Newly Implemented and Proposed Market Products and Reformulations: Pricing Implications, Analysis, and Enhancements

*Final Project Report*

M-39

Power Systems Engineering Research Center

*Empowering Minds to Engineer the Future Electric Energy System*
Newly Implemented and Proposed Market Products and Reformulations: Pricing Implications, Analysis, and Enhancements

Final Project Report

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Power Systems Engineering Research Center

The Power Systems Engineering Research Center (PSERC) is a multi-university Center conducting research on challenges facing the electric power industry and educating the next generation of power engineers. More information about PSERC can be found at the Center’s website: http://www.pserc.org.

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Executive Summary

Energy markets are ever evolving due to operational complexities as well as complications due to a changing resource mix (e.g., renewables). Independent system operators are introducing new market products and reformulations to accommodate the inherent complexity of the electricity production, transmission, and consumption. This research focuses on three core areas: (1) flexible ramping products and multi-period pricing policies, (2) enhanced generator contingency modeling, and (3) management of stochastic resources. This research evaluates these essential structural changes to energy markets and provide a comprehensive evaluation of the impacts on settlements. This research also provides suggestions on market reformulations to enhance market efficiency, pricing, and transparency. The report is presented in two parts.

Part I: Analyzing the Market Impacts of Ramping and Multi-Period Pricing Approaches, Contingency Modeling Enhancements, and Management of Stochastic Resources

Part I of this report focuses on: (1) day-ahead resource scheduling with enhanced flexible ramp product (FRP) design, (2) pricing implications and objective function evaluation for markets with stochastic formulations, (3) the conditions for inter-temporal ramp rate constraints causing uplift payments, and (4) the market implications of the explicit representation of generator contingencies. The research shows that:

- Accounting for 15-min net load changes in day-ahead flexible ramping product procurement enhances the quantity allocation of ramping capabilities by making relatively small changes in market models compared to the existing FRP design. This approach leads to less expected final operating cost in the fifteen-minute market, higher reliability as the power system gets close to real-time operation, and less discrepancy between day-ahead and real-time unit commitment decisions.
- With more accurate representation of contingencies in the extensive-form two-stage security-constrained unit commitment, N-1 security constraints are better satisfied with less need for out-of-market corrections while the value of services (contingency-based reserve) provided by generators is reflected in the locational marginal prices.
- The stochastic market design with expected objective function does not give solutions that ensure minimum realized operating costs at N-1 contingency states. Instead, the stochastic market design with base-case objective function had better performance compared to the market model with expected objective function in terms of the base-case costs (i.e., normal operating costs) and realized N-1 costs.
- Possible inaccuracies in scenario probability estimations significantly affect results of stochastic market models with expected cost minimization as the objective function. Thus, it defeats the purpose of such expected cost minimization models.
- Constant ramp rate constraints do not cause uplift payments if dispatch decisions and prices in all scheduled time intervals are financially binding. In contrast, dispatch-dependent ramp rates introduce nonconvexity into the market, which may cause uplift payments. Uplifts may occur when using a multi-period model for scheduling while using a single-period pricing run.
• The explicit representation of generator contingencies adds a new congestion component within the traditional locational marginal price, which reflects the influence of congestion during the post-contingency states for the modeled critical generator contingencies.

Part II: Pricing Multi-interval Dispatch Under Uncertainty

Part II of this report focuses on pricing multi-interval dispatch under operational uncertainty. In particular, we consider rolling-window look-ahead dispatch in the real-time market where inaccurate forecasts of future demand are used. This project aims to gain insights into the issue of lack of dispatch-following incentives that result in discriminative out-of-the-market settlements. It is known that all existing uniform pricing schemes, including the standard locational marginal pricing (LMP), require out-of-the-market uplifts when forecasts of demand and renewable generation are not accurate.

Main results of this paper include the following:

• We show that no uniform pricing exists that guarantees dispatch-following incentives. Therefore, discriminative out-of-the-market settlements are necessary for uniform pricing schemes. In other words, price discrimination is unavoidable for multi-interval look-ahead dispatch.
• We propose a non-uniform pricing scheme, referred to as temporal locational marginal pricing (TLMP), that eliminates out-of-the-market settlements under arbitrary forecasting uncertainties. In other words, TLMP provides full dispatch-following incentives by prepaying the opportunity costs of ramp-limited generators and, at the same time, penalizes generators with limited ramping capabilities. To this end, TLMP discriminates in-side-the market to guarantee dispatch following incentives, whereas all uniform pricing schemes discriminate though out-of-the-market settlements.
• We establish several attractive features of TLMP. In particular, TLMP has a natural decomposition as the sum of LMP and the ramping prices. In other words, TLMP is the sum of energy price, congestion price, and ramping prices.
• We conducted detailed empirical studies (with and without network constraints) to evaluate TLMP and existing pricing benchmarks for multi-interval look-ahead dispatches. Among the performance measures evaluated are the dispatch following and ramping revelation incentives, the revenue adequacy of the operator, generator profits, and demand payments.
• Empirical studies show no single pricing scheme dominates all others. In general, TLMP performed well in both dispatch-following and ramping revelation incentives, operator revenue adequacy, and consumer payments while resulted in lower generator payments. TLMP, however, tends to have lower generator profits for generators with limited ramping constraints.

Project Publications:


Part I

Analyzing the Market Impacts of Ramping and Multi-Period Pricing Approaches, Model Enhancements of Contingencies, and Management of Stochastic Resources

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# Nomenclature

## Acronyms

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<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>CAISO</td>
<td>California Independent System Operator</td>
</tr>
<tr>
<td>DA</td>
<td>Day-Ahead</td>
</tr>
<tr>
<td>DCOPF</td>
<td>Direct Current Optimal Power Flow</td>
</tr>
<tr>
<td>DDP</td>
<td>Desired Dispatch Point</td>
</tr>
<tr>
<td>EMS</td>
<td>Energy Management System</td>
</tr>
<tr>
<td>FERC</td>
<td>Federal Energy Regulatory Commission</td>
</tr>
<tr>
<td>FRP</td>
<td>Flexible Ramping Product</td>
</tr>
<tr>
<td>GDF</td>
<td>Generator Loss Distribution Factor</td>
</tr>
<tr>
<td>ISO</td>
<td>Independent System Operator</td>
</tr>
<tr>
<td>ISO-NE</td>
<td>ISO New England</td>
</tr>
<tr>
<td>KKT</td>
<td>Karush-Kuhn-Tucker</td>
</tr>
<tr>
<td>LMP</td>
<td>Locational Marginal Price</td>
</tr>
<tr>
<td>LODF</td>
<td>Line Outage Distribution Factor</td>
</tr>
<tr>
<td>LP</td>
<td>Linear Programming</td>
</tr>
<tr>
<td>MILP</td>
<td>Mixed-Integer Linear Programming</td>
</tr>
<tr>
<td>MIP</td>
<td>Mixed Integer Programming</td>
</tr>
<tr>
<td>MIRTM</td>
<td>Multi-Interval Real-Time Market</td>
</tr>
<tr>
<td>MISO</td>
<td>Midcontinent Independent System Operator</td>
</tr>
<tr>
<td>MMS</td>
<td>Market Management System</td>
</tr>
<tr>
<td>NERC</td>
<td>North American Electric Reliability Corporation</td>
</tr>
<tr>
<td>NREL</td>
<td>National Renewable Energy Laboratory</td>
</tr>
<tr>
<td>NAE</td>
<td>National Academy of Engineering</td>
</tr>
<tr>
<td>OMC</td>
<td>Out-of-Market Correction</td>
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</table>
PTDF  Power Transfer Distribution Factor
RAS  Remedial Action Scheme
RT  Real-Time
SCED  Security-Constrained Economic Dispatch
SCUC  Security-Constrained Unit Commitment
SOS2  Special Ordered Sets of type 2
UC  Unit Commitment
VOLL  Value Of Lost Load

Sets and Indices

\(c\)  Index of operating state; 0 for the base-case, non-zero for contingencies.
\(C^g\)  Set of generator contingencies.
\(C^{g_{\text{crit}}}\)  Subset of critical (credible) generator contingencies, \(C^{g_{\text{crit}}} \subseteq C^g\).
\(C_0, C_g, C_k\)  Set of scenarios representing base-case, generator, and line contingencies, respectively.
\(g\)  Index of generators, \(g \in G\).
\(g(n)\)  Set of generators connected to node \(n\).
\(k, \ell\)  Index of transmission lines, \(k, \ell \in K\).
\(K\)  Set of transmission assets.
\(K^{\text{crit}}\)  Subset of critical transmission assets, \(K^{\text{crit}} \subseteq K\).
\(n\)  Index for buses, \(n \in N\).
\(N\)  Set of nodes.
\(n'(c)\)  Node index for generator loss under contingency \(c\).
\(S^{FR}\)  Set of nodes that have generators with frequency response capability.
\(t\)  Index for time periods, \(t \in T\).
### Parameters and Constants

- \( c_g^P \): Variable cost of unit \( g \) ($/MWh).
- \( c_g^{NL} \): No-load cost of unit \( g \) ($).
- \( c_g^{SD} \): Shutdown cost of unit \( g \) ($).
- \( c_g^{SU} \): Startup cost of unit \( g \) ($).
- \( c_n \): Operating cost for the generator at node \( n \).
- \( DT_g \): Minimum down time of unit \( g \).
- \( \overline{D}_n \): Fixed real power demand at node \( n \).
- \( FR_{up,t} \): Upward ramping requirement in period \( t \).
- \( FR_{down,t} \): Downward ramping requirement in period \( t \).
- \( FR_{up,t,0min}^{ih} \): Upward 15-min ramping requirement for the first interval of period \( t \).
- \( FR_{up,t,15min}^{ih} \): Upward 15-min ramping requirement for the second interval of period \( t \).
- \( FR_{up,t,30min}^{ih} \): Upward 15-min ramping requirement for the third interval of period \( t \).
- \( FR_{up,t,45min}^{ih} \): Upward 15-min ramping requirement for the forth interval of period \( t \).
- \( FR_{down,t,0min}^{ih} \): Downward 15-min ramping requirement for the first interval of period \( t \).
- \( FR_{down,t,15min}^{ih} \): Downward 15-min ramping requirement for the second interval of period \( t \).
- \( FR_{down,t,30min}^{ih} \): Downward 15-min ramping requirement for the third interval of period \( t \).
- \( FR_{down,t,45min}^{ih} \): Downward 15-min ramping requirement for the forth interval of period \( t \).
- \( GDF_{n'(c),n} \): Generation loss distribution factor for contingency \( c \) at node \( n \).
- \( Load_{nt} \): Load at bus \( n \) during time period \( t \).
- \( LODF_{ref}^{kr} \): Line outage distribution factor representing the change in flow on line \( k \) for outage of line \( l \).
$N_{1_k}$  
N-1 contingency indicator of transmission line $k$; 0 for a contingency on line $k$; otherwise, 1.

$N_{1_g}$  
N-1 contingency indicator of generator $g$; 0 for a contingency on generator $g$; otherwise, 1.

$N_{L_t}$  
System’s net load in period $t$.

$N_{L_{t+1}^{\text{max}}}$  
Maximum system’s net load in period $t$.

$N_{L_{t+1}^{\text{min}}}$  
Minimum system’s net load in period $t$.

$\bar{P}_{gt}$  
Scheduled power output of unit $g$ in period $t$.

$p_{g}^{\text{max}}$  
Maximum output of unit $g$.

$p_{g}^{\text{min}}$  
Minimum output of unit $g$.

$p_{k}^{\text{max}}$  
Thermal rating of transmission line $k$.

$p_{k}^{\text{max},c}$  
Emergency thermal rating of transmission line $k$.

$PTDF_{cnk}^{\text{ref}}$  
Power transfer distribution factor during operating state $c$ for line $k$ for an injection at $n$ to the reference bus.

$\bar{r}_{gt}$  
Scheduled contingency reserve of unit $g$ in period $t$.

$p_{k}^{\text{max},a}$  
Normal capacity (i.e., rate A) for the corresponding transmission asset (thermal limit or stability limit). Typically, $p_{k}^{\text{min},a} = -p_{k}^{\text{max},a}$.

$p_{k}^{\text{max},c}$  
Emergency capacity (i.e., rate C) for the corresponding transmission asset. Typically, $p_{k}^{\text{min},c} = -p_{k}^{\text{max},c}$.

$p_{n}^{\text{max}}$  
Real power maximum capacity for the generator at node $n$.

$PTDF_{k,n}^{R}$  
Proportion of flow on transmission asset $k$ resulting from injection of one MW at node $n$ and a corresponding withdrawal of one MW at reference node $R$.

$R_{g}^{HR}$  
Hourly ramp rate of unit $g$.

$R_{g}^{15}$  
15-min ramp rate of unit $g$.

$R_{g}^{10}$  
10-min ramp rate of unit $g$.

$R_{g}^{SU}$  
Startup ramp rate of unit $g$.

$R_{g}^{SD}$  
Shutdown ramp rate of unit $g$. 
Minimum up time of unit $g$.

Scheduled commitment of unit $g$ in period $t$.

Probability of base-case operating state.

Recognizes the node with generator loss under contingency $c$ (1 if contingency node, else 0).

**Variables**

$\bar{d}_{nt}$ Demand at bus $n$ in period $t$.

$dr_{gt}$ Hourly ramp down capability of unit $g$ in period $t$.

$dr_{gt}^{15}$ 15-min ramp down capability of unit $g$ in period $t$.

$D_n$ Real power demand at bus $n$ (assumed to be perfectly inelastic).

$FL_{lt}^0$ Flow on transmission line $l$ in period $t$.

$F_k^-, F_k^+$ Pre-contingency flowgate marginal prices. Dual variables (or shadow prices) on transmission asset $k$’s normal capacity constraints; lower and upper bounds respectively.

$P_{k}^{c-}, P_{k}^{c+}$ Post-contingency flowgate marginal prices. Dual variables on critical transmission asset $k$’s emergency capacity constraints under critical contingency $c$; lower and upper bounds respectively.

$P_{gct}$ Power output of unit $g$ during operating state $c$ in period $t$.

$P_{gt}$ Power output of unit $g$ in period $t$.

$p_{nt}^{inj}$ Net real power injection at bus $n$ for operating state $c$ in period $t$.

$p_{nt}^{inj}$ Net real power injection at bus $n$ in period $t$.

$P_n$ Real power production from the generator at node $n$.

$r_{gt}$ Contingency reserve of unit $g$ in period $t$.

$ur_{gt}$ Hourly ramp up capability of unit $g$ in period $t$.

$ur_{gt}^{15}$ 15-min ramp down capability of unit $g$ in period $t$.\n

$u_{gt}$  Unit commitment binary variable for unit $g$ in period $t$.

$v_{gt}$  Startup variable for unit $g$ in period $t$.

$w_{gt}$  Shutdown variables for unit $g$ in period $t$.

$\alpha_n$  Dual variable on generator’s (at node $n$) capacity constraint; upper bound constraint.

$\delta$  Dual variable on system-wide power balance constraint (marginal energy component of LMP).

$\lambda_{nct}$  Locational marginal price at bus $n$ for operating state time $c$ in period $t$.

$\lambda_n$  Locational marginal price (LMP) at node $n$. Dual variable of power balance constraint.
1. Evaluation, Refinement, and Enhancement of Flexible Ramping Products

1.1 Introduction

The recent rapid integration of renewable resources, such as wind and solar, poses new challenges to operations in the power systems. For example, 26 percent of total California ISO (CAISO) supply in 2018 was served by non-hydro renewable generation connected directly to the ISO, an increase from 24, 22, and 18 percent in 2017, 2016, and 2015, respectively [1]. One emerging challenge due to this trending increase is the growth of the variability and uncertainty in the net load (i.e., actual system load minus the scheduled interchange and total renewable generation). This leads to a significant need for ramp-down capabilities during sunrise and ramp-up capabilities during sunset from flexible resources, as illustrated by the “duck curve” of the CAISO [2]. In this situation, if there is insufficient ramp capability in the system, the power balance violation will occur; there are not enough flexible generation resources in the system to follow the net load changes. The power balance violation can not only jeopardize the system’s reliability, but can also cause high penalty prices during the RT market processes and consequently create market inefficiency in the long run [3]-[4]. To meet the imbalance, one other alternative for the power system to rely on is the activation of other reserve products; however, this can affect the ability of the system to meet the other required reliability criterion such as the N-1 reliability. Furthermore, there has been acknowledgement that, in some markets such as the CAISO, the traditional regulation and contingency-based reserves are not able to provide sufficient ramping capabilities and therefore new market products are necessary [5]-[6]. As a result, some ISOs, e.g., CAISO [7] and MISO [8], have been augmenting their market models with ramping products in order to meet the net load’s variability and uncertainty and to attain higher responsiveness from the existing flexible resources. Furthermore, ISO New England (ISO-NE), in its strategic planning document, has recognized the necessity of ramp capabilities [9]. In this study, such products are called Flexible Ramp Products (FRPs), although they are also referred to “flexiramp” in the CAISO [7], and “ramp capability” in the MISO [8].

