarchical control structure consisting of a lower level controller (consolidator) that communicates with loads, together with a higher level controller that formulates coordinated responses to threats of voltage instability. It has been shown that model predictive control (MPC) provides a very effective higher-level control strategy.

MPC utilizes an internal model of the system to predict response to a disturbance. A dynamic embedded optimization problem is formulated to determine an appropriate load shedding response for restoration of voltages to acceptable levels. It has been shown that the use of trajectory sensitivities allows this optimization to be reduced to a linear programming problem, even though the system may exhibit complicated nonlinear non-smooth dynamics. This simplification, together with MPC model approximations, gives rise to discrepancies between predicted and actual system behaviour. However errors are corrected by periodic repetition of the MPC prediction/optimization algorithm. The MPC control strategy is therefore robust to significant levels of uncertainty.

These investigations suggest that the proposed hierarchical control strategy is practical for large-scale power system applications.

A Linear Discrete-Time System Model

A.1 Flows and trajectory sensitivities

An mentioned earlier, power system large disturbance response typically exhibits interactions between continuous dynamics and discrete events. Numerous models for such hybrid systems have been proposed [20, 21]. It is common for the continuous dynamics to be modelled using a differential-algebraic (DAE) representation. Discrete events are incorporated via impulsive mappings and switching within the DAE model.

Independent of the exact form of the underlying model, hybrid system dynamic behaviour can be described by the *flow*

$$x(t) = \phi(x_0, u_0, t) \tag{14}$$

together with the algebraic constraints

$$g(x(t), y(t), u_0) = 0 \tag{15}$$

where x and y are the dynamic and algebraic states respectively of the DAE model, x_0 is the initial value of x, so that $x_0 = x(0) = \phi(x_0, u_0, 0)$, and u_0 describes (constant) parameters. Examples of x states include generator fluxes, y states include bus voltages, and parameters include load magnitudes. It will be shown later that control is realized through piecewise variation of u_0.

The Taylor series expansion of (14) can be expressed as

$$\phi(x_0 + \Delta x_0, u_0 + \Delta u_0, t) = \phi(x_0, u_0, t) + \Phi_x(x_0, u_0, t)\Delta x_0 + \Phi_u(x_0, u_0, t)\Delta u_0 + \text{h.o.t.} \tag{16}$$

where $\Phi_x \triangleq \frac{\partial \phi}{\partial x_0}$ and $\Phi_u \triangleq \frac{\partial \phi}{\partial u_0}$ are trajectory sensitivities. Defining

$$\Delta x(t) = \phi(x_0 + \Delta x_0, u_0 + \Delta u_0, t) - \phi(x_0, u_0, t) \tag{17}$$

and neglecting higher order terms gives rise to

$$\Delta x(t) \approx \Phi_x(x_0, u_0, t)\Delta x_0 + \Phi_u(x_0, u_0, t)\Delta u_0. \tag{18}$$

Differentiating (15) results in

$$g_x \Delta x(t) + g_y \Delta y(t) + g_u \Delta u_0 = 0 \tag{19}$$

where $g_x \triangleq \frac{\partial g}{\partial x}$, $g_y \triangleq \frac{\partial g}{\partial y}$ and $g_u \triangleq \frac{\partial g}{\partial u}$.

It is shown in [22] that the trajectory sensitivities Φ_x, Φ_u are well defined for hybrid systems, provided the underlying flow ϕ is well defined. (This excludes phenomena such as algebraic singularity, sliding modes and Zeno effects.) Furthermore, if simulation utilizes an implicit numerical integration process, then very efficient computation of these sensitivities is possible [22, 23].

A.2 Model formulation

Based on the flow concept presented in Section A.1, prediction of behaviour forward from time t_k is possible with knowledge of the state $x_k \triangleq x(t_k)$, and control $u_k \triangleq u(t_k)$. Let T be the period associated with MPC operation, and NT the prediction horizon. Then

$$\begin{aligned} x_{k+N} \triangleq x(t_k + NT) &= \phi(x_{k+N-1}, u_k, T) \\ &= \phi(x_{k+N-2}, u_k, 2T) \\ &\ \vdots \\ &= \phi(x_k, u_k, NT) \end{aligned}$$

with the corresponding y given by $g(x_{k+i}, y_{k+i}, u_k) = 0$. In other words, the nominal discrete-time trajectory $(x_k, y_k), (x_{k+1}, y_{k+1}), ..., (x_{k+N}, y_{k+N})$ can be obtained by sampling the simulation that begins at the initial value x_k, and that runs for time NT. This trajectory is nominal in the sense that control is held constant at its initial value u_k. The aim of MPC is to determine control adjustments $\Delta u_k, \Delta u_{k+1}, ..., \Delta u_{k+N-1}$ that achieve desired behaviour in an optimal way.

It follows from (18) that perturbations from the sampled nominal trajectory are given by

$$\Delta x_{k+i+1} = \Phi_x(x_{k+i}, u_k, T)\Delta x_{k+i} + \Phi_u(x_{k+i}, u_k, T)\Delta u_{k+i}, \quad 0 \leq i \leq N-1, \tag{20}$$

where we have used the fact that $u_k = u_{k+1} = ... = u_{k+N-1}$. Deviations in algebraic states follow from (19),

$$\Delta y_{k+i} = -g_y^{-1}\big(g_x \Delta x_{k+i} + g_u \Delta u_{k+i}\big) \tag{21}$$

where g_x, g_y and g_u are all evaluated at t_{k+i}. (It is assumed that $\Delta u_{k+N} \equiv 0$.)

