Tools for Assessment of Bidding into Electricity Auctions

Final Project Report

Power Systems Engineering Research Center

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Final Project Report

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Power Systems Engineering Research Center

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Executive Summary

In many restructured electricity markets, transactions occur through frequently-repeated uniform-price auctions. For example, the Electric Reliability Council of Texas (ERCOT) balancing market, and the day-ahead and real-time markets in the Northeast and Midwest U.S. use uniform-price auctions. Such market mechanisms are justified, in part, by theoretical models that suggest these auctions facilitate efficient dispatch and send "correct" signals for future investment.

However, empirical analyses of offers into electricity spot auctions have uncovered evidence that actual offers by some market players can deviate significantly from theoretical models of profit-maximizing offers. In particular, smaller players face considerable uncertainty and appear to avoid participation in these markets, even when participation would increase profits and reduce system dispatch costs. Existing evidence from ERCOT suggests that these "sub-optimalities" lead to dispatch inefficiency in the balancing market. Unfortunately, these inefficiencies propagate outside the balancing market – many bilateral transactions are linked to the balancing price, and the balancing price affects investment signals.

This project developed a computational tool for analyzing offers into auctions. The tool has two major applications. First, market monitors can employ it to assess the competitiveness and efficiency of offers by comparing the actual offer of a market participant to a hypothetical perfectly competitive offer and to an ex post profit maximizing offer. Such analysis can be useful to market monitors who seek to evaluate the behavior of a particular market participant. Second, the tool can assist market participants, especially the smaller ones, in the formulation of offers in the face of the strategic complexity facing them. The tool and associated graphical user interface allows the ex post profit maximizing offer for a firm to be constructed on the basis of information about the aggregate offers of other market participants, the firm's own cost function, and zonal transmission constraints.

This project brings together work from both the economics and electrical engineering literatures. The tool operationalizes some of the recent theoretical developments in auction theory in a prototype tool that is available in the public domain to analyze offers. Moreover, the tool incorporates the effects of transmission constraints, providing a unique combination of economic and engineering analyses that is not available elsewhere. Finally, we sought to make the tool "user friendly" so an analyst or market participant can exploit the insights of recent academic analysis in the "real-time" time horizon that such analysts typically make decisions.

The academic grade tool is written in the Java language, and is released as open-source software under the terms of the GNU General Public License (GPL). If you are a PSERC industry member and would like to receive a copy of the tool in a .zip file, please send an email to either puller@econmail.tamu.edu or Ross.Baldick@engr.utexas.edu and we will be happy to send it to you. In the future, the tool may be publicly available. The example case study is based on the ERCOT zonal electricity balancing market.

Future work could add features to the existing tool that analyze other outcomes that are important to the efficient design of restructured electricity markets. For example, the tool could be expanded to calculate prices under scenarios such as: (a) firms exercise market power but there is a small expansion in transmission capacity, and (b) firms bid competitively. In addition, the tool could be modified to estimate the effect of a single firm's bids on the overall dispatch costs. Yet another area for future research is to tailor the tool to other markets with similar features in the procurement process. While the tool is specifically tailored to ERCOT's zonal market, the analytical technique below can be applied elsewhere. Finally, future research could extend the theoretical underpinnings to consider nodal markets.

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1 Introduction

In many restructured electricity markets, transactions occur through frequently-repeated uniform-price auctions. For example, the ERCOT balancing market and the day-ahead and real-time markets in the Northeast and Midwest US use uniform-price auctions. Day-ahead nodal markets with uniform-pricing are planned for California and for ERCOT. Such market mechanisms are justified, in part, by theoretical models that suggest these auctions facilitate efficient dispatch and send "correct" signals for future investment.

There has been considerable theoretical work aimed at modeling such markets, including Green (1999) Borenstein, Bushnell, and Stoft (2000), Baldick, Grant and Kahn (2004), Baldick and Hogan (2002 and 2004), and Hortacsu and Puller (2008). Empirical work includes Wolak (2003), Hortacsu and Puller (2008), Niu, Baldick and Zhu (2005), and Sioshansi and Oren (2006).

Empirical analyses of bidding in electricity spot auctions has uncovered evidence that actual bidding by some market participants leads to prices that distort the proper price signal. Generators that offer to generate at significantly above the incremental marginal cost of generation can inflate prices and send distorted signals as to the proper amount and location of new investment. Similarly, in balancing markets, firms that bid to reduce generation at prices that are below marginal cost can suppress energy prices and can therefore discourage investment. Evidence of both types of behavior has been found in several markets.

This report describes the development of theory and an academic grade tool that market monitors can utilize to access the degree to which specific market participants distort the efficient price signal. The tool calculates, for any individual market participant, what would have been the "best response" by the participant to the offers of all other market participants. "Best response" is from the perspective of maximizing the operating profit of the market participant, assuming that the offers of the other market participants are known to this market participant. The tool is not dependent on the way that the market participant would estimate the offers of its competitors. For simplicity, in this project, we use historical data and assume that the offers of the competitors remain the same. In this case, the tool can calculate, for each individual market participant, its ex post profit maximizing offer into the market. The ex post maximum profit is therefore an upper bound on the profit that could have been achieved in practice without the perfect foresight of knowing the offers of the other market participants.

The tool improves the ability of market monitors to test for market power, by enabling the market monitor to measure both the ability of a market participant to exercise market power and, through comparison with the actual offer, the exercise of that potential market power. That is, comparison of an actual offer to the polar extremes of a competitive offer and to the ex post profit maximizing offer allows assessment of the degree to which a market participant is exercising its market power. (In some cases, as discussed in Hortacsu and Puller (2008), offers may involve lower profits than a competitive offer, with the participant effectively offering in such a manner as to stay out of the market.) This tool is an improvement over current methods such as tests of whether certain generators are pivotal suppliers in the market because it explicitly considers the incentives to distort prices and explicitly considers the variability of realized demand.

Moreover, the tool aids in determining whether high prices reflect market power or scarcity rents by distinguishing competitive from supra-competitive profits. It is important that market monitors be able to make this distinction for several reasons. In particular, identifying and mitigating market power is important to ensure markets are competitive; however, allowing generators to recover true scarcity rents is necessary to promote resource adequacy.

A secondary benefit of this project is that the tool can increase market efficiency by facilitating increased participation in the balancing market. Analysis of the ERCOT balancing market has found that system dispatch costs in the balancing market are significantly increased because some small players do not participate in the balancing market. This lack of participation makes the market less competitive and causes efficient generators to be underutilized.

Hortacsu and Puller (2008), Sioshansi and Oren (2006), and Niu, Baldick, and Zhu (2005) identify the extent to which smaller generators avoid the ERCOT balancing market. For example, Hortacsu and Puller (2008) find that the considerable amount of price uncertainty in the market combined with a lack of trading experience discourage bidding even when participation would increase profits and reduce system dispatch costs. Hortacsu and Puller estimate that limited participation by smaller generators increases the dispatch costs in the balancing market by 22%. Unfortunately, these inefficiencies propagate outside the balancing market since many bilateral transactions are linked to the balancing price, and the balancing price affects investment signals.

The tool developed in this project can encourage smaller market participants to participate in the market by providing a conceptual and practical framework to formulate offers, evaluate offer strategies, and reduce uncertainty about market prices. Increased participation by smaller players can yield a “double dividend” – by better utilizing efficient generation owned by small firms and by reducing the potential market power of larger firms.

To summarize, the tool developed in this project provides market monitors with a framework to evaluate the offer behavior of individual market participants. It can measure the effect of specific behavior on prices and on the cost of dispatch. Also, it can help to decompose prices into market power and scarcity rents. Such a tool could be applied to many auction-based markets such as PJM, ISO-NE, NYISO, MISO and ERCOT.

The tool also provides smaller market participants with a framework to assess their offers into the market and so potentially increase their willingness to participate in auction-based electricity markets. Increased participation by small bidders is very likely to increase the competitiveness of these markets.