Two new market design variables, i.e., flexible ramp up and flexible ramp down capabilities, are introduced to the market models by implementing the FRPs. The FRPs enable withholding generators ramping capabilities from the flexible resources and better preposition them at time interval $t$ to be able to respond to the net load’s variations and uncertainty at time $t + 1$. The upward and downward ramping requirements are illustrated in Figure 1.1. The ramp capability product has been implemented in the CAISO’s RT market since Fall 2016, where requirements are set to manage the net load variations and uncertainty 5-min ahead [10]. For MISO, the ramp capability product has been implemented in its RT market since Spring 2016. For this market, requirements are set to manage net load variations and uncertainty 10-min ahead, while the RT dispatch is performed for 5-min intervals [11].
There have been intentions toward modeling and procuring some of the ramp capability in the DA market in order to ensure resource adequacy for meeting the ramping requirements in RT. For example, the CAISO has started market initiatives to add hourly FRP to its DA market to correctly position and commit resources to provide sufficient ramp capability in RT operation [13]-[14]. The hourly FRP in CAISO’s market is designed in such a way that resultant added ramp capability must be able to respond to foreseen and unforeseen changes in net load between the hour $t$ and the specified target hour $t+1$. However, it does not consider magnitude and duration of intra-hour net load changes in the following market processes which have shorter scheduling granularity (e.g., 15-min and 5-min intervals). The essential goal of designing hourly FRPs is to increase the system’s ramp capability to follow the realized net load in next RT market processes [13]. In this situation, the hourly FRPs may not be able to accommodate the steep realized RT net load changes as they happen in the shorter periods of time and their effects are not considered when the hourly FRP decisions are made in DA. With the above discussion, there is a need for better design of the hourly FRP in the DA market in order to ensure enough flexible units are online for improving both efficiency and reliability in the RT markets. CAISO also acknowledge the necessity of meeting 15-min ramping needs by hourly DA schedules [14].

1.2 Literature review

The new resource mix, e.g. renewable resources, are imposing operational complexities to modern power systems by intensifying the uncertainty and variability in the system net load. Energy markets are evolving in order to overcome such operational challenges. For instance, the ISOS have been motivated to institute and implement a new ancillary service product, ramping products, so as to handle variations in the forecasted net load (foreseeable changes), as well as uncertainties (unforeseeable changes). While ISOS may use different terms to refer to ramping products (e.g., ramp capability in the MISO and flexiramp in the CAISO [4]), the concept and motivations behind this new ramping requirements are essentially the same.
1.2.1 FRPs in real-time markets

A number of work [3], [4], [6], [15]–[19] has been focused on the RT markets since the main purpose of the FRPs is to improve the ramping capabilities in RT. Reference [3] can be regarded as the first attempt for introducing the FRPs into the RT economic dispatch problem. Reference [6] compares the dispatch, prices, settlements, and market efficiency of a deterministic market model, including the FRPs constraints (similar to ISO operations), with a stochastic market model in the RT economic dispatch problem. A simplistic case study is employed to fundamentally evaluate the performance of FRP markets. A similar study with the same intentions is conducted in [4]; however, the comparisons are done for a real-time unit commitment (RTUC) market model with 15-min granularity. Reference [16] discussed the FRP design for the RT economic dispatch in the MISO market, wherein the ramping requirements are set 10-min ahead to manage net load variations and uncertainty, while the RT dispatch is performed every 5-min. It has been shown that the 10-min ramping capabilities may be depleted in the first 5-min dispatch, so the system may not be able to follow the variability and uncertainty of the net load in the second 5-min dispatch. Therefore, a new FRP design is proposed in [16] to address this subtle issue and to maintain the reliability and efficiency of the system operation.

Reference [18] includes the FRPs constraints into a risk-limiting RT economic dispatch problem, wherein risk has been defined as the “loss-of-load probability.” Furthermore, the concept of deliverable robust FRPs is proposed in [19] for RT markets. The proposed method utilizes the application of the robust optimization to procure FRPs that are deliverable with respect to the physical line limits.

1.2.2 FRPs in day-ahead market

There also have been intentions in some works [20]–[23] toward implementing and procuring some of the ramp capability in the DA market so as to ensure resource adequacy for meeting ramping requirements in the RT market processes. Furthermore, the CAISO intends to add the FRPs to their DA market to address uncertainty and variability in the net load previously left to the RT market [10].

Reference [20] has proposed an optimization model for energy storage aggregators to maximize its profit by biding and procuring ramp up and down capabilities in the DA energy and reserve markets. However, it is pertinent to note that the FRPs are not biddable products [4], [24]. A wind power ramping product (WPRP) has been proposed by [21] to allow wind resources to participate in the FRP market. The WPRP is designed for responding to the ramping requirements that are implemented by ISOs in order to ensure sufficient ramp capability in the RT operations. However, due to their relative low operational costs, the units may not be good options to procure the ramp capabilities [25], [26].

Reference [22] has proposed an integrated stochastic DA scheduling model for natural gas transportation networks, so as to solve the dispatchability issue of fuel-constrained natural gas-fired units. The proposed model dispatches generation and load resources and allocates FRPs for managing the variability and uncertainty of the renewable energy system. A non-deterministic FRP design is proposed in [23] to adequately allocate the ramp capacities in the DA market. The proposed model which is an adjustable robust UC model is based on a cost-free ramp capacity
procurement procedure. In spite of appealing results, [22] and [23] have respectively utilized stochastic programming and robust optimization methods, both of which are under the umbrella of advanced stochastic programming techniques [27]. However, due to scalability issues, the FRPs were originally designed for the deterministic markets, not the stochastic markets [6]. Furthermore, in the real-world electricity markets (e.g., CAISO and MISO), submission of separate bids for procuring ramp capabilities are not allowed. Instead, the prices for ramp up and ramp down capabilities are only based on the opportunity cost [4], [24]. Therefore, in compliance with common practice, there should be proper incentives in terms of opportunity cost for the generators that participate in FRP markets; however, this step is ignored in [20]–[23].

Overall, such structural changes, e.g., inclusion of FRPs in the DA market formulation, requires detailed analysis and design in order to ensure adequate operational flexibility, market efficiency, pricing, and transparency. In the same direction of CAISO’s proposal [10], the focus of this chapter is on the efficient design of DA FRPs.

1.2.3 Different options for increasing system ramp capability

Some studies have attempted to procure flexible ramp capabilities through the emerging techniques including, but not restricted to, electrical vehicles [28], [29], demand response programs [22], [29], and energy storage systems [20], [29]. However, the lowest cost option for improving the flexibility is associated with the improved market and operational design, such as improved weather forecasting, shorter granularity for market processes, and enhanced designs of ancillary service products [30], as shown in Figure 1.2. The reason why it is cheaper is because this option makes relatively small market and operational changes while utilizing the existing infrastructures [24].

![Figure 1.2 Cost comparison between different options to improve operational flexibility from lower cost on the left to higher cost on the right [24].](image)

In some other studies, wind generators themselves participate in providing ramp capabilities [21], [31]; however, as mentioned in the previous section, wind resources may not be good choices for providing FRPs.

Based on the above literature review, in this chapter, firstly, a DA market model is formulated analogous to the CAISO’s DA market model with addition of FRP constraints. More specifically, we have incorporated existing RT FRPs design of CAISO [5], [10], and [26] into a DA market model based on the hourly time interval resolution. Then, it sheds light on a subtle issue that can potentially happen in the next market processes after DA market, as a result of procurement of DA ramp capabilities only based on the hourly ramping requirements. Then, a new FRP design is proposed to address this issue. In the proposed formulation, the DA FRP design is modified to capture the impacts of the 15-min net load variability and uncertainty on the hourly ramping capabilities in the DA market, which improves upon the existing industry models. Finally, to
effectively evaluate different FRP designs from reliability and market efficiency points of view, a validation methodology is proposed that is similar to the RTUC process of the CAISO. The main outcomes of this chapter, with respect to augmenting operating markets with enhanced FRP design, are:

- Propose a novel DA FRP market design to enhance its performance in the next RT market processes.
- Create a validation methodology that mimics the RT market operation of CAISO in order to evaluate both efficiency and reliability of the proposed FRP design against general FRP design. Please note that with DA general FRP design we mean modeling of FRPs with the existing CAISO’s RT FRP design [5], [10], and [26] into the DA market with hourly scheduling.
- Conduct a comprehensive evaluation of the proposed FRP design, including its impacts on the market efficiency and system reliability.

1.3 Model formulation

The FRP design within the DA market model is formulated in Section 1.3.1, which is based on the CAISO’s DA market model and the hourly ramping requirements [10] and [26]. Then, a subtle issue that can potentially happen in the next market processes after DA market with the shorter granularity or time resolution is discussed in Section 1.3.2. This subtle issue is the motivation behind the enhanced FRP hourly design proposed in this chapter. Finally, in order to improve the DA FRP design presented by CAISO from reliability and efficiency points of view, the enhanced DA FRP design is presented in Section 1.3.3.

1.3.1 DA market model with general FRP design

One potential formulation of DA market model with hourly FRP requirement is presented in (1.1)-(1.23). The objective function is to minimize total operating costs (i.e., variable operating costs and fixed costs – no-load costs, startup costs, and shutdown costs) as follows:

\[
\text{minimize } \sum_g \sum_t (c_g^P P_{gt} + c_g^{NL} u_{gt} + c_g^{SU} v_{gt} + c_g^{SD} w_{gt})
\]

(1.1)

In (1.1), there are not costs associated with the ramp capability products. The reason is that the generators are not allowed to bid for the FRPs, rather they are compensated based on the opportunity costs caused by withholding their capacity or ramping capability in order to procure the FRPs. The objective function is subject to generator constraints as well as network constraints presented through (1.2)-(1.21). Minimum up and down time constraints of the generators are enforced by (1.2) and (1.3), while constraints (1.4) and (1.5) ensure ramp rate limits. Constraint (1.6) guarantees the power injection and withdrawal balance at each bus, and constraint (1.8) ensures the energy balance between load and generation across the system. Constraints (1.8) and (1.9) model the transmission line limits. Constraints (1.10) and (1.11) models the relationship between the commitment variable, the startup variable, and shutdown variable. Constrains (1.12)-(1.14) model the binary commitment decision \((u_{gt})\) and the startup \((v_{gt})\) and shutdown \((w_{gt})\) decisions, respectively.

\[
\sum_{s=t-UT_g+1}^{t} v_{gs} \leq u_{gt}, \forall g, t \in \{UT_g, \ldots, T\}
\]

(1.2)
\[ \sum_{s=t-DT_{g}+1}^{t} w_{gs} \leq 1 - u_{gt}, \forall g, t \in \{DT_{g}, \ldots, T\} \quad (1.3) \]

\[ p_{gt} - p_{gt-1} \leq R_{g}^{HR} u_{gt-1} + R_{g}^{SU} v_{gt}, \forall g, t \geq 2 \quad (1.4) \]

\[ p_{gt-1} - p_{gt} \leq R_{g}^{HR} u_{gt} + R_{g}^{SD} w_{gt}, \forall g, t \geq 2 \quad (1.5) \]

\[ \sum_{g \in g(n)} p_{gt} - \text{Load}_{nt} = p_{nt}^{inj}, \forall n, t \quad (1.6) \]

\[ \sum_{n} p_{nt}^{inj} = 0, \forall t \quad (1.7) \]

\[ \sum_{n} p_{nt}^{inj} \text{PTDF}_{nk} \leq P_{k}^{max}, \forall k, t \quad (1.8) \]

\[ -P_{k}^{max} \leq \sum_{n} p_{nt}^{inj} \text{PTDF}_{nk}, \forall k, t \quad (1.9) \]

\[ v_{gt} - w_{gt} = u_{gt} - u_{g,t-1}, \forall g, t \quad (1.10) \]

\[ v_{gt} + w_{gt} \leq 1, \forall g, t \quad (1.11) \]

\[ 0 \leq v_{gt} \leq 1, \forall g, t \quad (1.12) \]

\[ 0 \leq w_{gt} \leq 1, \forall g, t \quad (1.13) \]

\[ u_{gt} \in \{0,1\}, \forall g, t \quad (1.14) \]

Constraints (1.15)-(1.22) are associated to DA FRPs constraints in the DA market model. The generator output limits including up and down FRPs are presented by (1.15) and (1.16). Constraints (1.17) and (1.18) limits the ramp up and down capabilities to the hourly generators’ ramp rate capability, respectively. The total ramp up and down capabilities meet the up and down the hourly FRP requirement through (1.19) and (1.20), respectively. Finally, the hourly up and down FRP requirements are formulated through (1.21) and (1.22).

\[ p_{gt} + u_{r_{gt}} \leq P_{g}^{max} u_{g,t}, \forall g, t \quad (1.15) \]

\[ p_{gt} - d_{r_{gt}} \geq P_{g}^{min} u_{g,t}, \forall g, t \quad (1.16) \]

\[ u_{r_{gt}} \leq \text{Ramp}_{g} u_{g,t}, \forall g, t \quad (1.17) \]

\[ d_{r_{gt}} \leq \text{Ramp}_{g} u_{g,t}, \forall g, t \quad (1.18) \]

\[ \sum_{g} u_{r_{gt}} \geq \text{FRup}_{t}, \forall t \quad (1.19) \]

\[ \sum_{g} d_{r_{gt}} \geq \text{FRdown}_{t}, \forall t \quad (1.20) \]
\[ FRu_t = \max\{NL_{t+1}^{\max} - NL_t, 0\}, \forall t \leq 23 \]  
\[ FRd_t = \max\{NL_t - NL_{t+1}^{\min}, 0\}, \forall t \leq 23 \]

In the above market model, other ancillary service products are not considered to specifically study impact of FRPs; however, the model can also be generalized to include these products.

1.3.2 DA resource adequacy with hourly FRP requirements

The market model with the FRP design presented by (1.1)-(1.22) is similar to the CAISO’s DA market while entailing the discussed DA ramping capabilities to cover the hourly net load variability and uncertainty [10] and [26]. This FRP formulation is designed in such a way that resultant added ramp capabilities be able to respond to foreseen (variability) and unforeseen (uncertainty) changes in the net load between the hour \( t \) and the specified target hour \( t + 1 \), without consideration of magnitude of the intra-hour net load changes. The general FRP design disregards the intra-hour net load changes, while the procured DA FRPs are supposed to increase the ramp capabilities to meet the imbalances in the next RT markets such as the fifteen-minute market (FMM) [13]. The FMM seeks to meet the balance between supply and 15-min net load. The net load variation between the 15-min intervals can potentially experience steeper slope, which happen in a shorter time than the hourly net load variations – see case I and case II in Figure 1.3. With this discussion in mind, (1.1)-(1.22) model may not ensure adequate quantification of DA up and down ramp capabilities to cover potential deviations between the DA and the FMM, i.e., the next closest RT market process.

![Diagram](image)

**Figure 1.3** Hourly FRP versus 15-min variability and uncertainty.

In this chapter, the DA ramping requirements are quantified more accurately so as to capture not only the hourly net load variation, but also DA to FMM net load variation. More specifically, unlike the conventional DA FRP design, which immune solutions against only the hourly net load variability and uncertainty, the proposed model immune the solutions against both hourly and
intra-hour 15-min net load variabilities and uncertainties. The idea for the new formulation in this chapter is to design an effective set of constraints to secure the DA ramp capabilities for the variability and uncertainty in the 15-min granularity. The goal of the additional constraints would be:

- Improve quantity determination of the DA FRP requirements in order to improve reliability and efficiency in the next market processes without adding too much complexity to the existing FRP design.
- Enable more consistency between DA and RT scheduling frameworks.

It is anticipated that the proposed model will lead to more consistency between DA and RT scheduling frameworks and will reduce the need for expensive adjustments in the FMM. Note that the DA scheduling framework will be kept based on the hourly time frame to be consistent with current industry practice.

### 1.3.3 Feasibility of DA FRPs against intra-hour 15-min variability and uncertainty

As discussed in section 1.3.2, apart from the conventional hourly FRP requirements, 15-min ramping requirements should also be considered in the DA FRP requirements so as to immune the DA solutions against the 15-min net load variability and uncertainty, that is the granularity of the FMM. In this chapter, the concept of the intra-hour ramping requirement is incorporated in the DA FRP design in this chapter. To do so, additional FRP constraints are imposed to ensure DA ramp capabilities are adequate for accommodating 15-min net load variability and uncertainty.