The linear time-varying discrete-time model therefore becomes

$$\Delta x_{k+i+1} = A_{k+i}\Delta x_{k+i} + B_{k+i}\Delta u_{k+i} \tag{22}$$

$$\Delta y_{k+i} = C_{k+i}\Delta x_{k+i} + D_{k+i}\Delta u_{k+i} \tag{23}$$

where the definitions of A, B, C and D follow directly from (20) and (21). This formulation relates quite closely to [16], though their starting point was a discrete-time model.

As mentioned earlier, implicit numerical integration allows efficient computation of Φ_x and Φ_u. Furthermore, such integration techniques require the formation of g_x, g_y and g_u, and factorization of g_y. Therefore the model (22),(23) can be compute efficiently, even for large-scale systems such as power systems.

It should be emphasized that the linear model (22),(23) is not a linearization around an equilibrium point, but rather a linearization around a (possibly) large disturbance nonlinear, non-smooth trajectory. During periods of normal power system operation though, when the system is close to equilibrium, the properties of trajectory sensitivities [22] ensure that the model (22),(23) effectively reverts to a time-invariant linearization around the equilibrium point. This model is therefore suited to both small disturbance regulation and large disturbance emergency control.

References

[1] C. Taylor, "Concepts of undervoltage load shedding for voltage stability," *IEEE Transactions on Power Delivery*, vol. 7, no. 2, pp. 480–488, April 1992.

[2] Cigré Working Group 37.03, "The potential impact of demand-side management on future electricity demand," *Electra*, vol. 138, pp. 127–145, October 1991.

[3] W. Uturbay, A. Simões Costa, A. De Marco, S. Nakanishi, J. Schier, A. Cassano, and S. Pacheco, "A load management program for large consumers of a distribution company," in *Proceedings of the IX SEPOPE*, Rio de Janeiro, Brazil, May 2004.

[4] M. Ilic, J. Black, and J.L.Watz, "Potential benefits of implementing load control," in *Proceedings of the PES Winter Meeting*, vol. 1, January 2002, pp. 177–182.

[5] B. Williams, W. Schmus, and D. Dawson, "Transmission voltage recovery delayed by stalled air conditioner compressors," *IEEE Transactions on Power Systems*, vol. 7, no. 3, pp. 1173–1181, August 1992.

[6] E. Agneholm and J. Daalder, "Cold load pick-up of residential load," *IEE Proceedings: Generation Transmission Distribution*, vol. 147, no. 1, pp. 44–50, January 2000.

[7] J. Eto, C. Goldman, G. Heffner, B. Kirby, J. Kueck, M. Kintner-Meyer, J. Dagle, T. Mount, W. Schultz, R. Thomas, and R. Zimmerman, "Innovative developments in load as a reliability resource," in *Proceedings of the IEEE PES Winter Meeting*, vol. 2, New York, January 2002, pp. 1002–1004.

[8] A. Agogino, "Intelligent commercial lighting: demand-responsive conditioning and increased user satisfaction," UC Energy Institute, Berkeley, CA, 2005.

[9] D. Backer, "Technologies for fast load control," in *Proceedings of the IEEE PES Winter Meeting*, New York, January 2002, pp. 999–1000.

[10] U.S.-Canada Power System Outage Task Force, *Final Report on the August 14, 2003 Blackout in the United States and Canada: Causes and Recommendations*, April 2004.

[11] S. Arnborg, G. Andersson, D. Hill, and I. Hiskens, "On undervoltage load shedding in power systems," *Electrical Power and Energy Systems*, vol. 19, no. 2, pp. 141–149, 1997.

[12] T. Van Cutsem and C. Vournas, *Voltage Stability of Electric Power Systems.* Kluwer Academic Publishers, 1998.

[13] C. Moors, D. Lefebvre, and T. Van Cutsem, "Load shedding controllers against voltage instability: A comparison of designs," in *Proceedings of the 2001 IEEE Porto Power Tech Conference*, Porto, Portugal, September 2001.

[14] J. Rawlings, "Tutorial overview of model predictive control," *IEEE Control Systems Magazine*, vol. 20, no. 3, pp. 38–52, June 2000.

[15] R. Findeisen, L. Imsland, F. Allgöwer, and B. Foss, "State and output feedback nonlinear model predictive control: an overview," *European Journal of Control*, vol. 9, pp. 190–206, 2003.

[16] Y. Lee, B. Kouvaritakis, and M. Cannon, "Constrained receding horizon predictive control for nonlinear systems," *Automatica*, vol. 38, pp. 2093–2102, 2002.

[17] C. Taylor (convenor), "Modeling of voltage collapse including dynamic phenomena," CIGRE TF38-02-11, Brochure No. 75, 1993.

[18] P. Kundur, *Power System Stability and Control.* EPRI Power System Engineering Series, McGraw Hill, 1994.

[19] P. Sauer and M. Pai, *Power System Dynamics and Stability.* Upper Saddle River, NJ: Prentice Hall, 1998.

[20] I. Hiskens, "Power system modeling for inverse problems," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 51, no. 3, pp. 539–551, March 2004.

[21] P. Barton, "Modeling, simulation and sensitivity analysis of hybrid systems," in *Proceedings of the 2000 IEEE International Symposium on Computer-aided Control System Design*, Anchorage, AK, September 2000, pp. 117–122.

[22] I. Hiskens and M. Pai, "Trajectory sensitivity analysis of hybrid systems," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 47, no. 2, pp. 204–220, February 2000.

[23] T. Maly and L. Petzold, "Numerical methods and software for sensitivity analysis of differential-algebraic systems," *Applied Numerical Mathematics*, vol. 20, pp. 57–59, 1997.