The tool is implemented in Java and requires information about offers into the market, a generation firm's costs, and the transmission constraints. It analyzes and displays the ex post optimal offer by the firm into the market, providing a summary of market outcomes. If you are a PSERC industry member and would like to receive a copy of the tool in a .zip file, please send an email to either puller@econmail.tamu.edu or Ross.Baldick@engr.utexas.edu and we will be happy to send it to you. In the future, the tool may be publicly available.

The example case study is based on the ERCOT zonal electricity balancing market. Future work includes improvements to the graphical user interface to improve usability and the extension of the theoretical underpinnings to consider nodal markets. The Java code is tailored to the formats for data on the ERCOT website. Future application in other markets would require some modification of this code.

The outline of the rest of this report is as follows. Section 2 describes the theoretical approach to calculating the ex post optimal offer. Section 3 describes the implementation in Java, including details about input and outputs. Section 4 presents a case study based on ERCOT. Section 5 contains the conclusions.

2 Theoretical approach to ex post optimal offer calculation

In this section, the theoretical approach to ex post optimal offer calculation is outlined, first in the absence of transmission constraints. Then the extension to consider transmission constraints is considered, including the detailed theoretical analysis in the case of transmission constraints.

2.1 No transmission constraints

In the absence of transmission constraints, the calculation of the ex post optimal offer of each firm is conceptually straightforward. We will discuss the situation from the perspective of a market monitor that wants to assess the offer of each market participant in turn. Suppose that the market monitor has access to data on system load, the offers of each generator, and an estimate of each firm's marginal cost of generation. Marginal cost data can be gathered from commercially available data on heat rates and the spot price of fuel.[1] (The situation for a particular firm assessing its own ex post optimal offer is similar, and only requires industry-wide offer data, the offer data relevant to the particular firm, the forward contract position, and the marginal cost data that is relevant to the particular firm.)

Using these data, the market monitor can use economic theory to predict each firm's offer if the firm were exercising unilateral market power. Comparing this theoretical offer to the actual offer and to a hypothetical competitive offer based on marginal costs provides an assessment of the competitiveness of the actual offer.

Figure 1 illustrates the economics of constructing these theoretical offer functions in detail. Firm i's marginal cost curve is given by $MC_i(q)$, and its forward contract position is labeled at QC_i.

[1] For example, Platts and Henwood provide these data and some market monitors subscribe to such services.

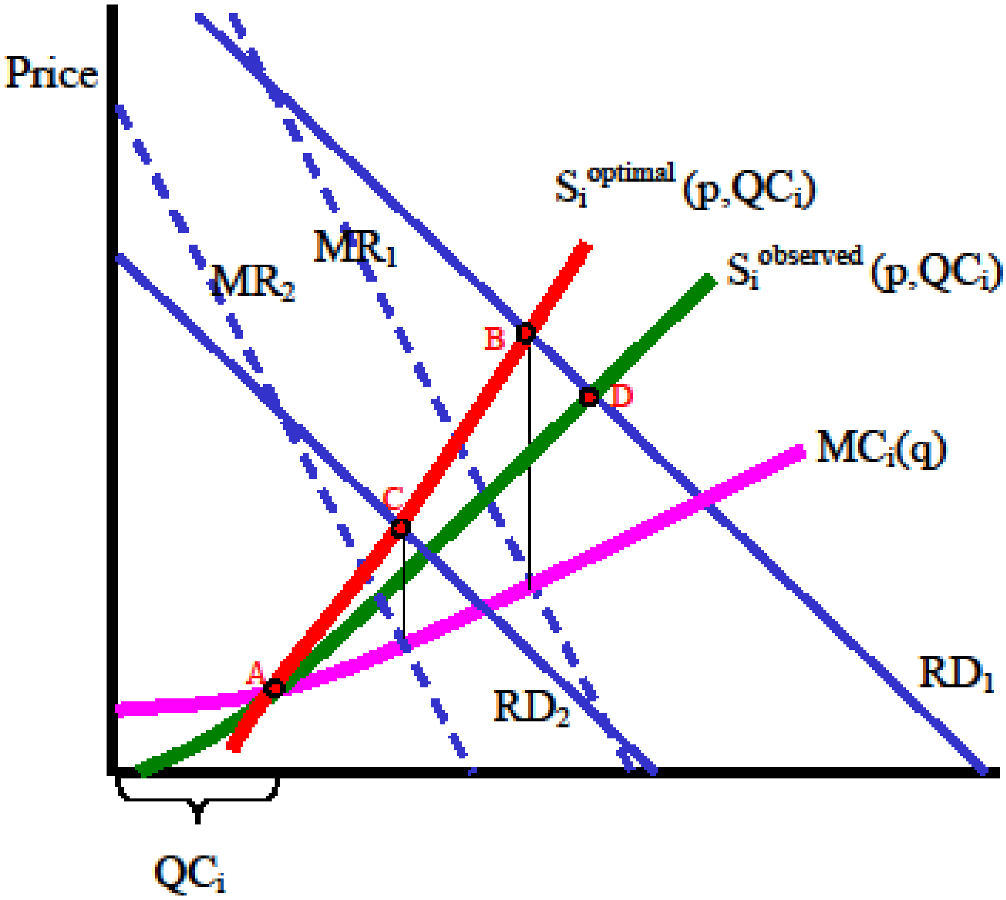


Figure 1 Example of constructing optimal offer function.

Referring to Figure 1, suppose that Firm *i* is observed to submit the supply schedule $S_i^{observed}$ (p,QC_i) in the balancing market. As explained in Hortacsu and Puller (2008), the forward contract position can be estimated as the quantity corresponding to the intersection of the observed supply and the marginal cost curve.

The market clearing price in the balancing market, and the actual amount of electricity that firm *i* will be called upon to generate, will be determined by the intersection of $S_i^{observed}$ (p,QC_i) and the residual demand (*RD*) curve faced by firm *i*. The *RD* curve is the sum of the supply schedules submitted by firms other than *i*, subtracted from the total market demand for electricity. The *RD* curve is uncertain from the perspective of firm *i* (since it depends on the realization of aggregate demand and rivals' bids).

Given a particular realization, RD_1, and given its actual supply schedule, $S_i^{observed}$ (p,QC_i), firm *i* would supply the quantity at the price given at point *D*. At this quantity, the firm would supply more electricity than it was previously contracted to sell (*D* is to the right of *A*), and its profits can be calculated as the profits from meeting its contract position, plus the profit made from providing additional power to the market.

However, firm i could have increased its profits if it had instead submitted a different offer. For the residual demand curve RD_1, firm i could calculate the marginal revenue curve given by MR_1. By equating marginal revenue and marginal cost (the solution to the first-order necessary condition for profit-maximization), the firm could then select point B, which maximizes its profits. This, of course, is what would happen if firm i knew that residual demand would be RD_1.

However, the residual demand may instead be RD_2. Firm i could then calculate the marginal revenue curve corresponding to this realization of residual demand, and find the profit-maximizing point C. This process can be repeated for various possible realizations of the residual demand, under the assumption that the different realizations are due to variation in demand quantity that are independent of price; that is, under the assumptions that the various realizations correspond to parallel shifts in the residual demand curve. By calculating the set of profit-maximizing points for all such parallel shifts in the residual demand curve, corresponding to various possible realizations of the demand, a bid function can be constructed that connects each of the points and maximizes profits under each possible realization of uncertainty, $S_i^{optimal}$.[2]

This theoretical bid function $S_i^{optimal}$ serves as a benchmark for exercising unilateral market power. If a specific firm is bidding very close to this benchmark, the market monitor can identify the firm as exercising market power. If the firms are bidding closer to marginal cost, the firm is behaving more competitively. For example, the firm in Figure 1 (with bid given by $S_i^{observed}$) is bidding above marginal cost but not fully exercising market power.