Here, the enhancement for the DA ramp up FRP design is discussed, and ramp down FRP design enhancement can be similarly developed. The 15-min ramp up requirements should accommodate the foreseen variability in the 15-min net let plus a required level of confidence that can happen between two successive intervals as follows:

\[
FR_{\text{up}}^{i\text{h}}_{t,0\text{min}} = \max \{NL_{t,15\text{min}}^{\text{max}} - NL_{t,0\text{min}}, 0 \}, \forall t
\]  
\[ (1.23) \]

\[
FR_{\text{up}}^{i\text{h}}_{t,15\text{min}} = \max \{NL_{t,30\text{min}}^{\text{max}} - NL_{t,15\text{min}}, 0 \}, \forall t
\]  
\[ (1.24) \]

\[
FR_{\text{up}}^{i\text{h}}_{t,30\text{min}} = \max \{NL_{t,45\text{min}}^{\text{max}} - NL_{t,30\text{min}}, 0 \}, \forall t
\]  
\[ (1.25) \]

\[
FR_{\text{up}}^{i\text{h}}_{t,45\text{min}} = \max \{NL_{t+1,0\text{min}}^{\text{max}} - NL_{t,45\text{min}}, 0 \}, \forall t
\]  
\[ (1.26) \]

In (1.23)-(1.26), the first three 15-min ramp up requirements are related to the two successive 15-min net loads occur within hour \( t \). However, the last 15-min ramp up requirement is associated to the last 15-min interval of hour \( t \) and first 15-min interval of hour \( t + 1 \) (see Figure 1.3 for clarity). The enhanced DA FRP formulation introduces a new market design variable that represent the 15-min ramp capability of a generator in response to the 15-min net load changes. The total 15-min ramp up capabilities should be greater than the biggest 15-min requirements as (1.27) enforces.

\[
\sum_{g \in G} w_{g,t}^{i\text{h}} \geq \max (FR_{\text{up}}^{i\text{h}}_{t,0\text{min}}, FR_{\text{up}}^{i\text{h}}_{t,15\text{min}}, FR_{\text{up}}^{i\text{h}}_{t,30\text{min}}, FR_{\text{up}}^{i\text{h}}_{t,45\text{min}}), \forall t
\]  
\[ (1.27) \]
Where the $ur_{gt}^{ih}$ is limited to 15-min ramp rate capability as follows:

$$ur_{gt}^{ih} \leq \text{Ramp}_{g}^{15} u_{gt}, \forall g, t$$

Finally, in order to immune the DA ramp up capability against the 15-min variability and uncertainty, the ramp up capability in the DA market for each generator should be greater than the corresponding the 15-min ramp up capability as follows:

$$ur_{gt}^{ih} \leq ur_{gt}, \forall g, t$$

Similar formulation can be proposed for the enhanced DA ramp down FRP design, which are presented in (1.30)-(1.36):

$$FR_{down_{t,0min}}^{ih} = \max\{NL_{t,0min} - NL_{t,15min}^{min}, 0\}, \forall t$$

$$FR_{down_{t,15min}}^{ih} = \max\{NL_{t,15min} - NL_{t,30min}^{min}, 0\}, \forall t$$

$$FR_{down_{t,30min}}^{ih} = \max\{NL_{t,30min} - NL_{t,45min}^{min}, 0\}, \forall t$$

$$FR_{down_{t,45min}}^{ih} = \max\{NL_{t,45min} - NL_{t+1,0min}^{min}, 0\}, \forall t$$

$$\sum_{g} dr_{gt}^{ih} \geq \max (FR_{down_{t,0min}}^{ih}, FR_{down_{t,15min}}^{ih}, FR_{down_{t,30min}}^{ih}, FR_{down_{t,45min}}^{ih}), \forall t$$

$$dr_{gt}^{ih} \leq \text{Ramp}_{g}^{15} u_{gt}, \forall g, t$$

$$dr_{gt}^{ih} \leq dr_{gt}, \forall g, t$$

Finally, the proposed DA market model incorporating the proposed FRP design is constructed by adding constraints (1.23)-(1.36) to (1.1)-(1.22). Thus, the complemented DA market model is:

$$\text{minimize} \sum_{g} \sum_{t} (c_{g}^p p_{gt} + c_{g}^{NL} u_{gt} + c_{g}^{SU} v_{gt} + c_{g}^{SP} w_{gt})$$

subject to:

(1.2)-(1.22) and (1.23)-(1.36),

1.4 Validation methodology

A RT validation methodology is needed to compare the performance of the proposed FRP design and the general FRP design. Some of the previous work have implemented RT economic dispatch performed every 5-minute as the validation methodology while ignoring the other market
processes, i.e., the FMM, that happen between the DA market and RT economic dispatch market. The FMM is the closest market to the DA market which makes the preliminary modifications to the posted DA market solutions for meeting the RT load. Therefore, this market can be regarded as the first market which experiences the impacts of DA market reformulations and enhancements. Therefore, it can be a good candidate to evaluate the performance of the DA market decisions from the flexibility, economic and reliability points of view.

This chapter creates a RT validation methodology which mimics FMM of the CAISO. This methodology is performed after the solutions of DA market are posted and is used to conduct comprehensive reliability and economic comparisons under out-of-sample 15-min net load scenarios. The flowchart of proposed methodology is presented in Figure 1.4. Please note that with DA general FRP design we mean modeling of FRPs with the existing CAISO’s RT FRP design [5], [10], and [26] into the DA market with hourly scheduling.

![Flowchart of proposed methodology.](image)

**Figure 1.4 Flowchart of proposed methodology.**

### 1.4.1 CAISO’s FMM [32]

The CAISO’s FMM includes four RTUC runs, a market process for committing fast-start and short-start units at the 15-minute intervals, which is performed on a rolling-forward basis for each trading hour. The four RTUC processes, i.e., RTUC#1, RTUC#2, RTUC#3, and RTUC#4, include time horizons of 60-105 minutes spanning from the previous trading hour and the current trading hour. The binding interval is the second interval of the RTUC run horizon, and the rest are advisory intervals.
1.4.2 FMM validation phase

For the sake of simplicity in the proposed validation methodology, without loss of generality, all the four RTUC runs are combined into a one-process RTUC run for each trading hour. The one-process RTUC run has the same number of intervals as RTUC#1 (i.e., seven 15-min intervals) and is performed in the same time schedule as the RTUC#1 is performed. It approximately starts 7.5 minutes prior to the first trading hour for $T - 45$ minutes to $T + 60$ minutes, where $T$ is the top of the trading hour. Since each original RTUC process has 1 binding interval, the one-process RTUC simulation would have four successive binding intervals out of 7 intervals. It is pertinent to note that the binding intervals of trading hour $T - 1$ that are also a part of intervals of the next trading hour $T$, are used and kept the same (in terms of both dispatch and commitment) in the one-process RTUC run of the trading hour $T$.

Overall, for each trading hour, only one RTUC process is solved with four binding intervals, and it will be continued on a rolling-forward basis for the next trading hours. After performing all one-process RTUC runs, the final schedule for the 96 intervals are achieved through putting together all the binding intervals. Furthermore, in each one-process RTUC run, power balance violations are allowed to occur if there is insufficient ramp capability to follow the sudden net load changes. The insufficient ramp capability is the result of situation, wherein offline fast-start units and online flexible units are not able to provide sufficient flexibility.

1.4.3 Data transferring from DA market to FMM

In the proposed validation methodology, the DA market solutions are transferred and embedded into the FMM for running the multiple one-process RTUC problems. The commitments obtained from DA market model are kept fixed for long-start units in the FMM. Furthermore, the dispatch modification of the committed units in the DA market model is limited to their 15-min ramp rate in the first interval of RTUC process so as to not deviate much from DA market dispatch decisions. Furthermore, the fast-start and short-start units can be further committed to follow the realized net load if the ramping shortage occurs.

1.5 Numerical studies and discussion

A 118-bus IEEE test system is employed for performing simulations. CPLEX v12.8 is utilized to solve the proposed DA and RTUC optimization models on a computer with an Intel Core i7 CPU @ 2.20 GHz, 16 GB RAM, and 64-bit operating system.

1.5.1 System Data and assumptions

The 118-bus IEEE test system has 54 generators, 186 lines, and 91 loads [33]. The uncertainty in this work is introduced by renewable resources such as the solar and wind. The hourly and 15-min forecasted net loads have been depicted in Fig 1.5. Note that the average of 15-min net loads during each hour is almost equal to the corresponding forecasted hourly net load. Simulation results are performed for two days. The first day has a peak load of 3019 MW, while the peak load of the second day is about 4519 MW. It is evident that the second day is prone to have more severe net load variability and uncertainty (see the hourly net loads presented in Figure 1.5 which are the
percentage of peak load). The error of the hourly net load forecast is assumed to have a Gaussian distribution with zero mean and ~5% standard deviation. Also, 95% confidence level, i.e., 1.96 standard deviations, is considered for hourly ramping requirements, i.e., (1.21) and (1.22). Based on the total probability theory [16], [34], the exact relationship between standard deviation of error of hourly and 15-min net load can be quantified, i.e., $\sigma_{\text{hourly}} = 2\sigma_{\text{15-min}}$. By using this formula, the 15-min ramping requirements formulated by (1.23)-(1.26) and (1.30)-(1.33) can be calculated. The confidence level for the 15-min ramping requirements is also 95%. Fig 1.6 illustrates the hourly net load bounds versus the 15-min net load bounds, both of which are based on the 95% confidence level. As it can be seen from this figure, the bounds of 15-min net load are located inside the hourly bounds.

![Hourly net load versus 15-min net load](image1)

**Figure 1.5** Hourly net load versus 15-min net load.

![Hourly net load bounds versus 15-min net load bounds](image2)

**Figure 1.6** Hourly net load bounds versus 15-min net load bounds for 95% confidence level.
In the validation methodology, 18 gas and oil generators which have the maximum capacity up to 30 MW are considered as the fast-start and short-start units. These units can be further committed and dispatched in the FMM. The violation is allowed to occur if the newly committed units are not fast enough to follow the realized net load. The penalty price is in term of VOLL, which is chosen as $10000/15min. Additionally, 500 different 15-min net load scenarios are generated based on the 15-min net load uncertainty for this out-of-sample validation phase.

1.5.2 Simulation results

Figure 1.7 compares the RTUC operating costs (excluding the VOLL cost) against the violation for the corresponding FRP design for the out-of-sample net load test scenarios. It is worth mentioning that if the operating costs include the violation cost (similar to what is usually done in the literature [15], [16], [25], [29], [31], [35]), the comparisons between the operating costs of different models may be subjective as the results are sensitive to the choice of VOLL. However, by the removing the cost associated with the VOLL from the operating costs, the comparisons can be more objectively conducted. According to Figure 1.7, it can be observed that the proposed approach is effectively capable of reducing the violation (caused by the system inability to follow the net load changes), since it preemptively takes into account the impacts of 15-min net load variability and uncertainty on the DA FRP decisions. In Figure 1.7, the results are presented not only for the FMM in which the FS units have the same bids as their bids in the DA market, but also for the FMM with increased bid (by 15% of the DA bids) of only the FS units. From the economic point of view, it can be seen that the CAISO’s proposed approach has operating costs that are more than or mainly comparable to those of proposed approach. In other words, although considerably less load has been served for CAISO’s proposed approach (load shedding ranged from 0 to 600 MW), only 36% and 15% of obtained RTUC operating costs are lower than those of the proposed approach for both original and increased bids. These results demonstrate the performance of proposed FRP design for improving the RT market efficiency from the economic aspects for different bids in the RT market. Furthermore, from differing the operating cost for different bids in the FMM while having violation, it can be inferred that the FS units have been committed to follow the load, but they have not been fast-enough to accommodate steep 15-min net load changes.

The operating costs (excluding the violation cost) versus the increased number of 15-min commitments of FS units for the proposed approach and CAISO’s proposed approach are compared in Figure 1.8. The increased number of FS units 15-min commitments represents the increase of the total commitment of FS units from DA market to RT market for the 15-min intervals. From Figure 1.8., it is apparent that the proposed approach reduces the need for the FS units that were not committed in the DA market to become committed in the FMM. The lower number for commitment increase is an indicative of less discrepancy between the DA and RT markets, which potentially can lead to a reduction of the necessity for expensive adjustments in the RT markets. Furthermore, according to the Figure 1.8, for both FRP designs, the scenarios with violation tends to have the highest increase in the commitment number, which shows that allowance for violation is regarded as the last option in the FMM because of its high cost to the system.
Figure 1.7 RTUC operating cost (excluding violation cost) versus violation for the first test day.

Figure 1.8 RTUC operating cost versus increased number of 15-min commitments of FS units for the first test day (“scens. with viol” stands for scenarios with occurrence of violation; “scens. without viol” stands for scenarios without occurrence of violation).

Figure 1.9 compares increased number of 15-min commitments of FS units against the violation for the out-of-sample net load test scenarios in the validation phase for the different FRP designs. It can be seen that for almost all the out-of-sample scenarios, the proposed approach provides pareto optimal solutions (with respect to increased number of 15-min commitments of FS units and the violation) compared to the general approach, wherein less violation and increased commitment numbers occurs in the FMM. These results show the efficiency of the proposed FRP design in quantifying more adaptive FRP requirements in the DA market with respect to the RT condition.
Figure 1.9 Increased number of 15-min commitment of the FS units versus violation for the first test day in RTUC.

Owing to the discounting the impacts of the nodal FRP deployment on the physical network limitations (e.g., congestion) in the DA market model, the proposed approach is not expected to fully remove additional commitment of FS units in the FMM. That is why in the Figure 1.9, the FS units are still required to be turned on to follow the net load. Future work should investigate how to address the deliverability issue associated with the post-deployment of FRPs in the RT markets.

Table 1.1 lists the results of DA markets with their corresponding results in the FMMs for both the FRP designs. According to the table, although, the proposed approach results in the higher DA operating costs in comparison to the other model, it has much fewer expensive adjustments in the FMM and has less final operating cost in the FMM, as can be observed by comparing the last three rows of Table 1.1. It is evident from this table that the DA operating costs have not been changed with respect to the corresponding average RTUC operating costs for the proposed approach, while huge difference can be seen for the other approach. The average of final operating costs of the general approach over all of the out-of-sample scenarios equals to $3060k, which is considerably higher than that of the proposed method, i.e., $841k. On the other hand, the operating cost standard deviation of the proposed approach is effectively less than that of the general approach, which demonstrates the robustness of the proposed approach performance in response to the different realized out-of-sample net load scenarios. Finally, from evaluating the results for increased bid of FS units in Table 1.1, it can be seen that the performance of the proposed approach is consistent with the non-increased bids results explained above.
Table 1.1. Results for DA market and FMM across 15-min net load scenarios for the first test day

<table>
<thead>
<tr>
<th>Approach</th>
<th>General FRP design</th>
<th>Proposed FRP design</th>
<th>General FRP design (with increased bid of FS units in FMM)</th>
<th>Proposed FRP design (with increased bid of FS units in FMM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DA operating cost (k$)</td>
<td>833</td>
<td>841</td>
<td>833</td>
<td>841</td>
</tr>
<tr>
<td>Final operating costs in RTUC</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ave (k$)</td>
<td>3060</td>
<td>841</td>
<td>3062</td>
<td>843</td>
</tr>
<tr>
<td>Standard deviation (k$)</td>
<td>1112</td>
<td>61</td>
<td>1112</td>
<td>55</td>
</tr>
<tr>
<td>Max (k$)</td>
<td>6431</td>
<td>1511</td>
<td>6432</td>
<td>1512</td>
</tr>
</tbody>
</table>

In Table 1.2, the average [violation] and standard deviation [violation] are statistical measures that are utilized to assess and compare the reliability extent that can be resulted from performing two approaches. It is evident that the lower value for statistical measures can be used as a gauge for determining the more reliable model. According the results, the proposed approach outperforms the CAISO’s proposed approach with respect to the both statistical measures because the proposed FRP design considers the 15-min net load changes impacts on the dispatch position and commitment obtained from the DA market to attain more adequate responsiveness from the existing flexible resources in RT (resource adequacy). Three additional reliability metrics, i.e., the sum of the violations in the 96 intervals of the RTUC over the out-of-sample scenarios (Σ violation), the number of scenarios with non-zero violations (# Scenarios with violation), and the maximum reported violation (max violation) are also reported from the RTUC runs. By comparing the first two metrics, it is apparent that the proposed approach generally tends to have solutions that are more reliable. Additionally, the max violation of the proposed approach is 67 MW which is effectively less than 560 MW for max violation of the CAISO’s proposed approach. Note that effectively decreasing the max violation is mainly useful when the ISO is interested in decreasing the worst-case violation.

The increased number of 15-min commitments of FS units presented in Table 1.3 compares the 15-min commitments number of FS units that are turned on in the RTUC to follow the 15-min net load changes, in addition to the FS units that were previously committed from the DA market. Four metrics, which are indicative of the discrepancy between DA and RT operations, are presented in this Table. It can be observed that the proposed model effectively reduces the need for the expensive FS units to be additionally committed in the RT market by turning on cheaper and potentially longer run units which are available in the DA market. Thus, it can be safe to conclude that, since the proposed model tends to generally give the RTUC decisions with fewer commitments of FS units, it more likely results in lower final operating cost.

Overall, according to the results from Tables 1.1, 1.2, and 1.3, it can be concluded that, the proposed approach enhances the quantity allocation of FRPs by making relatively small market changes compared to general approach, which leads to (i) less expected final operating cost, (ii)
higher reliability as the power system gets close to its RT operation, and (iii) less discrepancy between DA and RTUC decisions.

Table 1.2. Violation Comparison in RTUC across 15-min net load scenarios for the first test day

<table>
<thead>
<tr>
<th>Metric</th>
<th>General FRP design</th>
<th>Proposed FRP design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average [violation] (MW)</td>
<td>222</td>
<td>1</td>
</tr>
<tr>
<td>Standard deviation [violation] (MW)</td>
<td>111</td>
<td>5</td>
</tr>
<tr>
<td>Σ violation (MW)</td>
<td>111 178</td>
<td>355</td>
</tr>
<tr>
<td># Scenarios with violation</td>
<td>488</td>
<td>21</td>
</tr>
<tr>
<td>Max violation (MW)</td>
<td>560</td>
<td>67</td>
</tr>
</tbody>
</table>

Table 1.3. Results for increased number of 15-min commitments of FS units in RTUC across 15-min net load scenarios for the first test day

<table>
<thead>
<tr>
<th>Metric</th>
<th>General FRP design</th>
<th>Proposed FRP design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Σ increased commitment of fast-start units</td>
<td>93550</td>
<td>11865</td>
</tr>
<tr>
<td>Ave [Increased commitment of fast-start units]</td>
<td>187</td>
<td>24</td>
</tr>
<tr>
<td>Max increased commitment of fast-start units</td>
<td>302</td>
<td>155</td>
</tr>
<tr>
<td># Scenarios with increased commitment of fast-start units</td>
<td>500</td>
<td>490</td>
</tr>
</tbody>
</table>

The proposed approach is also assessed on the out-of-sample net load scenarios from one additional test day in order to evaluate its robustness. The second test day has higher peak demand in comparison to the first test day, so, within its the 15-min net load, more severe variabilities can potentially occur. Figure 1.10 compares the RTUC operating costs (excluding the VOLL cost) against the violation for the corresponding FRP design for the out-of-sample net load test scenarios. The operating costs (excluding the violation cost) versus the additional 15-min commitments of FS units for the proposed approach and general approach are elaborated in Figure 1.11. Figure 1.12 compares additional 15-min commitments of FS units against the violation for the out-of-sample net load test scenarios in the validation phase for the different FRP designs. Finally, Table 1.4 summarizes the corresponding results across 15-min net load scenarios for the second test day. Similar to the first test day, the proposed ramp capability products have tendency to be more reliable but result in the higher DA operating costs. Consistent with the first test day results, the proposed model outperforms the CAISO’s model by improving the reliability of the DA market solution while also reducing the final operating costs in the FMM.
Figure 1.10 RTUC operating cost (excluding violation cost) versus violation for the second test day.

Figure 1.11 RTUC operating cost versus increased number of 15-min commitments of FS units for the second test day (“scens. with viol” stands for scenarios with occurrence of violation; “scens. without viol” stands for scenarios without occurrence of violation).
Table 1.4. Results for DA market and FMM across 15-min net load scenarios for the second test day

<table>
<thead>
<tr>
<th>Approach</th>
<th>General FRP design</th>
<th>Proposed FRP design</th>
<th>General FRP design (with increased bid of FS units in FMM)</th>
<th>Proposed FRP design (with increased bid of FS units in FMM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DA operating cost (K$)</td>
<td>1315</td>
<td>1329</td>
<td>1315</td>
<td>1329</td>
</tr>
<tr>
<td>RTUC operating cost</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average (K$)</td>
<td>5752</td>
<td>1326</td>
<td>5754</td>
<td>1326</td>
</tr>
<tr>
<td>Standard deviation (K$)</td>
<td>1824</td>
<td>72</td>
<td>1825</td>
<td>80</td>
</tr>
<tr>
<td>Max (K$)</td>
<td>11370</td>
<td>2377</td>
<td>11371</td>
<td>2475</td>
</tr>
<tr>
<td>Violation Comparison in the RTUC</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma$ violation (MW)</td>
<td>221824</td>
<td>421</td>
<td>221855</td>
<td>448</td>
</tr>
<tr>
<td>Average [violation] (MW)</td>
<td>444</td>
<td>1</td>
<td>444</td>
<td>1</td>
</tr>
<tr>
<td>Standard deviation [violation] (MW)</td>
<td>182</td>
<td>7</td>
<td>183</td>
<td>8</td>
</tr>
<tr>
<td>Max violation (MW)</td>
<td>1007</td>
<td>105</td>
<td>1007</td>
<td>115</td>
</tr>
<tr>
<td># Scenarios with violation</td>
<td>499</td>
<td>17</td>
<td>499</td>
<td>16</td>
</tr>
<tr>
<td>Increased number of 15-min commitments of FS units in the RTUC</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma$ increased commitment of fast-start units</td>
<td>84004</td>
<td>2930</td>
<td>84282</td>
<td>2898</td>
</tr>
<tr>
<td>Average [increased commitment of fast-start units]</td>
<td>168</td>
<td>6</td>
<td>169</td>
<td>6</td>
</tr>
<tr>
<td>Max increased commitment of fast-start units</td>
<td>331</td>
<td>115</td>
<td>331</td>
<td>118</td>
</tr>
<tr>
<td># Scenarios with increased commitment of fast-start units</td>
<td>500</td>
<td>88</td>
<td>500</td>
<td>88</td>
</tr>
</tbody>
</table>
The high penetration of the renewable resources is imposing new challenges (e.g., ramp capability shortage) to modern power systems by intensifying the uncertainty and variability in the system net load. The shortage in the system’s ramp capabilities not only can jeopardize its reliability, but also can cause inefficiencies in the RT markets. Energy markets are evolving in order to overcome such challenges by implementing new ancillary service products called FRPs. There have been proposals to implement the FRPs into the practical DA markets such as CAISO’s DA market in order to ensure resource adequacy to respond to net load variability and uncertainty in the next market processes such as the FMM and the RT economic dispatch. The DA formulation with FRPs proposed by industry is based on the hourly ramping requirements with the aim to enhance ramping capabilities in the shorter scheduling granularity, e.g., 15 min. However, it ignores the impacts of steep 15-min net load changes in the procured hourly FRPs. To address this challenge, this chapter proposed a novel DA FRP design to reposition the flexible resources in order to better respond to the RT imbalances caused by ramping shortages. The proposed DA FRP formulation are designed in such a way that not only are the hourly net load variability and uncertainty taken into account, but also impacts of the 15-min net load variability and uncertainty are captured in the procured DA FRPs. The proposed model enhances the quantity allocation of FRPs by making relatively small market changes compared to CAISO’s proposed FRP design, which leads to less expected final operating cost in the FMM, higher reliability as the power system gets close to RT operation, and less discrepancy between DA and RTUC decisions.
2. Pricing Implications and Objective Function Evaluation for Markets with Stochastic Formulations

2.1 Introduction

Electric systems are considered the greatest achievement of the 20th century by the National Academy of Engineering [36]. Operational scheduling of this most sophisticated engineering system necessitates consideration of both economical and reliability aspects. However, due to its complexity, it is next to impossible to model all system components, capture detailed characteristics of all system assets (e.g. dynamics), and satisfy all reliability requirements all together. Hence, existing operational scheduling models are designed with approximations, e.g., DC approximation of power flow and approximations of the $N$-$1$ reliability criteria. The $N$-$1$ reliability criteria, defined as the loss of a single element such as a generator or non-radial transmission asset should not cause involuntary load shedding [37], is mandated by the North American Electric Reliability Corporation (NERC). It is worth noting that this mandate makes the underlying market model stochastic in nature.

Some of the existing electricity market operators solve day-ahead (DA) Security-Constrained Unit Commitment (SCUC) models with an approximation of the $N$-$1$ reliability mandate via a proxy reserve requirement [38], where total of contingency reserves across the power system is forced to be greater than a certain threshold. Such SCUC models do not account for and guarantee post-contingency reserve deliverability. To compensate for the various approximations in market models, the operator may intervene and make adjustments in the market solutions. Such interventions are referred to as out-of-market corrections (OMC) [39] or out-of-merit adjustments [40], [41] which include committing additional generation units or changing the dispatch of committed units. After an $N$-$1$ reliable dispatch solution is obtained, the settlements are calculated. The existing practice is to use the locational marginal prices (LMPs) from the DA SCUC (which have not been affected by the OMC) and the modified $N$-$1$ reliable dispatch solution to calculate market settlements. This pricing scheme may not be incentive compatible for market participants that provide reliability since it does not capture the true value of achieving $N$-$1$ reliability.

Furthermore, some ISOs, e.g. Midcontinent ISO (MISO), model zonal reserve requirements [42]. This model is unable to differentiate the generators within each zone regardless of their ability or inability to deploy reserve due to system congestion. Therefore, such a market model is also unable to account for the value of reserve provided by each generator in a nodal basis and consequently might not incentivize resources to do as directed by the market. These inabilitys to impact market prices will lead to the missing money problem (i.e., insufficient compensation received by generators) and cause a natural unfairness as market participants might not be dispatched fairly with these pricing schemes.

The pricing schemes are a critical part of an efficient market design; the market settlements based on an incentive-compatible pricing scheme motivate efficient operational and investment decisions [43]. However, in the electricity markets, compensation mechanisms are still a subject of debate. The LMPs are used to compensate for the provision of energy. Many ISOs use the duals of proxy reserve requirements to compensate the generation units that provides ancillary services [44]. However, there is still a controversy over the missing money problem which can be caused due to
different reasons. Some ISOs now have capacity market auctions in an attempt to satisfy market participants for the peak load reliability concerns that reflect an estimated demand at associated price levels [45]. Others, such as the MISO, use convex hull pricing, which is an alternative pricing scheme in non-convex markets to clear the market while also minimizing the total uplift payments [46]. While these approaches are developed in an attempt to improve compensation mechanisms, the issue still persists; existing pricing schemes do not sufficiently reflect the true value of providing energy in contingency periods, as these uncertain events are not explicitly included in the market models.

Explicit representation of the contingencies via a two-stage stochastic SCUC, extensive-form SCUC (ESCUC), enables inclusion of value of reliability services into the LMPs and can reduce the missing money problem. The ESCUC optimizes the recourse decision variable (or corrective actions) while explicitly considering the network constraints for the post-contingency state, which ensures nodal reserve deployment considering physical network limitations. It is pertinent to note that existing optimization solvers still have difficulties to handle stochastic programming problems within the suitable time for the large-scale systems.

In addition, some prior work proposed approaches based on the estimated post-contingency states using pre-determined participation factors. These approaches fill the gap between the traditional deterministic and the future stochastic models by representing generator or non-radial line contingencies explicitly without any second-stage recourse decisions. The primary influence of these changes can include new congestion components within the traditional LMPs, which represent the impacts of congestion during the post-contingency states for the modeled contingencies.

With the explicit modeling of contingency events within the state-of-the-art market auction models, the industry is actually moving away from the deterministic program formulations to the stochastic program formulations. The anticipated pricing implications with this transition include market prices, i.e., LMPs, that potentially can better reflect the quality of service provided by market participants in response to contingencies. However, there are unsolved issues regardless of the choice of uncertainty modeling: the impact of such contingency modeling on market pricing and how the generators should be compensated for providing $N$-1 reliability services.

Also, with the existing proposals to change the market auction models from a deterministic structure to a stochastic structure, efficiency and incentive-compatibility of such market designs should be closely studied. Majority of prior work, which have proposed a stochastic market design, adopt an objective function that optimizes expected cost of scenarios, in which several uncertain events are modeled along with their corresponding probabilities. However, there is a number of reasons why optimizing over an expected cost may not be the best choice. Firstly, it is difficult to accurately predict the probabilities of outages, which itself can lead to different pricing implications and market solutions. Furthermore, during emergency conditions in real-time, the operator may not exactly follow the proposed corrective actions since the intention during an emergency condition is not to minimize cost; rather, the goal is to recover from the event as quickly as possible to prevent future unforeseen problems that could lead to cascading outages. However, limited studies have analyzed and included the market design evaluation related to formation of objective function.
2.2 Literature review

The basic UC model, disregarding contingencies, is well-studied; related efforts were overviewed by [47], [48] and more recently by [27]. After the 2003 Northeast blackout in North America, the SCUC models with enhanced contingency constraints have received increasing attentions from both industry and academics.

Different approaches for the SCUC problem in the context of managing uncertain contingencies can be mainly categorized into three subgroups. The first subgroup is the SCUC problem with proxy reserve policies that implicitly determine reserve requirements to respond to the contingencies. The second subgroup for meeting the N-1 reliability mandate is through using the stochastic programming approaches, e.g., the two-stage stochastic SCUC or the ESCUC, wherein the two stages represent the pre-contingency and post-contingency states. The third subgroup is associated with the estimation of the post-contingency state of line flows due to the contingencies. In the following subsections, a literature survey related to these approaches is presented.

2.2.1 Existing proxy reserve policies

Proxy reserve requirement and reserve zones are predominantly employed by a majority of the system operators in their DA SCUC market model, in attempt to achieve the N-1 reliability criteria. Today, the existing reserve policies mostly make some approximations in the market model. The above contingency reserve requirements are a result of approximating the N-1 reliability criteria. They focused on quantitative aspects of reserves, rather than modeling the N-1 contingency events explicitly within the SCUC market model.

2.2.1.1 Proxy reserve requirements

The proxy reserve requirements simply entail the contingency reserve to be greater than a certain percentage of the peak load and/or to be adequate in order to compensate for the loss of any single generation unit. Moreover, there are some other rules, such as the 3+5 rule suggested by the National Renewable Energy Laboratory (NREL), which simply suggests that the system-wide reserve should not be less than 5% of the short-term forecasted wind power or less than 3% of the load [49].

Although the proxy reserve criteria are generally easy to implement, they ignore the intrinsic stochastic nature of such models and rely on approximation of the N-1 mandate. With such approximations, where post-contingency operating states are not taken into account when making the scheduling decisions, there is no guarantee that procured reserves by the market will be deliverable without violating transmission constraints. After the market is cleared, the operator may change the market solutions to ensure the N-1 reliability criterion. In the industry, these specific actions which happen outside the market are referred to as out-of-merit energy/capacity [50], manual dispatches [51], security corrections [52], uneconomic adjustments [53], and exceptional dispatches [54]. These actions for making the market solution N-1 reliable are called the OMC [55] in this chapter. Newly committed units during OMC do not directly influence LMPs and they are either compensated based on the market LMP or based on their market bid. If needed, the operator will compensate them with an uplift payment. This inability to impact market prices
will cause a natural unfairness as market participants might not be dispatched fairly with this mechanism.

### 2.2.1.2 Regional reserve requirements

As discussed in section 2.1.1.1., the proxy reserve policies may not guarantee reliable operations because they are only based on the quantitative rules. In order to enhance deliverability issue associated with the contingency-based reserve, the zonal reserve model is being adopted by some ISOs (e.g., MISO) [42]. However, this zonal reserve model still includes approximations, such as treating all the locations inside a zone as the same. Consequently, it is unable to differentiate the generators within each zone regardless of their ability or inability to deploy reserve due to system congestion. It is pertinent to note that discounting intra-zonal congestion can itself lead to inaccurate inter-zonal flow calculations; potentially causing nodal reserve deliverability issue. Thus, the ISOs are usually forced to perform the OMC actions, such as manually disqualify generators or turning on additional units in local areas to account for modeling approximations and inaccuracies. Accordingly, such a market model is also unable to properly account for the value of reserve provided by each generator.

### 2.2.2 Stochastic programming

A more accurate solution to handle the uncertain disturbances is to solve a stochastic programming. The stochastic programming has been utilized in the optimization market models (e.g., SCUC) to address discrete disturbances (e.g., asset outages) and continuous disturbances (e.g., renewable resources and demand uncertainty). For example, in the context of discrete disturbances, uncertain contingency events are explicitly captured in a two-stage stochastic SCUC model, where the two stages are the representation of the pre-contingency and post-contingency states [56], [57]. For the modeled events, the contingency-based reserves are guaranteed to be deliverable since the network constraints and corrective actions are explicitly formulated. It is pertinent to note that no pre-defined reserve requirements are necessary because of the endogenous acknowledgement of uncertainty and more accurate modeling of the pre-contingency and post-contingency operating states.

A two-stage stochastic SCUC model that includes uncertain contingencies is presented in [56], wherein the focus is on the stochastic programming formulation and effective numerical methods. Reference [57] proposes a two-stage stochastic program approach to handle uncertain contingencies. Then, the benefits of combining both stochastic methods and reserve requirements are assessed in efficiently managing uncertainty in the stochastic model. However, in [56], [57], it has not been discussed how generators should be compensated for their procured energy and N-1 reliability services in such models.

Pricing analyses for stochastic security-constrained approaches in the energy and reserve markets are presented in [58]–[61]. Reference [58] investigates a method to compensate generators for the energy and reserve that they provide through an optimization problem. The model accounts for the UC, corrective security actions, and the transmission line limits, where reserves are offered by both generation units and loads. The authors in [58] derive a pricing mechanism where the generators are compensated for the modeled N-1 scenarios. However, results are shown for only a single time period while the formulated model allows for load shedding. It is worth noting that the
fixed cost of load shedding is hard to estimate since: (1) it is not necessarily proportional to bids submitted to the market as energy bids, and (2) it is not the same (fixed cost) for different sectors (industry, domestic, commercial). Furthermore, [58] does not consider other complex constraints, such as minimum-up and minimum-down time constrains.

The authors in [59], [60] formulate a multi-period stochastic SCUC model that takes into account the post-contingency states for pre-selected contingencies. The authors state that there is a “set of random generator and line outages with known historical failure rates.” The proposed stochastic program optimizes the base-case along with the expected cost of the post contingency states. However, their model allows for load-shedding and “pre-selected” contingencies. In [61], the authors utilize a two-stage stochastic linear program to propose different methods to compensate generators. The authors use a linear programming formulation, not mixed-integer linear programming. The models presented in [59]–[61] allow for load-shedding through the value-of-lost-load (VOLL); however, this approach is subjective since the obtained results are sensitive to the choice of VOLL.