2.2 Transmission constraints

The methodology described in the last section can be expanded to incorporate transmission congestion, so it can be applied to markets with either zonal or nodal pricing. In particular, if the residual demand in the discussion so far is replaced by a calculation of the transmission-constrained residual demand at a zone or a bus, then a profit-maximizing offer can be calculated for firm i located in that zone or bus. As will be discussed below, the transmission-constrained residual demand can be calculated if the supply bids in each zone together with the demand in each zone are known. The residual demand for firm i, evaluated at a particular market clearing condition, can be calculated from the results of transmission-constrained economic

[2] See Hortacsu and Puller (2008) for a proof that this bid function is a solution to the equilibrium bid of a multi-unit, uniform-price auction under uncertainty regarding total load and rival contract positions. They prove that under reasonable assumptions about uncertainty, the set of profit-maximizing points can be connected with a monotonic function. Empirical tests using historical bid data suggest that this assumption very nearly holds for the ERCOT balancing market.

dispatch. The full residual demand function can be traced out by varying the power injected by firm *i* at its zone or bus.

There are some complications in practice, however. The first is that as injection at a zone (or bus) varies, the congestion status of a line in the system may vary. For example, suppose that a zone is a net importer. Then for small values of injection by the firm in that zone it may be the case that there is a binding transmission constraint on imports. In this case, the residual demand for the zone will be steep: small changes in injected quantity lead to large changes in price. At higher levels of injection the line will be unconstrained and the residual demand will be less steep. The implication is that the profit-maximizing optimal offer may not be monotonically increasing. That is, due to the endogenous congestion status, there may be no single monotonic supply function that can be constructed ex ante from the bids of the other firms that will maximize profits under all realized outcomes of the market.

The lack of a monotonic best response is a significant issue from a theoretical perspective. However, experience with generator capacity constraints, which in several ways are analogous to transmission capacity constraints in this context, suggests that this issue may not be very significant in practice (see Baldick and Hogan (2002)). As will be discussed below, the empirical results of the ERCOT example case bear this out.

The second practical complication is that if a firm owns generation capacity in multiple zones or at multiple buses, then the residual demands depend on injections by the firm in these multiple zones or buses. This complicates the calculation by requiring calculations of what are, essentially, cross-elasticities between various zones or buses. In a nodal market, it is likely that most firms would own generators at several buses. In the context of a tool for small firms in the ERCOT zonal market, however, the assumption of a firm being wholly within a particular zone is not unrealistic. The development in the balance of this report assumes that a firm is wholly within a zone.

To summarize, the calculation of the optimal response for the case of no transmission congestion can be extended to the case of congestion under assumptions that are likely to be reasonable for ERCOT in practice. The analysis under these assumptions is carried out in the following subsections.

2.3 Transmission-constrained optimal offer

For simplicity, assume the market participant under consideration does not have any forward bilateral contracts. The transmission-unconstrained optimal offer in a supply function model is characterized as:

$$S_i(p_i) = -\left(p_i - C_i'\left(S_i(p_i)\right)\right)R_i'(p_i), \tag{1}$$

where:

- p_i is the locational marginal price for the generation firm located at bus i,
- $C_i'(\bullet)$ is the marginal cost function of the generation firm located at bus i,
- $S_i(\bullet)$ is the optimal supply function of the generation firm located at bus i; that is, the inverse of its optimal offer,
- $R_i'(\bullet)$ is the residual demand derivative faced by the generation firm located at bus i.

For generation firms, an individual generation firm's optimal supply (and offer) function depends on its residual demand derivative (RDD) $R_i'(p_i)$ as characterized in (1), so the residual demand derivative plays a key role in constructing optimal bidding strategies. If the residual demand derivative is available, the optimal offer strategy can be constructed based on (1) by evaluating at price p_i the supply quantity $S_i(\bullet)$ that satisfies (1). By evaluating the residual demand derivative at various prices, the optimal supply function can be found. The inverse of the optimal supply function is the optimal offer function.

2.4 Residual demand derivative calculation

The theoretical underpinning of the transmission-constrained residual demand calculation was carried out as part of this project and reported in Xu and Baldick (2007). The following paraphrases the analysis in Xu and Baldick (2007). Further details, including illustrative examples, are reported in Xu and Baldick (2007).

The offer-based market is cleared by solving the following simplified DC Optimal Power Flow (OPF) problem:

$$\min_{\mathbf{q}} \quad \sum_{i=1}^{n} O_i(q_i), \tag{2}$$

$$\text{s.t.} \quad \mathbf{Hq} \le \mathbf{Z}, \tag{3}$$

$$q_n^{\min} \le q_n \le q_n^{\max}, \tag{4}$$

$$\mathbf{1}^T \mathbf{q} = \mathbf{0}, \tag{5}$$

where:

- $O_i\left(q_i\right)$ is the net offer function at bus *i*; that is, to simplify notation, we have combined each generation firm's offer function with the demand curve at the same bus by treating demand as negative supply,
- bus n is the slack bus,
- $\mathbf{q}=\left[q_1 \quad q_2 \quad \dots \quad q_n\right]^T$ is the nodal power injection quantity vector,
- (3) consists of the transmission constraints and the generation capacity constraints for non-slack buses (suppose there are totally m of them),
- $\mathbf{H}$ is an $m \times n$ matrix consisting of the submatrix of power transfer distribution factors (PTDFs) corresponding to the transmission constraints and the submatrix representing the capacity constraints for non-slack buses,
- $\mathbf{Z}$ consists of the transmission capacity limits and the generation capacity limits for non-slack buses,
- $\mathbf{1}=\underbrace{\left[1 \quad 1 \quad \dots \quad 1\right]}_{n}^T$,
- (4) is the generation capacity constraint, that specifies the upper and lower limits of the domain of the offer cost function at the slack bus, and
- (5) is the energy balance constraint.

Without loss of generality, we calculate the RDD for the slack bus, *n*, as a whole, i.e. the local actual demand at bus n has been combined with the supply at the same bus.

The RDD is a function of the LMP at the slack bus, $\hat{\lambda}$,

$$\frac{d\tilde{R}_n}{d\lambda}\left(\hat{\lambda}\right)=-\overline{\mathbf{1}}^{\mathrm{T}}\mathbf{\Lambda}\overline{\mathbf{1}}+\overline{\mathbf{1}}^{\mathrm{T}}\mathbf{\Lambda}\overline{\mathbf{H}}_{\mathrm{b}}^{\mathrm{T}}\left(\overline{\mathbf{H}}_b\mathbf{\Lambda}\overline{\mathbf{H}}_{\mathrm{b}}^{\mathrm{T}}\right)^{-1}\overline{\mathbf{H}}_{\mathrm{b}}\mathbf{\Lambda}\overline{\mathbf{1}}, \tag{29}$$

where $\overline{\mathbf{H}}_b$ is the PTDF matrix for binding transmission constraints and generation capacity constraints evaluated at the solved DC OPF. By varying load at bus *i*, the RDD at various prices can be evaluated, providing the data needed for constructing the optimal offer.

3 Implementation of the Java tool

This section discusses the implementation of the Java tool. Figure 2 shows the overall architecture of the tool. Offer data and transmission data is obtained from web sources. This data is used as input for the DC OPF calculation, which provides information for calculating the residual demand derivative. Then the residual demand derivative together with marginal cost and contract data is used to construct the ex post optimal offer, which is then displayed on the graphical user interface.