A SCUC auction model for energy and contingency-based ancillary services is presented in [62] in order to optimize reserve requirements by explicitly simulating contingencies rather than implementing the fixed reserve requirements. The original auction problem is the decomposed into a master problem along with subproblems for addressing the computational complexity.

However, a comprehensive economic evaluation for the stochastic two-stage SCUC and its comparison with other contingency modeling approaches have neither been included nor analyzed into the studies [56]–[62]. Specifically, this evaluation can include an assessment of the generation cost, generation revenue, generation rent, load payment, and congestion rent.

Apart from economic evaluation, it is also important to assess performance of stochastic market formulation. Generally, multiple scenarios are modeled in the two-stage stochastic SCUC models, wherein the objective function is an expected cost of the base-case along with the expected cost for corrective actions (or recourse decisions). The base-case itself is a scenario with high-probability, wherein “the power system is in normal steady-state operation, with all components in service that are expected to be in service” [63]. Traditionally, two-stage stochastic programs are formulated in a way that the objective function is optimized over an expectation [56], [57], [59]–[61], [64]–[66]. There are a number of reasons why optimizing over an expected cost may not be preferred. First, it is difficult to accurately predict the probabilities of outages. Second, the amount of discretionary, ad-hoc corrections made between the day-ahead market and the real-time market are not accurately captured. Third, there is also the issue that market operators in real-time may not implement the proposed corrective actions; as a fast recovery of the system security is more of a concern compared to the least cost path to recovering the system security. Finally, when the objective function is modified to include the probabilities and to optimize over the expected cost, there are different pricing implications than when the system is optimizing only over the base-case cost. There are other studies [58], [62], [63] that minimize the base-case cost as their objective functions, while the model is a stochastic two-stage SCUC problem including explicitly modeling of the post-contingency constraints. Therefore, a comprehensive study is needed to examine the implications of an expected cost objective function versus what is commonly used today in the industry, which only optimizes the base-case cost.
2.2.3 State-of-the-art market auction models with estimated post-contingency states

The gap between the traditional deterministic and future stochastic models is filled by the use of market models with the pre-determined participation factors. The aim of these approaches is to explicitly represent generator and/or non-radial line contingencies without any second-stage recourse decisions.

In [67] line outage distribution factors (LODFs) is used to explicitly model the transmission line contingencies in the DA SCUC models. Another example is the CAISO, which intends to enhance its scheduling model to explicitly enforce the post-contingency transmission constraints for the generator contingencies using generator loss distribution factors (GDF) [68].

Reference [69] proposes a set of G-1 security constraints, thereby, contingency reserves are allocated more efficiently in the system with respect to post-contingency dispatch feasibility. The proposed model disqualifies the undeliverable contingency-based reserves in the post-contingency states and assigns those reserves at other locations to guarantee a market solution that is G-1 reliable on a locational basis. A reserve response set model in order to enhance existing proxy reserve policies is proposed in [70]. This model is aimed to address the deliverability issues related to proxy reserves by modeling the estimated post-contingency impacts of nodal reserve deployment for a few critical transmission elements. The performance of the proposed reserve model is compared against an ESCUC model and contemporary proxy reserve policies from only operating costs and reliable point of views.

Among the above models, transmission line contingencies within state-of-art market models based on LODFs have been well implemented in industry practice; however, there has been limited efforts for investigating its pricing implications and market settlements compared to other contingency modeling approaches.

Based on the above literature survey, this chapter investigates:

- Impacts of uncertainty modeling strategies on electricity market outcomes, pricing, and settlements are analyzed. The LMP comprises three components, i.e. energy, congestion, and loss components. This chapter uses a method based on the duality theory to shed light on the LMP calculation in a stochastic market model; this theoretical method confirms that the value of providing N-1 reliability services can be reflected in the LMPs of such stochastic market models. This pricing scheme is then compared to the two state-of-the-art market auction models, where the corrective actions to achieve an N-1 reliable solution is postponed to OMC. Also, the market settlements of these models are calculated and compared. With these analyses, this chapter seeks to inform market stakeholders about the impacts of contingency modeling approaches in the DA process and their implications on pricing and settlements.

- The choices of objective function for the stochastic market models are analyzed; a stochastic market design with an expected cost objective function is examined and compared with the base-case costs minimization objective function from two aspects: (i) realized cost during N-1 contingencies and (ii) effects of inaccurate calculation of the probabilities on market outcomes.
2.3 Model Formulation

The uncertainties in the power systems are caused by discrete disturbances (e.g., generator, transmission line, and transformer outages) or continuous disturbances (e.g., forecasting error of loads and renewable resources). In this work, just the former is considered as the source of the uncertainty. Previous studies in the area of managing uncertainty associated with contingencies in the SCUC problem can be categorized as follows: (i) the proxy reserve policies, (ii) modeling the system response via participation factors, e.g. LODF and GDF, (iii) stochastic programming approaches, e.g., ESCUC, and (iv) chance-constrained optimization and robust optimization. The main focus of this work is on uncertainty modeling using (i), (ii), and (iii). In the following three subsections, model formulations related to these approaches are presented.

2.3.1 SCUC with deterministic proxy reserve requirement

A SCUC market model with deterministic proxy reserve requirement is presented in (2.1)-(2.19), which is similar to the model in [70]. The objective function, minimizing total operating costs, is presented in (2.11). In this formulation, constrains (2.12) and (2.13) model the relationship of the unit commitment variables with the startup and shutdown variables, respectively. Constrains (2.14)-(2.17) model the binary commitment \((u_{gt})\) decision and the startup \((v_{gt})\) and shutdown \((w_{gt})\) decisions, respectively. Minimum up and down time constraints are enforced by (2.18) and (2.19), while constraints (2.10) and (2.11) ensure ramp rate limits. Constraint (2.12) guarantees the balance between the power injection and withdrawal at every bus and constraint (2.13) ensures the energy balance between load and generation across the system. Constraint (2.14) models the transmission line limits. The generator output limits are presented by (2.15) and (2.16), while constraint (2.17) limits the spinning reserve to the 10-minute generators’ ramp rate capability. Finally, proxy reserve requirements are modeled through (2.18)-(2.19).

\[
\text{minimize} \sum_g \sum_t \left( c_g^p P_{gt} + c_g^{NL} u_{gt} + c_g^{SU} v_{gt} + c_g^{SD} w_{gt} \right) \quad (2.1)
\]

Subject to

\[
v_{gt} \geq u_{gt} - u_{gt-1}, \forall g, t \geq 2 \quad (2.2)
\]

\[
w_{gt} \geq u_{gt-1} - u_{gt}, \forall g, t \geq 2 \quad (2.3)
\]

\[
v_{gt} \geq u_{gt}, w_{gt} = 0, \forall g, t = 1 \quad (2.4)
\]

\[
0 \leq v_{gt} \leq 1, \forall g, t \quad (2.5)
\]

\[
0 \leq w_{gt} \leq 1, \forall g, t \quad (2.6)
\]

\[
u_{gt} \in \{0,1\}, \forall g, t \quad (2.7)
\]

\[
\sum_{s=t-UT_g+1}^{t} v_{gs} \leq u_{gt}, \forall g, t \geq UT_g \quad (2.8)
\]
\[
\sum_{s=t-DT_g+1}^{t} w_{gs} \leq 1 - u_{gt}, \forall g, t \geq DT_g
\] (2.9)

\[
P_{g_0t} - P_{g_0t-1} \leq R^H_g u_{gt-1} + R^S_{g} v_{gt}, \forall g, t
\] (2.10)

\[
P_{g_0t-1} - P_{g_0t} \leq R^H_{g} u_{gt} + R^D_{g} w_{gt}, \forall g, t
\] (2.11)

\[
\sum_{g \in g(n)} p_{g_0t} - \text{Load}_{nt} = p_{n_{0t}}^{\text{inj}}, \forall n, t \quad [\lambda_{n_{0t}}]
\] (2.12)

\[
\sum_{n} p_{n_{0t}}^{\text{inj}} = 0, \forall c, t
\] (2.13)

\[
-p^\text{max}_k \leq \sum_{n} p_{n_{0t}}^{\text{inj}} P^TDF^{\text{ref}}_{0nk} \leq p^\text{max}_k, \forall k, t
\] (2.14)

\[
P_{g_0t} + r_{gt} \leq p^\text{max}_g u_{gt}, \forall g, t
\] (2.15)

\[
p^\text{min}_g u_{gt} \leq P_{g_0t}, \forall g, t
\] (2.16)

\[
0 \leq r_{gt} \leq R^1_{g} u_{gt}, \forall g, t
\] (2.17)

\[
\sum_{j} r_{jt} \geq P_{g_0t} + r_{gt}, \forall g, t
\] (2.18)

\[
\sum_{g} r_{gt} \geq \eta \% \sum_{n} \text{Load}_{nt}, \forall t
\] (2.19)

The above deterministic proxy reserve policies procure reserve solutions, which may not be deliverable in the event of generator or non-radial line contingency realizations. Thus, the market solution may not meet reliability criteria and, therefore, OMC actions are required to achieve an N-1 reliable solution.

### 2.3.2 SCUC with transmission contingency modeling using LODF

Today, some ISOs use LODF to explicitly model non-radial line contingencies in the DA SCUC model without adding any second-stage recourse decisions [67]. The LODFs are participation factors, which indicate redistribution of flow on the transmission lines (e.g. line \(k\)) after outage of line \(\ell\) [39].

The SCUC model that incorporates explicit representation of the transmission contingency using LODF is presented below.

\[
\text{minimize} \sum_{g} \sum_{t} (c^P_g P_{g_0t} + c^L_{g} u_{gt} + c^S_{g} v_{gt} + c^D_{g} w_{gt})
\] (2.20)

Subject to

Constraints (2.2)-(2.19)
It is worth noting that although the market solution of the above formulation is reliable for transmission line contingencies, it may not be reliable for generator contingencies. Similar to the SCUC model presented in previous section, OMC actions are needed to make the solution reliable for generator contingencies.

2.3.3 ESCUC market model

The ESCUC problem is formulated as a two-stage stochastic program to manage discrete disturbances (i.e., for N-1 reliability). Explicit scenarios represent base-case pre-contingency scenario plus contingency scenarios (i.e., the loss of non-radial transmission line and generator) that have their corresponding probabilities. This market model is defined by (2.23)-(2.36)

\[
\begin{align*}
\text{minimize} & \sum_g \sum_t \pi_{BC} c^p \sum_n p_{g0}^n, & \sum_g \sum_t (c_g^N L + c_g^S L + c_g^S D) + \sum_g \sum_{c \in C} \sum_t \pi_c c^p \sum_n p_{gct}^n \\
\text{Subject to} & \sum_{g \in g(n)} p_{gct} - d_{nt} = p_{nct}^n, \forall n, t & [\lambda_{nct}] \\
\sum_{g \in g(n)} p_{gct} - d_{nt} = p_{nct}^n, \forall n, c \in C_g, t & [\lambda_{nct}] \\
\sum_{g \in g(n)} p_{gct} - d_{nt} = p_{nct}^n, \forall n, c \in C_k, t & [\lambda_{nct}] \\
d_{nt} = Load_{nt}, \forall g, t & [\lambda_{nt}^{\text{secured}}] \\
\sum_n p_{nct}^n = 0, \forall c, t & \\
-p_k^{\text{max,c}} \leq \sum_n p_{nct}^n PTDF_{0nk} \leq p_k^{\text{max}}, \forall k, c \neq C_k, t & (2.23) \\
-p_k^{\text{max,c}} \leq \sum_n p_{nct}^n PTDF_{cnk} \leq p_k^{\text{max,c}}, \forall k, c \in C_k, t & (2.30) \\
p_g^{\text{min,u}} N1_g \leq p_{gct} \leq p_g^{\text{max,u}} N1_g, \forall g, c, t & (2.31) \\
\sum_{g \neq c} p_{gct} - p_{gct} \leq R_{g}^{10} u_{gt}, \forall g: g \neq c, c, t & (2.32) \\
\sum_{g \neq c} p_{gct} - p_{gct} \leq R_{g}^{10} u_{gt}, \forall g: g \neq c, c, t & (2.33) \\
\sum_{g \neq c} p_{gct} - p_{gct} \leq r_{gt}, \forall g: g \neq c, c, t & (2.34)
\end{align*}
\]
\( P_{g_{ot}} - P_{g_{ct}} \leq r_{g_{t}}, \forall g: g \neq c, c, t \)  

(2.36)

In the above formulation, the objective is to minimize the expected operating cost over a set of uncertain scenarios. The node balance constraint (see (2.25)- (2.27)) is separated to distinguish when the constraint represents the base-case (2.25), G-1 generation contingency scenarios (2.26), and finally T-1 transmission contingency scenarios (2.27). Constraint (2.29) enforces energy balance between the supply and the demand at the system level. Transmission line capacity limit for the base-case scenario and the G-1 generation contingency scenarios is constrained by (2.30), whereas that for T-1 transmission contingency scenarios is imposed by (2.31). The generator output limit constraint is represented by (2.32). Finally, deviation of an online generator output level (see (2.33)-(2.36)) from the base-case dispatch to the post-contingency dispatch is limited either by its reserve dispatch decision \( r_{g_{t}} \) from the first stage or by its 10-minute ramp rate \( R_{g}^{10} \).

### 2.4 Pricing implications of contingency modeling approaches

The majority of the large balancing authorities implement proxy reserve requirements in an attempt to achieve \( N-1 \) reliability. The market model presented in Section 2.3.1 is based on this practice. Another practice (i.e., market model described in Section 2.3.2) is to estimate and limit the post-contingency line flows through participation factors (i.e., LODF). To achieve \( N-1 \) reliable solutions, the market operators implement OMC on their market solutions [40]. To replicate this practice, this report implements OMC on the output of market models from Sections 2.3.1 and 2.3.2. The OMC approach used in this work is similar to [40] and [70]. In this approach, the generation units that are committed in the DA SCUC market model are not allowed to be de-committed, and their dispatches are limited to the original approximated DA solution by their 10-minute ramp rate limit. However, modifying the dispatch and the commitment of additional units is allowed in order to ensure reliable operation.

After an \( N-1 \) reliable dispatch solution is obtained using OMC approach, the settlements are calculated. However, these settlements may not reflect the true value of \( N-1 \) reliability services due to the discrepancy between LMP calculation and final dispatch solution. Thus, such practice may not be incentive compatible for market participants, especially those who provide reliability services.

On the other hand, since all contingencies are represented endogenously in the ESCUC market model presented in Section 2.2.3, the obtained solution is expected to be \( N-1 \) reliable with respect to non-radial transmission line and generator contingencies. For this model, a pricing mechanism can be obtained to properly incentivize all market participants for providing energy and ancillary services (i.e., contingency reserve in this report).

In this report, with the market implication comparison of the deterministic proxy criterion, market auction model with estimated post-contingency state, and the ESCUC model, the concept of the securitized-LMP (SLMP) is presented to better represent the contingency-reserve prices. The SLMP is the dual variable of (2.28), i.e., \( \lambda_{nt}^{\text{securitized}} \), in the ESCUC model. After deriving the dual formulation from the primal problem of (2.23)-(2.36), equation (2.37) will be obtained, which shows the relationship between LMPs from the base-case (2.25), contingency scenarios (2.26)-(2.27), and SLMP.
The SLMP is the dual variable of (2.28), i.e., $\lambda_{n,t}^{\text{securitized}}$. Since ESCUC model is a mixed-integer linear program, its dual formulation is not well-defined. However, after fixing the binary variables to their values at the best solution found, the linear model of ESCUC is achieved, which has a well-defined dual formulation. Equation (2.37) is obtained by deriving the dual formulation from the ESCUC linear primal problem, which shows the relationship between LMPs from the base-case (2.25), contingency scenarios (2.26)-(2.27), and the SLMP.

$$\lambda_{n,t}^{\text{securitized}} = \lambda_{n0,t} + \sum_{c \in C_g} \lambda_{nct} + \sum_{c \in C_k} \lambda_{nct}, \forall n, t$$

(2.37)

It is worth mentioning that to obtain the relation presented equation (34), the demand is treated as a variable in (2.25)-(2.27), and the model enforces $d_{nt} = \text{Load}_{nt}$ in (2.28). The first term in the right-hand side of (2.37) represents energy and congestion components of SLMP in pre-contingency state, while the second and third terms represent energy and congestion components of SLMP in post-contingency state for the generators and non-radial transmission lines contingencies, respectively. Therefore, the SLMP inherently captures the true values of reserves in the post-contingency state on a locational basis.

It is pertinent to note that in (2.37), $\lambda_{n0,t}$ and $\lambda_{nct}$ include both energy and congestion components since the model explicitly enforce a nodal power balance for each operational state. The pricing scheme presented here from the ESCUC market model has the advantage that it permits the ISOs to gauge how the market participants should be compensated for providing contingency-based reserve. Figure 2.1 illustrates the process of comparing three above market auction models.

![Figure 2.1 Process flowchart for comparing pricing implication of three market auction models.](image)

2.4.1 Testing & Results of pricing implication

CPLEX v12.8 are used to perform all simulations on a computer with an Intel Core i7 CPU @ 2.20 GHz, 16 GB RAM, and 64-bit operating system. A modified 118-bus IEEE test system [71] is used to implement the market auction models, which has 54 generators, 186 lines (177 non-radial), and 91 loads. Set $C_g$ and $C_k$ include $N$-1 contingencies for all generators and non-radial
transmission line elements, respectively. Consequently, there are 232 scenarios modeled in the ESCUC market auction model, including the base-case scenario, 54 generator outages and 177 non-radial transmission line outages. The probability of contingencies is calculated from historical failure rates [57]. The probability of base-case is considered to be 0.946 (i.e., \( \pi_{BC} = 0.946 \)) in order to make the summation of probabilities over all scenarios equal to 1. The relative MIP gap for different SCUC models is set to 0%. The three market auction models, i.e., SCUC with Proxy reserve requirements (abbreviated as “SCUC-Prxy” in the figures), SCUC with transmission contingency modeling based on the LODF (abbreviated as “SCUC-LODF” in the figures), and ESCUC (abbreviated as “SCUC-Extsv” in the figures), are compared from the potential operational efficiency, incentive compatibility, market transparency and market settlements.

Figure 2.2 Final cost comparison for \( N - 1 \) reliable solutions obtained from different approaches.

Figure 2.2 compares the final costs (i.e., the cost after the OMC phase) for the different market auction models. This cost includes the SCUC cost and OMC cost. It is clear that there is no need for OMC actions with the ESCUC model as it includes explicit representation of contingencies through modeling recourse decisions. The solution of the ESCUC market auction model has the lowest final cost (benchmark solution) since originally deliverable reserve is assigned to generators by considering post-contingency states. The SCUC with transmission line contingency modeling through LODF results in higher SCUC cost compared to the SCUC model with proxy reserve, but it requires less discretionary changes or uneconomic adjustments (OMC actions) to achieve \( N-1 \) reliability; thus, the SCUC model with LODF results in less OMC cost. From the reliability point of view, it can be concluded that the SCUC with transmission line contingency modeling through LODF provides a solution that is closer to \( N-1 \) reliable ESCUC solution.

LMPs of the market models are studied in Figure 2.3 for hour 22 across the buses. Based on this figure, as the market model moves away from SCUC-Prxy toward capturing more accurate representation of the contingency events, the prices are increased from bus #67 to bus #109, and also are mostly higher in bus #1-67. The difference between prices is due to the new elements of LMP, i.e., marginal security elements, which represent the value of reserve provision in the modeled contingencies. More specifically, the deterministic model, which utilizes proxy reserve to achieve \( N-1 \) reliability, does not capture the true value of achieving \( N-1 \) reliability due to the fact that the obtained market LMPs do not adequately reflect the value of delivering reserve in the post-contingency state. Accordingly, the LMPs tend to be lower in this model. In this case, the new committed units after OMC may not be fully compensated for providing ancillary services, which can be a reason of missing money issue. This will cause a natural unfairness in market
strategy as market participants might not be compensated fairly with this mechanism. On the other hand, the SCUC-Extsv inherently captures the different values of reserves offered by various entities, as it reflects the value of delivering reserve in the post-contingency state on a locational basis, so the SLMPs tend to be higher. This result occurs because the model explicitly checks to see whether the reserve is deliverable for each contingency. Overall, these analyses confirm that with more accurate representation of contingencies in the market auction models, the reliability and associated products are priced more accurately. This would result in fair and accurate market signals for market participants and improve overall market efficiency.

Figure 2.3 Pricing implication of transition from SCUC with deterministic proxy requirements to more explicit representation of contingency scenarios (ESCUC).

Figure 2.4 Settlements for different market action models.

Figure 2.4 compares the market auction models with respect to market settlements. It can be seen that the generators revenue and load payments have the highest values with the ESCUC model, while they have the lowest value with the SCUC with proxy reserve requirements. The generation rent is calculated from subtracting variable cost of units from their revenues, which is also increases as the models have more explicit and accurate representation of the contingency events.
From these results it can be inferred that more accurate modeling of N-1 requirement in market models result in increased profit and potentially decreased need for uplift payments.

2.5 Objective function and formulation evaluation for stochastic market models

ESCUC model presented by (2.23)-(2.36) is solved for minimizing over expectation of all scenarios (called ESCUC-expected model in this chapter), wherein each scenario has an assigned probability, as shown in (2.23). However, there are some issues that can cause disruption from proposed DA solution to the implemented real-time dispatch as follows:

- During emergency conditions in real-time, the system operators implement corrective actions, which are expected at avoiding load shedding and other violations. These actions aim at eliminating violations as quickly as possible in order to recover from the events and to prevent future unforeseen problems rather than the lowest cost option. Thus, minimizing post-contingency cost in the DA SCUC may result in a solution, which anyways will not be fully implemented.
- Inaccurate prediction of the probabilities of the element outages can lead to different pricing implications and market outcomes.

These reasons combined with the fact that by minimizing expected cost, the base-case solution which has the highest probability may deviate from its optimal value and increase, create a need for detailed examination of the objective function and formulation for the stochastic market models.

An alternative option for objective function formulation is to minimize only the base-case costs including generator production costs, the startup costs, and the shutdown costs as shown in (2.38).

\[
\minimize \sum_g \sum_t \left( c^g_P P_{gt}^g + c^N_{g} u_{gt} + c^{SU}_{g} v_{gt} + c^{SD}_{g} w_{gt} \right)
\]  

(2.38)

Subject to

Constraints (2.24)-(2.36)

(2.39)

The above model for stochastic market is called ESCUC-base market model throughout this chapter. This model minimized (2.38) while searching for a feasible solution during post-contingency states. The two ESCUC models (i.e., ESCUC-expected and ESCUC-base models) are compared from aforementioned aspects, which are presented in more detail in the following subsections. The goal is to determine the more beneficial stochastic market design that can result in more effective DA solution for the conditions that there is imprecision in prediction of the probabilities and the system gets closer to real time operation.

2.5.1 Realized N-1 final operating cost

In emergency conditions, it is hard to predict exact corrective actions that will be chosen by operator. This is because the operator does not exactly follow the proposed corrective actions of ESCUC market since the intention during an emergency condition is not to minimize cost; rather,
the goal is to recover from the event as quickly as possible by minimizing violation to prevent future unforeseen cascades. In order to obtain actual N-1 costs which are realized after N-1 contingency scenarios, contingency analysis with violation minimization is performed, which mimics the actions that operators follow in the emergency conditions. Contingency analysis performed in the energy management systems (EMSs) prepares the system operators to respond to disturbances in the post-contingency state by taking certain actions, thereby, potential security violations (e.g., line overloads or load shedding) can be minimized and even avoided. In this section, the realized N-1 operating cost for the two ESCUC models is calculated after using the dispatch from contingency analysis tool; this cost reflects the corrective generation dispatch actions after an N-1 contingency. Then, the realized N-1 cost for the dispatch from two ESCUC models are compared. Figure 2.5 shows the procedure of this analysis.

The mathematical formulation of contingency analysis which is a linear programming problem is given below.

\[
\text{minimize} \sum_n \sum_{c \in C_o} \sum_t (Ls^+_{net} + Ls^-_{net})
\]  

subject to

\[
-P_{gt} \leq \bar{r}_{gt} - \bar{p}_{gt}, \forall g: g \neq c, c \in C_g, C_k, t
\]  (2.41)

\[
P_{gt} \leq \bar{r}_{gt} + \bar{p}_{gt}, \forall g: g \neq c, c \in C_g, C_k, t
\]  (2.42)

\[
p^\text{min}_{gt} u_{gt} N_{1g} \leq p_{g,c,t} \leq p^\text{max}_{gt} u_{gt} N_{1g}, \forall g, c \in C_g, C_k, t
\]  (2.43)

\[
P_{nct} = \sum_{g \in G_n} p_{gt} - \text{Load}_{nt} + Ls^+_{net} - Ls^-_{net}, \forall n, c \in C^n, C_k, t
\]  (2.44)

\[
\sum_n p^\text{inj}_{nct} = 0, \forall c \in C_g, C_k, t
\]  (2.45)

\[
-P^\text{max}_{ct} \leq \sum_n \text{PTDF}^\text{ref}_{cnk} p^\text{inj}_{nct} \leq p^\text{max}_{ct}, \forall k, c \in C_g, t
\]  (2.46)

\[
-P^\text{max}_{ct} N_{1k} \leq \sum_n \text{PTDF}^\text{ref}_{cnk} p^\text{inj}_{nct} \leq p^\text{max}_{ct} N_{1k}, \forall k, c \in C_k, t
\]  (2.47)

Positive slack variables, i.e., \(Ls^-_{net}\) for shedding and \(Ls^+_{net}\) for load surplus, indicate the post-contingency security violations. Consequently, the contingency analysis objective (2.40) is to minimize the load shed and the load surplus, when an outage occurs. Constraints (2.41) and (2.42) restrict the deviation of the power generation from the pre-contingency to the post-contingency by the scheduled reserve obtained from the DA ESCUC models. The generator output limit constraint in post-contingency state is represented by (2.43). The node balance constraint in the post-contingency state is ensured by (2.44), while (2.45) ensures power balance at system level. Constraints (2.46) and (2.47) limit the post-contingency transmission line flows to be within the emergency limits for generation and transmission contingencies, respectively.
2.5.2 Impacts of imprecise probabilities

The second issue that should be investigated when it comes to design a stochastic market model is the implications of inaccuracy in the estimation of outages’ probability. These analyses are also very necessary to be performed as it is difficult to exactly estimate the probabilities of outages, which itself can lead to different pricing implications and market solutions. In order to realize the impact of this inaccuracy, procedure shown in Figure 2.6 is followed in this chapter.

2.5.3 Testing & Results of objective function evaluation

IEEE 118-bus test system that was explained in detail at section 2.4.1 is used to perform the simulations. First, ESCUC-base and ESCUC-expected models are solved with relative MIP gap set to 0% to compare their benchmark solutions. The formulations are evaluated based on the following metrics:
• DA base-case cost: this cost is calculated through equation (36) based on the DA base-case scenario dispatch and DA commitment decision of ESCUC models.

• Original DA expected cost: this cost is calculated through equation (24) based on the DA scenarios’ dispatch and DA commitment decision of ESCUC models.

• Original DA scenario cost: this cost includes the expected variable cost of generators for post-contingency scenarios (all scenarios excluding the base-case scenario) calculated based on the DA dispatch of ESCUC models.

• Realized N-1 cost: this cost is the realized operation cost during N-1 contingency scenarios (i.e., contingency analysis with violation minimization that mimics the operator’s actions in the emergency conditions).

Figure 2.7 (a) compares the two different costs, i.e., original expected costs versus realized N-1 costs for two market auction models. It is clear that the original DA expected costs of the ESCUC-expected model are lower than those of ESCUC-base model, as the objective function of the ESCUC-expected is to minimize the costs over all scenarios while the ESCUC-base minimizes just the base-case costs. However, the realized N-1 costs of two models are almost the same (the difference is only 0.002 percent). In practice, the operators’ objective in an emergency condition is to eliminate post-contingency violations. These results reveal that minimizing post-contingency costs in the ESCUC-expected does not represent operators’ actions and may not result in lower realized costs in the emergency conditions. Similar results are obtained when the scenario costs of the two models are compared as shown in Figure 2.7 (b). It is pertinent to note that the scenario costs include the expected variable cost of generators during post-contingency state (all scenarios excluding the base-case scenario).

Moreover, the industry practice of considering a 1% MIP gap is implemented here to achieve 30 various solutions for the two ESCUC models for the sake of further comparison. By pairing the solutions of two ESCUC models, the total number of cases is equal to 30×30=900, each of which could be a possible markets outcome from the two different models to be compared.

Table I lists the percentage of cases that the ESCUC-base model results in lower cost compared to the other model. It can be observed that, as expected, the DA base-case costs in 85 percent of cases is lower by the market model formulation based on the ESCUC-base compared to the other market model. Figure 2.8 (a) presents the histogram of costs difference calculated from subtracting the base-cost cost of ESCUC-base from that of ESCUC-expected. It can be seen that the maximum difference is about $8000, and the density of the cases tends to be toward positive. Moreover, Table I shows that the original expected costs of ESCUC-base are lower in 30 percent of the cases (see Figure 2.8 (b) for illustration of difference in original expected costs) as expected. Finally, Table I shows that in almost 86 percent of cases, the realized N-1 costs of ESCUC-base model are less than those of ESCUC-expected model during N-1 contingency scenarios. Figure 2.8 (c) illustrates realized N-1 costs difference (i.e., realized {N − 1} costs ESCUC-expected − realized {N − 1} costs ESCUC-base) for the 900 cases, in which the maximum costs difference for a number of cases reach to around $8000. The summarized results in Table I as well as Figure 2.8 (b) and Figure 2.8 (c) confirm that the realized N-1 costs from the DA dispatch of ESCUC-base are lower than the realized N-1 costs of ESCUC-expected in most of the cases regardless of N-1 cost minimization in
the ESCUC-expected. Moreover, the base-case costs of ESCUC-base are lower than those of the ESCUC-expected in most cases. As one can see, the ESCUC-base performs better in general compared to the ESCUC-expected model based on the base-case costs and realized N-1 final costs.

Figure 2.7 Cost comparison for N-1 reliable solutions obtained from ESCUC-expected and ESCUC-base models: (a) expected costs (b) scenario costs.

Table 2.1. Percentage of cases with lower cost for the ESCUC-base model compared to the ESCUC-expected model.

<table>
<thead>
<tr>
<th>Type of cost</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>DA base-case costs</td>
<td>85%</td>
</tr>
<tr>
<td>Original expected costs</td>
<td>30%</td>
</tr>
<tr>
<td>Realized N-1 costs (violation minimization)</td>
<td>86%</td>
</tr>
</tbody>
</table>
For exploring the impacts of inaccuracy of the probabilities’ estimation, the error in the estimation of probabilities is assumed to follow a Gaussian distribution with zero mean, and the standard deviation of 20% and 40%. 2000 scenarios are generated for each standard deviation, each of which includes a set of 231 probabilities (for generator and non-radial transmission line contingencies) excluding the base-case scenario. Then, the probability of base-case scenario is calculated through following equation:

$$
\pi_{BC,s} = 1 - \sum_{c \in C_k} \pi_{c,s} - \sum_{c \in C_G} \pi_{c,s}, \forall s
$$

(2.48)

where $s$ is the index of scenario, and $\pi_{BC,s}$ and $\pi_{c,s}$ are probability of base-case scenario and contingency event $c$ at the scenarios $s$. Since the range of probability of base-case scenario is more known for the system operators for a specific electric system, the scenarios that lead to a base-case probability out of the range 0.95 and 0.94 have been eliminated (perfect estimation has a base-case probability of 0.946). The scenarios are applied to each of the 30 solutions of ESCUC-base and ESCUC-expected models mentioned earlier. Therefore, there will be obtained $30 \times 2000 = 60,000$ solutions (costs) for each market model. By pairing the solutions of two different stochastic market model, the total number of cases in this condition would be equal to $60,000 \times 60,000 = 36,000,000,000$. It is worth mentioning that each case has two market outcomes, one from ESCUC-base model and one from ESCUC-expected that can be compared. 2.9 illustrates the impacts of error in estimation of probabilities on the percentage of the cases that ESCUC-base model has lower cost (original expected cost and scenario cost) compared to the ESCUC-expected model. From Figure 2.9, it can
be seen that the percentage of cases where ESCUC-base model has lower original expected cost increase from 30% to above 32% and 35% as the accuracy in estimation of probabilities moves from being perfect to have 20% and 40% estimation errors, respectively. The percentage of cases in which the ESCUC-base model has lower scenario cost in comparison with the other model increases from 0% for perfect estimation to 13% and 25% for 20% and 40% error in estimation, respectively. These results demonstrate that after the solutions of two models are affected by the inaccuracy in the probabilities estimation, the likelihood that the ESUUC-base outperforms the other model in having less original expected cost and scenario cost increases.

Figure 2.9 Effects of error in estimation of probabilities on percentage of the cases that ESCUC-base model has lower original expected cost (on the left axis) and scenario cost (on the right axis) compared to the ESCUC-expected model.

2.6 Conclusion

The ISOs generally set market prices based on the original market solution, not the resulting $N$-1 reliable schedule after performing OMC. At this stage, it is unclear whether these procedures enable opportunities for market exploitation. Current practices of operators (using a deterministic SCUC model following with OMC) can be translated as a hybrid market model. Newly committed generators during OMC do not have direct impact on the value of LMP for their location. Instead, they will be paid based on market LMP before OMC. If needed, the operator will compensate them with an uplift payment to make them even. Using this mechanism, the market model is not purely a pool; instead, it will be a combination of a pool and pay-as-bid model. As operators use more modifications and approximations in market solutions, this problem will become worse. In the current market model, this practice is limited and not transparent for all market participants. Therefore, the market participants will not change their bidding strategy accordingly (as they would in a pay-as-bid model). This will cause a natural unfairness in market strategy as market participants might not be dispatched fairly with this mechanism. Although this is an accepted market manipulation, some participants might receive less than deserved benefits and some might receive more. However, with more accurate representation of contingencies in the ESCUC
compared to SCUC models with approximation on $N$-1 security criteria, $N$-1 grid security requirements are originally captured, thereby, the value of service (contingency-based reserve) provided by generators is reflected to achieve grid security. In other words, if the market SCUC includes the reliability criterion more adequately, prices can better reflect the true marginal cost associated with the provision of the reliable electricity.