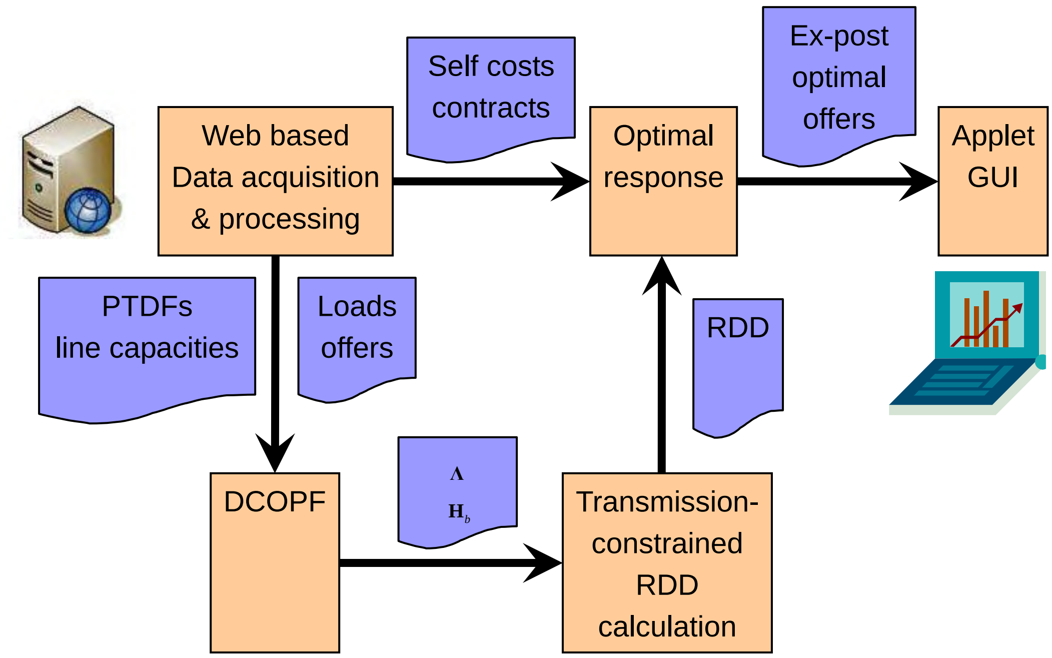


Figure 2 Overview of tool design

The Java tool can be run both in server mode, with web based inputs and outputs, and in stand-alone application mode using text data files for input and output together with an applet for graphing the ex post optimal response. As well as developing Java specific to this project, we also utilized and modified the DCOPFJ package developed at Iowa State University for solving the DC OPF and the PTPLOT plotting package developed at the University of California, Berkeley to plot the ex-post best offers, and cost functions.

In the following sections, the processing of the input data, the calculation of the OPF, modifications to the residual demand derivative calculation to enhance efficiency, and the output data and graphical user interface will be described.

3.1 Input data

The various inputs for the calculation of the optimal response are as follows.

- Line Capacity Data
- PTDF Data
- Generation Offer Data
- Self-offer Data
- Self-cost Data

The line capacity data and PTDF data are assumed to be available from a public website. In the context of the ERCOT case study, we developed Java code to retrieve branch capacity data and PTDFs from the ERCOT website that was then written to a file. For other sources of data, the Java could be modified as necessary to retrieve data in another format.

Generator aggregate offer data is also assumed to be available, but is typically released after a delay. We built a Java tool to download zipped files of historical aggregated data from the ERCOT website for particular days and hours. Since the offer data typically consists of many segments that have very similar prices or very similar quantities, the data is sampled according to user-specified tolerances to generate a summarized version. The user-specified tolerances are on the minimum:

- difference of adjacent offer curve slope (delta_slope), and
- difference of adjacent quantity (delta_MW).

The purpose of data sampling is to summarize the aggregate offer data in order to speed up downstream software running time.

Figure 3 shows the user interface for acquiring the generator aggregate offer data and summarizing it based on the user-specified tolerances. The user can choose the date, hour, and tolerance values. Clicking the "run" button generates the summarized aggregate offer data, writing it to a file.

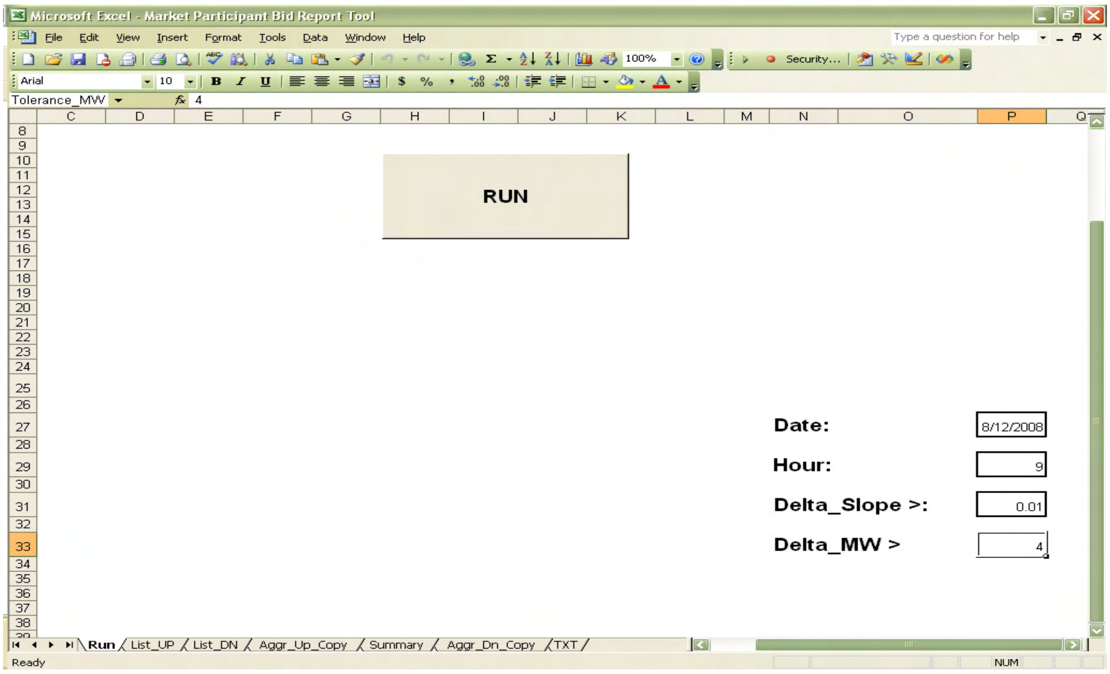


Figure 3 Interface for downloading generator aggregate offer data

To analyze the residual demand faced by a firm, its own offer, that is, its "self-offer" must be removed from the aggregate offers of the other firms. A firm's own costs, its "self-costs," are also needed for the calculation. It is assumed that self-offer and self-cost data is available in convenient format or can be entered by the user through the graphical user interface.

3.2 DC optimal power flow engine

The calculation of the residual demand derivative requires a solved OPF. DCOPFJ is a Java-based DC OPF solver developed at Iowa State University (http://www.econ.iastate.edu/tesfatsi/DCOPFJHome.htm). The operation of the Java code is outlined in Figure 4.

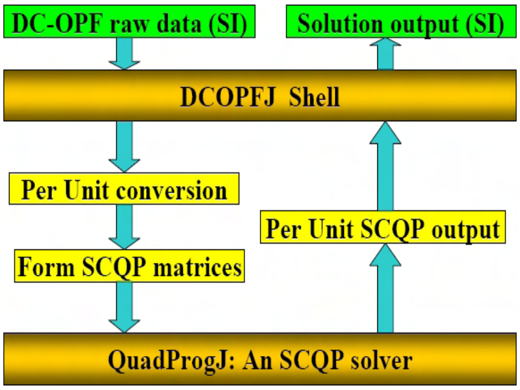


Figure 4 Operation of Java Based DC optimal power flow.

(Source: This image is taken from http://www.econ.iastate.edu/tesfatsi/DCOPFJHome.htm.)

The Jave code can solve a DC OPF model with full DC power flow equations as constraints. It is common that advanced OPF solvers separate the optimization and the power flow for the purpose of calculation efficiency and speed. In the optimization part, the power flow equations are represented by PTDFs relating to possibly binding transmission constraints, and in the power flow part, the power flow is solved to find possibly binding transmission constraints and calculate the PTDFs for them. The optimization part and the power flow part iterate until the process converges.

For the purposes of RDD calculation, it is also more convenient to separate the optimization and power flow. This is because, for a zonal electricity market like ERCOT, zonal PTDFs are calculated by the ISO in advance and are used to clear the market. Moreover, the PTDF data are available publicly. For example, ERCOT publishes the PTDFs at http://mospublic.ercot.com/ercot/jsp/commercially_significant_constraints.jsp.

To deal with the PTDFs directly, the DCOPFJ solver was modified so that it has the capability to take PTDFs as inputs to solve the optimization problem. For each RDD calculation for firm *i*, its zone is singled out and a value of net demand or generation is specified in place of the firm's offer at that zone. The DCOPF is then solved for this net demand or generation, together with the branch capacities, the PTDFs, and other offer data.

3.3 Performance enhancement

Although the discussion in the previous sections is theoretically correct, in practice the performance of the algorithm would be slow if it were applied literally as described. The reason for this is that the offer functions are described as piecewise linear functions. Practically, any offer function can be reasonably approximated by a piecewise linear function; however, in practice this means that there are a large number of segments describing each offer, even when the summarized offer data is used.

The $\mathbf{H}_\mathrm{b}$ matrix does not only contain the binding transmission constraints but also includes the binding generation capacity constraints. For a piecewise linear offer function representation there are effectively a large number of individual generation capacity constraints, one corresponding to each segment. Moreover, for each physical generator, there is at most one such linear segment that is not binding. That is, we expect a large number of binding generation capacity "segment" constraints. The matrix $\left(\bar{\mathbf{H}}_\mathrm{b}\mathbf{\Lambda}\bar{\mathbf{H}}_\mathrm{b}^\mathrm{T}\right)$ includes all of these constraints and so can be very large, depending on the number of binding segments, making it difficult to invert.

To handle this problem, we partition $\bar{\mathbf{H}}_\mathrm{b}$ into $\begin{bmatrix}\bar{\mathbf{H}}_\mathrm{b,tran}\\ \bar{\mathbf{H}}_\mathrm{b,gen}\end{bmatrix}$, where $\bar{\mathbf{H}}_\mathrm{b,tran}$ represents the binding transmission constraints, and $\bar{\mathbf{H}}_\mathrm{b,gen}$ represents the binding generation capacity segment constraints. Also, partition $\bar{\mathbf{q}}$ into $\begin{bmatrix}\bar{\mathbf{q}}_\mathrm{v}\\ \bar{\mathbf{q}}_\mathrm{f}\end{bmatrix}$, where $\bar{\mathbf{q}}_\mathrm{v}$ represents the generation outputs that are not at their segment bounds, and $\bar{\mathbf{q}}_\mathrm{f}$ represents the generation outputs that are at their segment bounds. Accordingly, partition $\boldsymbol{\mu} = \begin{bmatrix}\boldsymbol{\mu}_\mathrm{tran}\\ \boldsymbol{\mu}_\mathrm{gen}\end{bmatrix}$, $\mathbf{O}'' = \begin{bmatrix}\mathbf{O}''_\mathrm{v} & \mathbf{0}\\ \mathbf{0} & \mathbf{O}''_\mathrm{f}\end{bmatrix}$, and $\bar{\mathbf{H}}_\mathrm{b} = \begin{bmatrix}\bar{\mathbf{H}}_\mathrm{b,tran,v} & \bar{\mathbf{H}}_\mathrm{b,tran,f}\\ \bar{\mathbf{H}}_\mathrm{b,gen,v} & \bar{\mathbf{H}}_\mathrm{b,gen,f}\end{bmatrix}$. We have:

$$\begin{bmatrix}\mathbf{O}''_\mathrm{v} & \mathbf{0}\\ \mathbf{0} & \mathbf{O}''_\mathrm{f}\end{bmatrix}\begin{bmatrix}\dfrac{d\tilde{\bar{q}}_\mathrm{v}}{d\lambda}\\ \dfrac{d\tilde{\bar{q}}_\mathrm{f}}{d\lambda}\end{bmatrix} - \begin{bmatrix}\bar{\mathbf{H}}_\mathrm{b,tran,v} & \bar{\mathbf{H}}_\mathrm{b,tran,f}\\ \bar{\mathbf{H}}_\mathrm{b,gen,v} & \bar{\mathbf{H}}_\mathrm{b,gen,f}\end{bmatrix}\begin{bmatrix}\dfrac{d\tilde{\bar{\mu}}_\mathrm{v}}{d\lambda}\\ \dfrac{d\tilde{\bar{\mu}}_\mathrm{f}}{d\lambda}\end{bmatrix} = \begin{bmatrix}\mathbf{1}_\mathrm{v}\\ \mathbf{1}_\mathrm{f}\end{bmatrix}$$

$$\begin{bmatrix}\bar{\mathbf{H}}_\mathrm{b,tran,v} & \bar{\mathbf{H}}_\mathrm{b,tran,f}\\ \bar{\mathbf{H}}_\mathrm{b,gen,v} & \bar{\mathbf{H}}_\mathrm{b,gen,f}\end{bmatrix}\begin{bmatrix}\dfrac{d\tilde{\bar{q}}_\mathrm{v}}{d\lambda}\\ \dfrac{d\tilde{\bar{q}}_\mathrm{f}}{d\lambda}\end{bmatrix} = \begin{bmatrix}\mathbf{0}_\mathrm{tran}\\ \mathbf{0}_\mathrm{gen}\end{bmatrix}$$

where $\bar{\mathbf{H}}_\mathrm{b,tran,f} = \mathbf{0}$, $\bar{\mathbf{H}}_\mathrm{b,gen,v} = \mathbf{0}$, and $\bar{\mathbf{H}}_\mathrm{b,gen,f} = \mathbf{I}$.

Therefore,

$$\frac{d\tilde{\bar{q}}_\mathrm{v}}{d\lambda} = \mathbf{0},$$

$$\mathbf{O}''_{\mathrm{v}} \frac{d\tilde{\tilde{q}}_{\mathrm{v}}}{d\lambda} - \overline{\mathbf{H}}_{\mathrm{b,tran,v}} \frac{d\tilde{\tilde{\mu}}_{\mathrm{v}}}{d\lambda} = \mathbf{1}_{\mathrm{v}} ,$$

$$\frac{d\tilde{R}_n}{d\lambda}\left(\hat{\lambda}\right) = -\overline{\mathbf{1}}_{\mathrm{v}}^{\mathrm{T}} \mathbf{\Lambda}_{\mathrm{v}} \overline{\mathbf{1}}_{\mathrm{v}} + \overline{\mathbf{1}}_{\mathrm{v}}^{\mathrm{T}} \mathbf{\Lambda} \overline{\mathbf{H}}_{\mathrm{b,trans,v}}^{\mathrm{T}} \left(\overline{\mathbf{H}}_{\mathrm{b,trans,v}} \mathbf{\Lambda}_{\mathrm{v}} \overline{\mathbf{H}}_{\mathrm{b,trans,v}}^{\mathrm{T}}\right)^{-1} \overline{\mathbf{H}}_{\mathrm{b,trans,v}} \mathbf{\Lambda}_{\mathrm{v}} \overline{\mathbf{1}}.$$

Note that the new RDD formula only depends on binding transmission constraints and the non-binding generation outputs. The matrix $\left(\overline{\mathbf{H}}_{\mathrm{b,trans,v}} \mathbf{\Lambda}_{\mathrm{v}} \overline{\mathbf{H}}_{\mathrm{b,trans,v}}^{\mathrm{T}}\right)$ is much smaller and easier to invert than $\left(\overline{\mathbf{H}}_{\mathrm{b}} \mathbf{\Lambda} \overline{\mathbf{H}}_{\mathrm{b}}^{\mathrm{T}}\right)$, so the performance of RDD calculation is improved.

3.4 Graphical user interface and output

Users operate the bidding tool via a user-friendly graphical user interface. Upon opening the tool, the previously compiled values for Branch Capacity Data, PTDF Data, and Generation Offer Data are read in. The user has the option to change one or all of these values together with the data specifying the self-offer and self-costs.

The self-offer can be represented by a large number of "offer points" that specify break-points in the offer curve. First, choose the number of offer points at the top of the left column "No. of Observations." The tool will create a set of fields to enter each offer point on the right side of the interface. For each offer point, enter the generator and node identifier. (If the "Enforce Same ID" box is checked, these two fields are automatically filled with the generator and node ID). Then enter the offer price ($/MWh) and incremental quantity of power offered at that price (MW). For example, if a 100MW generator submits two (price, quantity) pairs: ($100/MWh, 25 MWh) and ($150/MWh, 75 MW), then the bid function is interpreted as a piecewise linear function that connects each of the bidpoints.

After all parameters are entered, the user can save the parameters to an external file. Then, the tool calculates the optimal offer functions and presents the output in both graphical and numerical form. The outputs produced by the calculation engine are as follows:

- Generation Dispatch
- Locational marginal prices
- Residual demand derivative.
- Optimal offer

4 ERCOT case study

In this section, we describe a case study based on the ERCOT balancing market, demonstrating the use of publicly available information to calculate the ex post optimal offer in the ERCOT electricity market.