Furthermore, it was shown that the stochastic market design with expected objective function does not give solutions that ensure minimum realized operating costs at the $N$-1 contingency states. Instead, the stochastic market design with base-case objective function had better performance compared to the aforementioned market model with respect to the base-case costs and realized $N$-1 costs. Moreover, the inaccuracy in estimated probability of scenarios results in larger differences in the original expected costs and the costs of scenarios of ESCUC-base and ESCUC-expected, where ESCUC-base further outperforms ESCUC-expected. It can be concluded that evidently the stochastic market model design with base-case objective function can be more efficient compared to the stochastic market design with expected objective function.
3. Causation of Uplift Payments due to Inter-Temporal Ramping

3.1 Introduction

Renewable energy resources, such as wind and solar, have intermittent and uncertain power output. Generation from renewable energy resource also varies throughout the day. Because generation and load in power systems must be balanced in real time, there is an increasing demand for conventional generators to ramp up or down to follow the fluctuations. As a famous example, the duck curve [72] from California ISO shows that the net load curve (demand minus generation from renewable energy resources) will have more steep ramps. To fully utilize renewable energy resources, more ramping capabilities in power systems are needed. Ramp rate constraints of generators define how fast generators can adjust their generation. As a result, the ramp rate constraints of generators are binding more often than before. However, in some market designs, ramp rate constraints may cause uplift. Uplift payments are made to generators so that generators don't lose money from participating in electricity markets. They also incentivize generators to follow system operator's dispatch, thereby creating an incentive compatible market design. Uplift payments are determined outside of markets; thus, they distort price signals. Reference [73] analyzed a design of multi-interval real-time market (MIRTM) where uplift payments may be needed when ramp rate limits are binding.

In an ideal market, the economic surplus is maximized at the market clearing price. Market participants maximizing their own surplus by following the price signals. Electricity markets are usually designed to be non-confiscatory: the market participants do not lose money from participating in the markets. The market operators of electricity markets should also be revenue neutral. To achieve the above goals, additional payments are made to generators, besides the payments paid at the market clearing price. These payments are called uplift payments. A general definition of uplift could be found in [74]. At the same time, loads are charged for these uplift payments. Thus, the prices no longer fully reflect the marginal cost when uplift exists, i.e., uplifts distort price signals.

In this chapter, the conditions for ramp rate constraints causing uplift are analyzed from two aspects: market design and the convexity of ramping constraints. Only the uplift caused by ramp rates are discussed here; other factors that affect uplift, such as nonconvexity caused by unit commitment, are not discussed.

There are some papers that discuss incentives in MIRTM, which are closed related to uplift. Reference [75] proposed a pricing method for the MIRTM that provides incentives to generators for following dispatch, by considering the losses in previous market intervals. Reference [76] proposed a multi-period market design that is incentive compatible. Reference [77] compared ramp capability constraints and multi-period market models from an incentive perspective.

3.2 Market Design

Three common market designs for real-time markets are discussed: single-period model and price clearing, multi-period model with single-period pricing, and multi-period model with multi-period pricing. Generators are assumed to have constant ramp rates.
3.2.1 Single-Period Model and Price Clearing

The real-time market optimizes decision variables only for the upcoming interval. The drawback of this market design is that, without ramping capability constraints or flexible ramping products, the ramping capability of generators is not included in the market. Thus, reliability and economic efficiency are reduced in power systems that are often constrained by ramp rate constraints. Ramping capability constraints and flexible ramping products could alleviate this problem. However, they have difficulty modeling ramp rate constraints across several intervals. In contrast, multiple-period models can model these inter-temporal constraints.

3.2.2 Multi-Period Models

The market optimizes decision variables for multiple intervals in a look-ahead window. Multi-period models are used to model MIRTM.

1) Multi-Period Model with Single-Period Pricing: Only the dispatch and prices for the first interval are used for settlement. The dispatch and prices for the other intervals are advisory. Because this market design ignores inter-temporal influences from the interval settled on other intervals, ramp rate constraints may cause uplift payments [77]. Reference [73] analyzed this market design and concluded that uplift for ramp rate constraints is needed.

2) Multi-Period Model with Multi-Period Pricing: The dispatch and prices for all intervals are settled. Therefore, the inter-temporal influences are included in the settlements. It will be shown in the next subsection that in this market design, ramp rate constraints do not cause uplift under some assumptions.

3.2.3 Linear Programming Formulation of Multi-Period Model

To study the connection between ramp rates and uplift, a simplified MIRTM is considered with the following assumptions:

- Dispatch decisions and prices in all time intervals are all financially binding.
- Generators have inter-temporal ramping constraints with constant ramp rates, and they have zero minimum generation levels.
- Generators have linear generation costs.
- The load forecasts within the market intervals are perfect.

With the above assumptions, the market is modeled as the following linear programming problem:

Minimize: \( \sum_t \sum_g c_g P_{gt} \) \hspace{1cm} (3.1)

Subject to:

\[ \sum_g P_{gt} = d_t \hspace{1cm} \forall t \hspace{1cm} (\lambda_t) \] \hspace{1cm} (3.2)

\[ -P_{gt} \geq -P_{gt}^+ \hspace{1cm} \forall g, t \hspace{1cm} (\alpha_{gt}) \] \hspace{1cm} (3.3)
\( P_{gt} - P_{g,t+1} \geq -R^+_g \)  
\( \forall g, t \in \{1, \ldots, T - 1\} \) \( (\beta^+_g) \) (3.4)

\( P_{g,t+1} - P_{gt} \geq -R^-_g \)  
\( \forall g, t \in \{1, \ldots, T - 1\} \) \( (\beta^-_g) \) (3.5)

\( P_{gt} \geq 0 \)  
\( \forall g, t \) (3.6)

Indices

\( g \) Generator.
\( t \) Time period; \( t = 1, \ldots, T \).

Parameters

\( c_g \) Production cost for generator (or value of load) \( g \); generally \( c_g > 0 \).
\( d_t \) Real power load (fixed) for period \( t \).
\( P^+_g \) Max generation of generator \( g \) for period \( t \).
\( R^+_g, R^-_g \) Max ramp rate in the up and down direction for generator \( g \).
\( T \) Number of periods.

Variables

\( P_{gt} \) Real power supply from generator \( g \) in period \( t \).

The dual variables are shown next to the constraints. Constraint (3.2) represents the power balance constraints. Constraint (3.3) represents the maximum generation constraints. Constraints (3.4) and (3.5) represent ramp rate constraints with constant ramp rates.

There are mainly two types of uplift payments in electricity markets [79]:

- make-whole payments: the payments to generators for making them break-even when following dispatch instructions results in losing money.
- lost opportunity cost: the difference between the maximum profit a generator can get by optimizing its dispatch decision variables given market prices, and the profit it gets when following dispatch instructions.

Both types of uplift payments were proved to be zero for the market model presented. The results could be generalized to simplified MIRTM\( s\) that include line power flow limits.

**3.2.4 Make-Whole Payments**

To analyze make-whole payments, the total profit (profit over all periods) of each generator needs to be obtained. Given the above market, the total profit of generator \( g \) is given by

\[
\sum_t (\lambda_t P_{gt} - c_g P_{gt}) = \sum_t (\lambda_t - c_g) P_{gt}.
\] (3.7)

Make-whole payments are zero if total profit is non-negative. Based on this observation, the following theorem regarding the total profit of each generator was proved. It shows that even when
inter-temporal ramping constraints are present, the total profit of each generator is non-negative if computed over the whole time horizon. As a result, the make-whole payments are zero.

**Theorem 1:** Under Assumptions 1–4, the total profit for each generator is non-negative: \( \Sigma_t(\lambda_t - c_g)P_{gt} \geq 0. \)

**Proof:** First the dual of the primal problem is derived:

Maximize: \( \Sigma_t d_t \lambda_t - \Sigma_t \Sigma_g P_g^+ \alpha_{gt}^+ - \Sigma_{t=\{1,\ldots,T-1\}} \Sigma_g (R_g^+ P_{gt} + R_g^- \beta_{gt}^-) \) \hspace{1cm} (3.8)

Subject to:

\[
\begin{align*}
\lambda_t - \alpha_{gt}^+ + \beta_{gt}^+ - \beta_{gt}^- & \leq c_g \quad \forall g, t = 1 \quad (P_{g1}) \\
\lambda_t - \alpha_{gt}^+ + \beta_{gt}^+ - \beta_{gt}^- - \beta_{g,t-1} + \beta_{g,t-1}^- & \leq c_g \quad \forall g, t \in \{2, \ldots, T - 1\} \quad (P_{gt}) \\
\lambda_t - \alpha_{gt}^+ - \beta_{g,t-1} + \beta_{g,t-1}^- & \leq c_g \quad \forall g, t = T \quad (P_{gT})
\end{align*}
\]

\( \alpha_{gt}^+, \beta_{gt}^+, \beta_{gt}^- \geq 0 \quad \forall g, t \)

LMP, free \hspace{1cm} \forall t

Variables

\( \alpha_{gt}^+ \): Marginal value of increasing the max capacity level of generator \( g \) in period \( t \).

\( \beta_{gt}^+, \beta_{gt}^- \): Marginal value of increasing the up (down) ramp rate of generator \( g \) in period \( t \).

\( \lambda_t \): Marginal cost (value) of increasing (decreasing) consumption in period \( t \).

By applying complementary slackness to (3.9) – (3.11), summing over \( t \), and rearranging we have:

\[
\begin{align*}
\Sigma_t(\lambda_t - c_g)P_{gt} &= \Sigma_t \alpha_{gt}^+ P_g - \Sigma_{t=\{1,\ldots,T-1\}}(\beta_{gt}^+ - \beta_{gt}^-)P_{gt} + \Sigma_{t=\{2,\ldots,T\}}(\beta_{g,t-1}^+ - \beta_{g,t-1}^-)P_{gt} \\
&= \Sigma_t \alpha_{gt}^+ P_g - \Sigma_{t=\{1,\ldots,T-1\}}(\beta_{gt}^+)P_{gt} + \Sigma_{t=\{2,\ldots,T\}}(\beta_{g,t-1}^+)P_{gt} \\
&\quad + \Sigma_{t=\{1,\ldots,T-1\}}(\beta_{gt}^-)P_{gt} - \Sigma_{t=\{2,\ldots,T\}}(\beta_{g,t-1}^-)P_{gt} \quad (3.12)
\end{align*}
\]

Note that:

\[
- \Sigma_{t=\{1,\ldots,T-1\}}(\beta_{gt}^+)P_{gt} + \Sigma_{t=\{2,\ldots,T\}}(\beta_{g,t-1}^-)P_{gt} = - \Sigma_{t=\{1,\ldots,T-1\}}(P_{gt} - P_{g,t+1})\beta_{gt}^+ \quad (3.13)
\]

And:

\[
+ \Sigma_{t=\{1,\ldots,T-1\}}(\beta_{gt}^-)P_{gt} - \Sigma_{t=\{2,\ldots,T\}}(\beta_{g,t-1}^-)P_{gt} = + \Sigma_{t=\{1,\ldots,T-1\}}(P_{gt} - P_{g,t+1})\beta_{gt}^+ \quad (3.14)
\]

Thus, (3.12) is equal to (3.15) below:

\[
\Sigma_t \alpha_{gt}^+ P_g - \Sigma_{t=\{1,\ldots,T-1\}}(P_{gt} - P_{g,t+1})\beta_{gt}^+ + \Sigma_{t=\{1,\ldots,T-1\}}(P_{gt} - P_{g,t+1})\beta_{gt}^- \quad (3.15)
\]
By applying complementary slackness to (3.4) and (3.5), we have:

\[(P_{gt} - P_{g,t+1})\beta^+_{gt} = -R^+_g \beta^+_{gt} \quad \forall g,t \in \{1, \ldots, T - 1\} \tag{3.16}\]
\[(P_{g,t+1} - P_{g,t})\beta^-_{gt} = -R^-_g \beta^-_{gt} \quad \forall g,t \in \{1, \ldots, T - 1\} \tag{3.17}\]

Since \(\beta^+_{gt}\) and \(\beta^-_{gt}\) are non-negative variables and since \(R^+_g\) and \(R^-_g\) are non-negative parameters, note that both the right hand side and the left hand side of both (3.16) and (3.17) are non-positive.

We can re-arrange (3.16) and (3.17) to have (3.18) and (3.19), which are non-negative:

\[-(P_{gt} - P_{g,t+1})\beta^+_{gt} = R^+_g \beta^+_{gt} \geq 0 \quad \forall g,t \in \{1, \ldots, T - 1\} \tag{3.18}\]
\[(P_{gt} - P_{g,t+1})\beta^-_{gt} = R^-_g \beta^-_{gt} \geq 0 \quad \forall g,t \in \{1, \ldots, T - 1\} \tag{3.19}\]

Therefore, by comparing (3.18) to the second summation term in (3.15) and by comparing (3.19) to the third summation term in (3.15), we know that the following statements are true:

\[-\sum_{t=1,\ldots,T-1}(P_{gt} - P_{g,t+1})\beta^+_{gt} \geq 0 \tag{3.20}\]
\[\sum_{t=1,\ldots,T-1}(P_{gt} - P_{g,t+1})\beta^-_{gt} \geq 0 \tag{3.21}\]

Finally, we know that the first summation term in (3.15) is non-negative since the dual variable \(\alpha^+_{gt}\) is non-negative and the primal variable \(P_{gt}\) is non-negative. Thus:

\[\sum_t \alpha^+_{gt} P_{gt} \geq 0 \tag{3.22}\]

This proves that (3.15) is non-negative and (3.15) is equal to the generator g’s total profit over all periods. Thus, each generator is guaranteed to have a non-negative total profit.

Theorem 1 can be proven from another perspective. Compared with the above proof, this method requires the proof of zero lost opportunity cost.

Proof: Compare the profits of the generator g in the following cases given \(\lambda_t\):

- \(\pi_1\): \(P_{gt} = 0\) for all \(t\). \(\pi_1 = 0\) regardless of the prices \(\lambda_t\), since \(\pi_1 = \sum_t(\lambda_t - c_g) * 0 = 0\).
- \(\pi_2\): the maximum profit by locally optimizing over \(P_{gt}\).
  \[\pi_2 = \max_{P_{gt} \in S_g} \sum_t(\lambda_t - c_g) P_{gt}, \text{ where } S_g \text{ represents constraints (3.3) – (3.6)}.\]
- \(\pi_3\): the profit by following the dispatch.

Because \(P_{gt} = 0 \in S_g\), according to the definition of \(\pi_2, \pi_1 \leq \pi_2\).

Since lost opportunity cost is zero (see Section 3.2.5), \(\pi_2 = \pi_3\). Thus,

\(\pi_3 = \pi_2 \geq \pi_1 = 0\).
This proof could be applied to the generalization of Theorem 1 for simplified MIRTMs that include line power flow limits.

### 3.2.5 Lost Opportunity Cost

Another type of uplift payments is lost opportunity cost. The economic dispatch problem under Assumptions 1–4 is a special case of the generalized economic dispatch problem in [80]. Reference [80] shows that for the generalized economic dispatch problem, the prices derived from dual variables support dispatch decisions, i.e., generators do not have incentives to deviate the dispatch decisions. Thus, the lost opportunity cost is zero according to its definition. The above analysis could be generalized for simplified MIRTMs that include line power flow limits.

### 3.2.6 Numerical Example

1) **System Data:** A single bus system that contains two generators is considered. It has a solar farm and a gas generator. The system operator dispatches these two generators for two time periods: time interval 1 and time interval 2 that are 10 minutes long each. The load is 8 MW for both time intervals. The generation cost of the solar farm is $0.1/MWh. Assume that according to the forecast, the generation from the solar farm is less than or equal to 10 MW during time interval 1, and 3 MW during time interval 2. The generation cost of the gas generator is $20/MWh. Its capacity is 6 MW. Its ramp rate is 2 MW per 10 min.

2) **Linear Programming Formulation:** The multi-period economic dispatch is represented by the following linear programming problem:

Minimize: $0.1P_{11} + 0.1P_{12} + 20P_{21} + 20P_{22}$

s.t.:  
$P_{11} + P_{21} = 8$  
$P_{12} + P_{22} = 8$  
$P_{11} \leq 10$  
$P_{12} \leq 3$  
$P_{21} \leq 6$  
$P_{22} \leq 6$  
$P_{22} - P_{21} \leq 2$  
$P_{21} - P_{22} \leq 2$  
$P_{11}, P_{12}, P_{21}, P_{22} \geq 0$  

in which $P_{11}, P_{12}$ represents generation from the solar farm at time interval 1 and 2 respectively; $P_{21}, P_{22}$ represents generation from the gas generator at time interval 1 and 2 respectively. Constraint (3.23) is the ramp up constraint of the gas generator, and constraint (3.24) the ramp down.