4.1 Branch and PTDF data

ERCOT currently implements a zonal electricity balancing market. The zones are redefined annually. There are four zones in 2008, namely Houston, North, South, and West. There are five inter zonal transmission constraints, namely North-Houston, North-South, North-West, South-North, and West-North.

To represent the zonal balancing market, we define a 5-node system using data downloaded from the ERCOT website. Nodes 1 to node 4 correspond to the four zones, and the node 5 corresponds to the slack bus. The format in the data file to repesent these zones is as follows:

```
//    ERCOT Node Definition:
//    Node        Zone
//    1           Houston
//    2           North
//    3           South
//    4           West
```

The corresponding branch definition is as follows. Branches 1 to 5 correspond to the 5 inter zonal constraints.

```
//    Branch Definition
//    Branch      ZoneFrom     ZoneTo
//    1           North        Houston
//    2           North        South
//    3           North        West
//    4           South        North
//    5           West         North
```

The geographical arrangement of the zones is depicted in Figure 5.

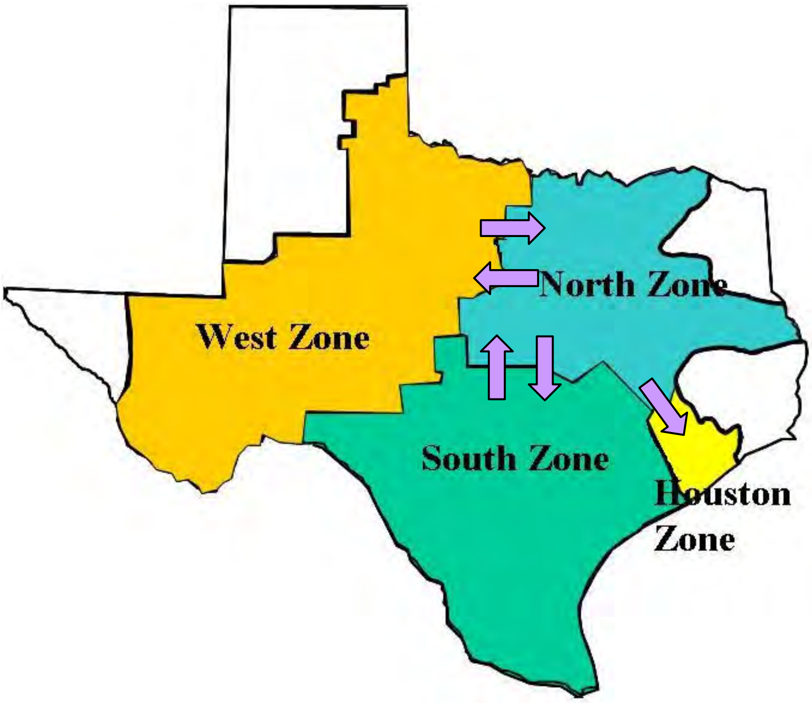
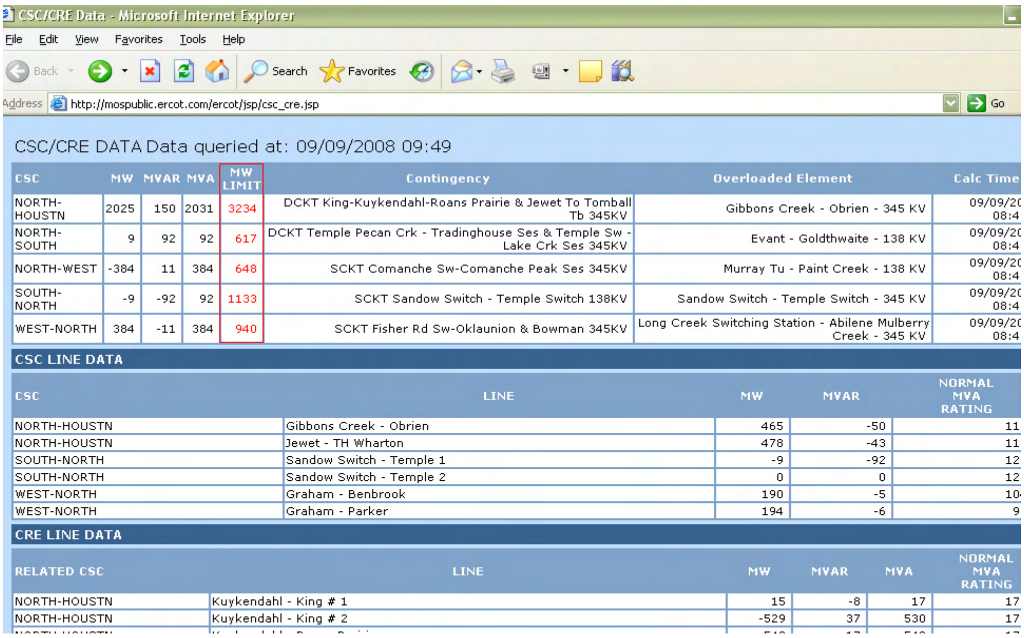


Figure 5 ERCOT zones and inter-zonal constraints

The inter zonal transmission constraint limits can be found at http://mospublic.ercot.com/ercot/jsp/csc_cre.jsp. A snapshot from that web page is shown in Figure 6. The Java code described in Section 3.1 downloads this page and extracts the constraint data.



CSC/CRE DATA Data queried at: 09/09/2008 09:49

CSC	MW	MVAR	MVA	MW LIMIT	Contingency	Overloaded Element	Calc Time
NORTH-HOUSTN	2025	150	2031	3234	DCKT King-Kuykendahl-Roans Prairie & Jewet To Tomball Tb 345KV	Gibbons Creek - Obrien - 345 KV	09/09/20 08:4
NORTH-SOUTH	9	92	92	617	DCKT Temple Pecan Crk - Tradinghouse Ses & Temple Sw - Lake Crk Ses 345KV	Evant - Goldthwaite - 138 KV	09/09/20 08:4
NORTH-WEST	-384	11	384	648	SCKT Comanche Sw-Comanche Peak Ses 345KV	Murray Tu - Paint Creek - 138 KV	09/09/20 08:4
SOUTH-NORTH	-9	-92	92	1133	SCKT Sandow Switch - Temple Switch 138KV	Sandow Switch - Temple Switch - 345 KV	09/09/20 08:4
WEST-NORTH	384	-11	384	940	SCKT Fisher Rd Sw-Oklaunion & Bowman 345KV	Long Creek Switching Station - Abilene Mulberry Creek - 345 KV	09/09/20 08:4

CSC LINE DATA

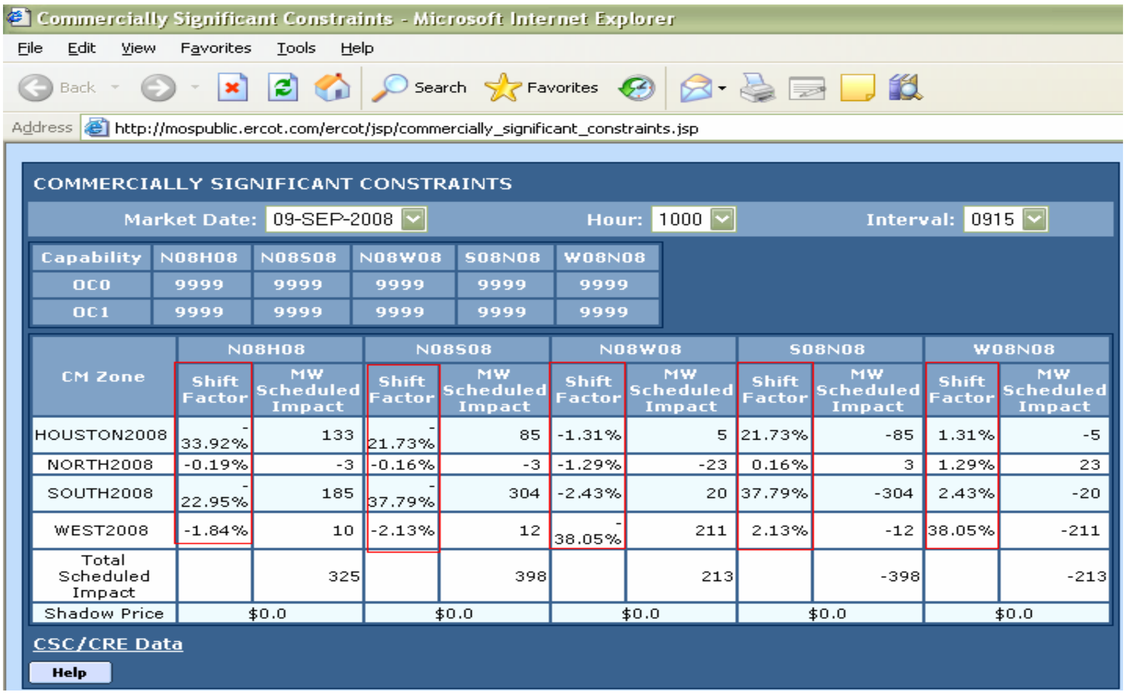
CSC	LINE	MW	MVAR	NORMAL MVA RATING
NORTH-HOUSTN	Gibbons Creek - Obrien	465	-50	11
NORTH-HOUSTN	Jewet - TH Wharton	478	-43	11
SOUTH-NORTH	Sandow Switch - Temple 1	-9	-92	12
SOUTH-NORTH	Sandow Switch - Temple 2	0	0	12
WEST-NORTH	Graham - Benbrook	190	-5	10
WEST-NORTH	Graham - Parker	194	-6	9

CRE LINE DATA

RELATED CSC	LINE	MW	MVAR	MVA	NORMAL MVA RATING
NORTH-HOUSTN	Kuykendahl - King # 1	15	-8	17	17
NORTH-HOUSTN	Kuykendahl - King # 2	-529	37	530	17

Figure 6 Web page showing inter-zonal transmission limits

ERCOT also publishes the PTDFs corresponding to these inter-zonal limits at: http://mospublic.ercot.com/ercot/jsp/commercially_significant_constraints.jsp. A snapshot for September 9, 2008 is shown in Figure 7. The Java code described in Section 3.1 downloads this page and extracts the constraint data.



Capability	N08H08	N08S08	N08W08	S08N08	W08N08
OC0	9999	9999	9999	9999	9999
OC1	9999	9999	9999	9999	9999

CM Zone	N08H08		N08S08		N08W08		S08N08		W08N08	
	Shift Factor	MW Scheduled Impact	Shift Factor	MW Scheduled Impact	Shift Factor	MW Scheduled Impact	Shift Factor	MW Scheduled Impact	Shift Factor	MW Scheduled Impact
HOUSTON2008	-33.92%	133	-21.73%	85	-1.31%	5	21.73%	-85	1.31%	-5
NORTH2008	-0.19%	-3	-0.16%	-3	-1.29%	-23	0.16%	3	1.29%	23
SOUTH2008	-22.95%	185	-37.79%	304	-2.43%	20	37.79%	-304	2.43%	-20
WEST2008	-1.84%	10	-2.13%	12	-38.05%	211	2.13%	-12	38.05%	-211
Total Scheduled Impact		325		398		213		-398		-213
Shadow Price	$0.0		$0.0		$0.0		$0.0		$0.0	

Figure 7 Web page showing PTDFs for inter-zonal constraints

4.2 Offer data

In the current ERCOT zonal market, each generation company makes a portfolio offer for all its generation in each zone. ERCOT publishes historical aggregated zonal offers from all generation companies at http://ercot.com/mktinfo/agg_bid/.

A snapshot is shown in Figure 8. The Java code described in Section 3.1 downloads this page and extracts the constraint data.

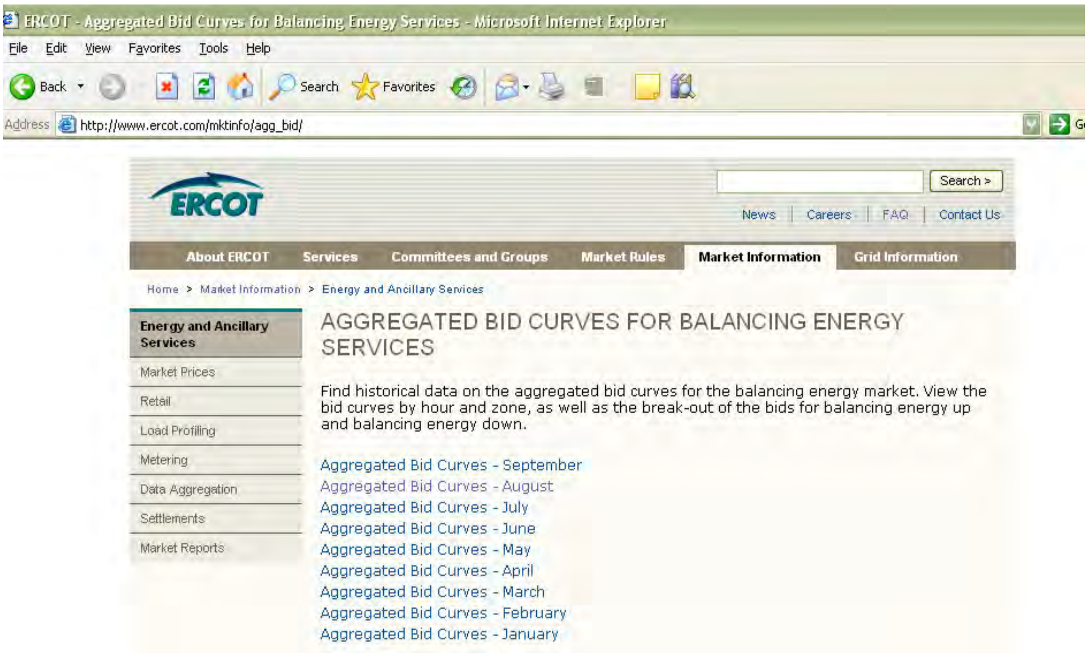


Figure 8 Web page showing offer data

Each generation company should know its own portfolio self-offers, its self-costs, and its own contracts. An independent system operator performing this calculation also has access to offers and can estimate costs and contracts. For the purposes of the cases study, generic cost data based on historical information was used. Finally, a range of various net generation and demand situations can be used to "explore" the ex post optimal offer.

4.3 Results of ERCOT case

Transmission and offer data based on April 2, 2008 was used in the case study. At that time, the North-Houston inter-zonal constraint was binding. The optimal offer for a large firm in the Houston zone was analyzed. The optimal offer was calculated and is shown in Figure 9 for the case of Houston load varying from 500 MW to 2150 MW. The optimal offer is coarse in the price range from \$130/MWh to \$200/MWh. To have a better view of the optimal offer in this price range, we plot the case of Houston load varying from 1250 MW to 2090 MW with smaller step sizes in Figure 10. The actual offer in this case study deviates significantly from the optimal offer and from marginal costs.

In section 2.2, it was observed that due to changes in the congestion status as injection changes, it might be the case that the constructed offer was not monotonic. However, in this case, as shown in Figures 9 and 10, the offer is monotonic and so it could be submitted as a valid offer.

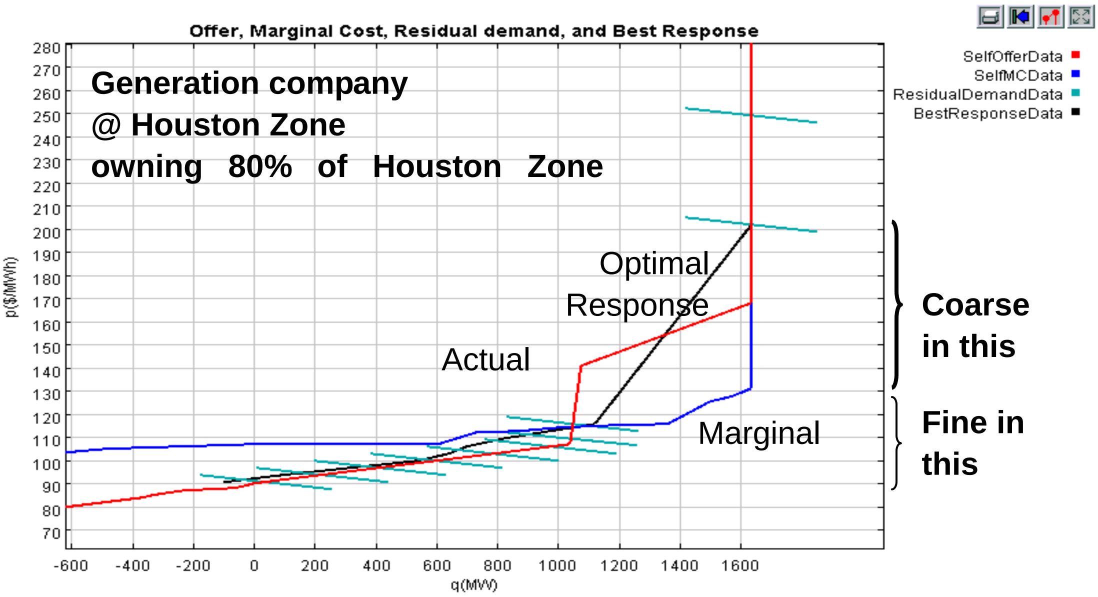


Figure 9 Optimal offer for April 2, 2008, Houston zone (load 500 MW – 2150 MW)

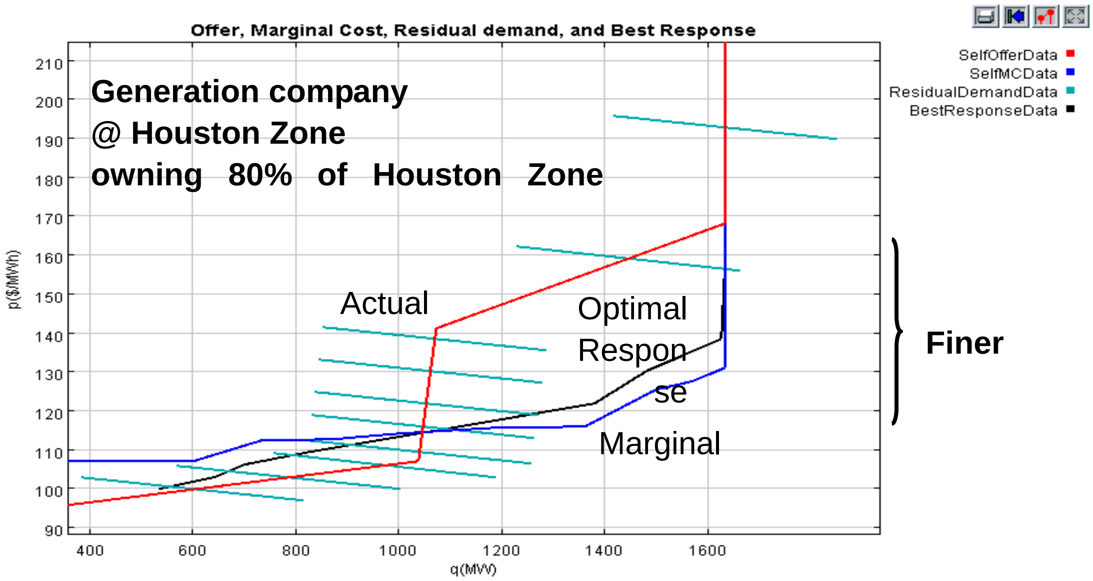


Figure 10 Optimal offer for April 2, 2008, Houston zone (load 1250 MW – 2090 MW)

5 Conclusion

In this report we have described the theory underlying the calculation of the ex post optimal offer by a generating firm in the presence of transmission constraints. The principal contribution is the calculation of the transmission-constrained residual demand derivative and the incorporation of this calculation into a profit maximization framework. A Java based tool was built to implement the calculations and display them and a case study of ERCOT performed.

We believe this tool to be an important first step to allowing market participants to assess the impact of bidding decisions on a variety of outcomes including profits, prices, and economic efficiency. In future research, this tool can be expanded along several dimensions.

One area for future research is to calibrate the tool for other markets with similar features of the procurement process. While the tool is specifically calibrated to ERCOT's zonal market, the analytical technique described in sections 2.3-2.4 can be applied elsewhere.

In addition, the tool could include additional diagnostics that may be useful to a market monitor in assessing the impact of a bidder's decisions on the overall functioning of the market. In particular, the tool could calculate prices under several counterfactual scenarios including a scenario where the firm in question submitted perfectly competitive offers. Alternatively, one could compute prices that would arise under alternative transmission capacity scenarios (with firms exercising market power). Such an analysis would serve as a means to assess the outcome of (small) grid expansions on expected market outcomes while accounting for possible market power. Finally, the tool could be expanded to estimate the effect of a single firm's bids on the overall dispatch costs.

Project publications

Lin Xu and Ross Baldick, “Transmission-constrained Residual Demand Derivative in Electricity Markets,” *IEEE Transactions on Power Systems,* 22(4):1563-1573, November 2007.

Puller, Steven L., (forthcoming), “Competitive Performance in the ERCOT Wholesale Market”, *Electricity Restructuring: The Texas Story*, AEI Press, eds. Lynne Kiesling and Andrew Kleit.

Project presentations

Ross Baldick at Seoul International Conference on Electricity Markets, June 19, 2008, Seoul, South Korea. “Tools for Assessing Offers into Electricity Auctions”.

Steven L. Puller to MIT Center for Energy and Environmental Policy Research, May 2, 2008, Cambridge, MA. “Understanding Strategic Bidding in Multi-Unit Auctions: A Case Study of the Texas Electricity Spot Market”.

Steven L. Puller to American Enterprise Institute conference on “Electricity Deregulation, Texas-Style”, January 25, 2008, Washington, DC. “Competitive Performance of the ERCOT Wholesale Market”.

References

Baldick, R., Grant, R., and Kahn, Edward, (2004) “Theory and Application of Linear Supply Function Equilibrium in Electricity Markets", *Journal of Regulatory Economics"*, 25(2):143-167, March.

Baldick, R. and Hogan, W. W., (2002) “Stability of supply function equilibria: Implications for daily versus hourly bids in a poolco market,” *Proceedings of the Seventh Annual POWER Research Conference,* pages 3.1-3.19, University of California Energy Institute, Berkeley, California, March.

Baldick, R. and Hogan, W. W., (2004) “Polynomial Approximations and Supply Function Equilibrium Stability,” Presented at the 6th IAEE European Conference, Modeling in Energy Economics and Policy, Zurich, Switzerland, September.

Borenstein, S., Bushnell, J., and Stoft, (2000) “The Competitive Effects of Transmission Capacity in a Deregulated Electricity Industry,” *RAND Journal of Economics*, 31(2):294-325, Summer.

Green, R., (1999) “The electricity contract market in England and Wales,” *The Journal of Industrial Economics*, XLVII(1):107-124, March.

Hortacsu, Ali, and Steven L. Puller, (2008) “Understanding Strategic Bidding in Multi-Unit Auctions: A Case Study of the Texas Electricity Spot Market,” *RAND Journal of Economics*, 39(1):86-114, Spring.

Niu, Hui, Ross Baldick, and Guidong Zhu, (2005) “Supply Function Equilibrium Bidding Strategies with Fixed Forward Contracts,” *IEEE Transactions on Power systems*, 20(4):1859-1867, November.

Sioshansi, Ramteen, and Shmuel Oren, (2006) “How Good are Supply Function Equilibrium Models: An Empirical Analysis of the ERCOT Balancing Market,” Manuscript, April.

Wolak, Frank, (2003) “Measuring Unilateral Market Power in Wholesale Electricity Markets: The California Market 1998—2000, *The American Economic Review"*, 93(2):425-430, May.

Xu, Lin, and Ross Baldick, (2007) “Transmission-constrained Residual Demand Derivative in Electricity Markets,” *IEEE Transactions on Power Systems,* 22(4):1563-1573, November.