3) **Dispatch Results:** By solving the above problem, the optimal dispatch is shown in Table 3.1. The prices are derived from the solution of dual variables. Table 3.2 shows the profits of the generators and load payment. It shows that the generators have nonnegative profit.
Table 3.1 Economic Dispatch Constant Ramp Rates

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price ($/MWh)</td>
<td>0.1</td>
<td>39.9</td>
</tr>
<tr>
<td>Dispatched Generation of the Solar Farm (MW)</td>
<td>5.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Dispatched Generation of the Gas Generator (MW)</td>
<td>3.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Load (MW)</td>
<td>8.0</td>
<td>8.0</td>
</tr>
</tbody>
</table>

Table 3.2 Profits and Payments* with Constant Ramp Rates

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>1</th>
<th>2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar Farm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Revenue ($)</td>
<td>0.5</td>
<td>119.7</td>
<td>120.2</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>0.5</td>
<td>0.3</td>
<td>0.8</td>
</tr>
<tr>
<td>Profit ($)</td>
<td>0.0</td>
<td>119.4</td>
<td>119.4</td>
</tr>
<tr>
<td>Generator with Constant Ramp Rates</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Revenue ($)</td>
<td>0.3</td>
<td>199.5</td>
<td>199.8</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>60.0</td>
<td>100.0</td>
<td>160.0</td>
</tr>
<tr>
<td>Profit ($)</td>
<td>-59.7</td>
<td>99.5</td>
<td>39.8</td>
</tr>
<tr>
<td>Load</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Payment ($)</td>
<td>0.8</td>
<td>319.2</td>
<td>320</td>
</tr>
</tbody>
</table>

*All values need to be divided by 6 to get the corresponding 10-minute values.

Given the prices in Table 3.1, the generators do not have incentive to deviate from the system dispatch. The solar farm is generating at its maximum capacity at time interval 2 because the price is above its cost, and it is indifferent about the generation level at time interval 1 because the price equals its cost. The gas generator's profit maximization problem is

Maximize: $0.1p_{21} + 39.9p_{22} - 20p_{21} - 20p_{22}$

s.t.: $p_{21} \leq 6$
$p_{22} \leq 6$
$p_{22} - p_{21} \leq 2$
$p_{21} - p_{22} \leq 2$
$p_{21}, p_{22} \geq 0$

The optimal value of the above problem is 39.8, which is equal to the profit when following the dispatch. This shows the opportunity cost of the gas generator is zero.
4) Determining Prices from Incremental Costs: The prices at each time interval can also be obtained by perturbing the load. When the load at time interval 1 is 8 MW, and the load at time interval 2 is increased to 8.1 MW, the optimal dispatch is shown in Table 3.3. The incremental cost to serve load at time interval 2 is \((-0.1*0.1+0.1*20+0.1*20)/(8.1-8) = 39.9 \$/MW\).

The incremental cost to serve load at time interval 1 can be obtained similarly. The incremental costs match with the prices obtained from dual variables in Table 3.1.

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dispatched Generation of the Solar Farm (MW)</td>
<td>4.9</td>
<td>3.0</td>
</tr>
<tr>
<td>Dispatched Generation of the Gas Generator (MW)</td>
<td>3.1</td>
<td>5.1</td>
</tr>
<tr>
<td>Load (MW)</td>
<td>8.0</td>
<td>8.1</td>
</tr>
</tbody>
</table>

3.3 Convexity of Ramping Constraints

Ramp rates are physical characteristics that depend on the generator. They are broadly classified into two types: constant ramp rates and dispatch-dependent ramp rates. In the previous section, it is assumed that generators have constant ramp rates. Combined-cycle units have dispatch-dependent ramp rates. Their ramp rates change with generation levels, because different generation levels correspond to different operation combinations of gas and steam generators [81].

Dispatch-dependent ramp rates could introduce nonconvexity into the problem, similar to binary variables in unit commit problems, which could cause uplift. This section shows that dispatch-dependent ramp rates may cause uplift, even with the market design of multi-period model with multi-period pricing.

3.3.1 Modeling Dispatch-Dependent Ramp Rates

Dispatch-dependent ramp rates are often modeled as piecewise linear functions of generation. References [81] and [82] used Special Ordered Sets of type 2 (SOS2), a method for modeling piecewise linear functions, to model dispatch-dependent ramp rates. For demonstration purposes, a Mixed Integer Programming (MIP) formulation without SOS2 was used in the study, which is mathematically equivalent to a formulation that uses SOS2.

As an example, assume generator 2 has dispatch-dependent ramp rates and \(T = 2\). A two-segment piecewise function for the ramp rate curve is given by

\[
R^+_g = R^-_g = \begin{cases} 
  r_1, & \text{for } 0 \leq P_{g1} < a_1 \\
  r_2, & \text{for } a_1 \leq P_{g1} \leq a_2 
\end{cases}
\]

(3.25)

where \(g = 2\), \(P_{g1}\) is the generation level at time interval 1, the two generation segments are \([0, a_1]\) MW and \( [a_1, a_2]\) MW, and \(r_1\) and \(r_2\) are ramp rates in these two segments.
To model the dispatch-dependent ramp rates defined by (3.25), a binary variable $Z$ is introduced to indicate the segment of the dispatch. The following constraints are added to the problem:

$$P_{21} - (a_2 - a_1)Z \leq a_1$$  \hspace{1cm} (3.26)
$$-P_{21} + a_1 Z \leq 0$$  \hspace{1cm} (3.27)

When $Z$ is 0, $P_{21}$ is restricted to the interval $[0, a_1]$; when $Z$ is 1, $P_{21}$ is restricted to the interval $[a_1, a_2]$.

The ramp rates, $R_2^+$ and $R_2^-$, are replaced with $r_1 + (r_2 - r_1)Z$.

The economic dispatch problem with dispatch-dependent ramp rates is then given by:

Minimize: Objective (3.1)

Subject to:
Constraints (3.2), (3.3) and (3.6)

Constraints (3.26) and (3.27)

$$P_{21} - P_{22} + (r_2 - r_1)Z \geq -r_1$$  \hspace{1cm} (3.28)
$$P_{22} - P_{21} + (r_2 - r_1)Z \geq -r_1$$  \hspace{1cm} (3.29)

$Z$ binary  \hspace{1cm} (3.30)

where constraints (3.26), (3.27), and (3.28) – (3.30) are the ramp rate constraints with dispatch-dependent ramp rates. They replace constraints (3.4) and (3.5), the ramp rate constraints with constant ramp rates.

3.3.2 Numerical Example

A single bus system that contains two generators is considered. It has a solar farm and a gas generator. The system operator dispatches these two generators for two time periods: time interval 1 and time interval 2 that are 10 minutes long each. The load is 8 MW for both time intervals. The generation cost of the solar farm is $0.1/MWh. Assume that according to the forecast, the generation from the solar farm is less than or equal to 10 MW during time interval 1, and 3 MW during time interval 2. The generation cost of the gas generator is $20/MWh. Its capacity is 6 MW. The gas generator has dispatch-dependent ramp rates. When generation is from 0 to 4 MW, the ramp rate is 0.5 MW per 10 min; from 4 MW to 6 MW, the ramp rate is 2 MW per 10 min.

The corresponding economic dispatch problem is:

Minimize: $0.1P_{11} + 0.1P_{12} + 20P_{21} + 20P_{22}$
Subject to:
$$P_{11} + P_{21} = 8$$
\[ P_{12} + P_{22} = 8 \]
\[ P_{11} \leq 10 \]
\[ P_{12} \leq 3 \]
\[ P_{21} \leq 6 \]
\[ P_{22} \leq 6 \]
\[ P_{22} - P_{21} - 1.5Z \leq 0.5 \]
\[ P_{21} - P_{22} - 1.5Z \leq 0.5 \]
\[ P_{21} - 2Z \leq 4 \]
\[ -P_{21} + 4Z \leq 0 \]
\[ P_{11}, P_{12}, P_{21}, P_{22} \geq 0 \]
\[ Z \text{ binary} \]

The above problem is solved, and the solution for the binary variable \( Z \) is 1. By fixing \( Z \) to the optimal value 1, the MIP problem is converted to a linear programming problem. It is then solved again to get the prices. The solutions of the dispatch and prices are summarized in Table 3.4. The profits of the generators and the load payments are shown in Table 3.5. In this case, the total profit of generator 2 is negative, and make-whole payments are needed.

Table 3.4 Economic Dispatch with Dispatch-Dependent Ramp Rates

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price ($/MWh)</td>
<td>0.1</td>
<td>20.0</td>
</tr>
<tr>
<td>Dispatched Generation of the Solar Farm (MW)</td>
<td>4.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Dispatched Generation of the Gas Generator (MW)</td>
<td>4.0</td>
<td>5.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load (MW)</td>
<td>8.0</td>
<td>8.0</td>
</tr>
</tbody>
</table>

Table 3.5 Profits and Payments* with Dispatch-Dependent Ramp Rates

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>1</th>
<th>2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar Farm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Revenue</td>
<td>0.4</td>
<td>60.0</td>
<td>60.4</td>
</tr>
<tr>
<td>Cost</td>
<td>0.4</td>
<td>0.3</td>
<td>0.7</td>
</tr>
<tr>
<td>Profit</td>
<td>0.0</td>
<td>59.7</td>
<td>59.7</td>
</tr>
<tr>
<td>Revenue</td>
<td>0.4</td>
<td>100.0</td>
<td>100.4</td>
</tr>
<tr>
<td>Cost</td>
<td>80.0</td>
<td>100.0</td>
<td>180.0</td>
</tr>
</tbody>
</table>
Constant ramp rates do not cause uplift if dispatch decisions and prices in all time intervals are all financially binding. The main principle used for the proof is that convex markets do not require uplift payments. Dispatch-dependent ramp rates may result in a nonconvex market; whenever a market nonconvexity exists, that nonconvexity may cause uplift payments. Finally, whenever you have a sequential decision making process, that is, you solve a collection of intervals (say periods 1-4) and then repeat by solving the following intervals (now 2-5) in a rolling horizon process, there is no guarantee that uplifts may not occur due to the inefficiency of breaking the real world time periods into separate mathematical programs. In other words, when you solve periods 1-4, those four schedules and settlements will not cause uplift if the (i) market is convex and (ii) you use multi-period scheduling with multi-period pricing. Once you then take the planned dispatch setpoints from the first run and impose restrictions on the next market process (periods 2-5), at that point you cannot guarantee a non-confiscatory market even if you have a market that is convex and you use multi-period scheduling and multi-period settlements; the only way to fix this is to extend the time periods within each iterative decision making process. While this situation is essentially unavoidable, the causation of uplifts is far less due to this problem whereas uplift is more so caused by: (i) nonconvexities, (ii) lost opportunity costs, and (iii) single period scheduling and pricing or multi-period scheduling with single period pricing.

Our recommendation is to go with a multi-period scheduling and multi-period pricing structure. We also recommend that causation of uplift payments be appropriately identified. Ramp rates have been characterized as a cause of uplift, including in market structures where the ramp rates are not causing the uplift, which can be mathematically proven; instead, the cause is an inadequate market structure (e.g., single period scheduling and pricing). Yes, ramp rates may cause uplift payments depending on how they are modeled. Ultimately, it is critical to properly identify causation of uplift payments in order to know the true problem and fix it accordingly.

### 3.4 Conclusions

<table>
<thead>
<tr>
<th>Generator with Dispatch Dependent Ramp Rates</th>
<th>Profit</th>
<th>0.0</th>
<th>-79.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
<td>Payment</td>
<td>0.8</td>
<td>160.0</td>
</tr>
</tbody>
</table>

*All values need to be divided by 6 to get the corresponding 10-minute values

Traditional electric energy markets do not explicitly model generator contingencies. To improve the representation of resources and to enhance the modeling of uncertainty, existing markets are moving in the direction of including generator contingencies and remedial action schemes within market auction models explicitly. This research contributes to the market design realm by providing detailed analysis of impending changes, it provides insightful guidance in understanding the market implications, and it provides recommendations on necessary changes to ensure a fair and transparent market structure. A primal (and the corresponding dual) formulation that accounts for the proposed changes to the auction model is provided to enable a theoretical analysis of the anticipated changes including the effect on market prices, settlements, and revenues. The derivation of the prices and the dual formulation are based on leveraging duality theory from linear optimization theory. A comparison to existing market structures is also included. The primary impact of the proposed changes includes the addition of a new congestion component within the traditional locational marginal price, which reflects the influence of congestion during the post-contingency states for the modeled critical generator contingencies.

4.1 Introduction

ISOs maintain a continuous, reliable, and economically efficient supply of electric energy with the assistance of energy management systems and market management systems (MMS). One key feature within the MMS is the determination of the generation dispatch and ancillary services schedule while respecting complex operational requirements and strict physical restrictions. The transmission planning standard (TPL-001-4), set by the NERC is an instance of one such requirement, which stipulates system performance requirements under both normal and emergency conditions [83]. Particularly, the system is required to recover from the loss of any single bulk element, e.g., a generator or a non-radial transmission element, without inconveniencing customers (involuntary load shedding). This rule is more commonly referred to as the N-1 reliability requirement and makes the underlying problem stochastic in nature. However, modeling such uncertain events within resource scheduling tools presents two practical barriers: (1) computational complexity of the resulting stochastic optimization problem and (2) market barriers primarily due to the complications associated with pricing in a stochastic market environment. Consequently, most of the contemporary power system operational frameworks rely on deterministic approaches and utilize numerous approximations to handle uncertainties to meet the N-1 mandate.

Today, ISOs model critical transmission contingencies in the market explicitly without utilizing second-stage recourse decision variables; post-contingency line flows are represented using shift factors, such as line outage distribution factors (LODFs), for a subset of critical transmission contingencies. Decomposition techniques are leveraged to manage the complexity of the overall mathematical program by acknowledging only the constraints deemed to be critical. Such approaches enable an efficient handling of critical transmission contingencies within MMS. A generator outage can also constrain the transmission system considerably. Generator contingencies are not modeled explicitly within state-of-the-art market auction models; instead, system or zonal operating reserve requirements are formed to ensure the system is reliable against generator contingencies. For instance, common industry practices, to approximate the N-1 mandate for
generator contingencies, include simplistic policies that require a MW level of contingency reserve to be acquired somewhere in the system [84], [85]. However, such policies do not ensure reliable operations (or reserve deliverability) since they only capture a quantitative aspect [86], [87]. Moreover, such approximate, deterministic approaches require out-of-market corrections (OMCs) to adjust resource schedules to account for modeling inaccuracies [88], [89]. Consequently, there is a push in the industry to include an explicit representation of generator contingencies in the market auction models.

Two-stage scenario-based stochastic programs are often proposed to improve operations by optimizing the system response, e.g., reserve activation, in the post-contingency states. However, recent industry movement to model generator contingencies suggests using pre-determined factors, such as generator loss distribution factors (GDFs) and zonal reserve deployment factors [90], [91], to approximate the system response to a generator contingency; such factors are analogous to the more familiar participation factors that are used today in real-time contingency analysis when simulating generator contingencies. CAISO recently proposed to update its market auction models to recognize the impact of generator contingencies and remedial action schemes (RAS) in the market, explicitly, without using second-stage recourse decision variables [90]. Furthermore, MISO augmented their market auction models by modeling the loss of the largest generator for each zone and the corresponding system response in the post-contingency state, explicitly, without using second-stage recourse decision variables [91]. MISO’s approach approximates post-contingency congestion on critical transmission interfaces due to the deployed zonal reserves. Moreover, the system response is modeled via zonal aggregated sensitivity factors and pre-determined zonal reserve deployment factors. With the explicit modeling of generator contingencies within the market auction models, the industry is moving away from deterministic program formulations to a stochastic program structure. The anticipated impacts include market prices that better reflect the quality of service provided by generators in response to a generator contingency. The main purpose of this chapter is to provide a theoretical analysis of the recent changes in market auction models while focusing on its influences on market clearing prices, i.e., locational marginal prices (LMPs). It investigates the impact that the explicit inclusion of generator contingencies will have on the market pricing structure using duality theory. Primal and dual formulations of market auction models, with and without explicit generator contingency modeling, derivation of the corresponding LMPs to demonstrate how the proposed changes affect market prices, settlements and revenues, are presented.

The remainder of the chapter is organized as follows. Section 4.2 introduces a theoretical analysis of a contemporary market auction model. Section 4.3 investigates the anticipated changes by providing an enhanced primal formulation for the market auction model and an economic interpretation of the corresponding dual problem, its variables, and its constraints. One goal is to investigate CAISO’s newly proposed (and related) payment structure in greater detail. Note that, in the following discussions, GDFs are used to model the corrective actions approximately without using a recourse decision variable. Finally, Section 4.4 concludes the chapter and summarizes potential future work.
4.2 Dual Problems of Electric Energy Market Formulations

4.2.1 Background on Duality Theory for Linear Optimization

In linear optimization, there is the primal problem, the problem at hand. In this case, the problem of interest is the direct current optimal power flow (DCOPF) problem or a security-constrained economic dispatch (SCED) problem. Each primal problem then has a corresponding dual problem and together they form what is known in linear optimization theory as a primal-dual pair. The dual problem can be interpreted as an optimization problem that is searching for the tightest lower bound (when the primal is a minimization problem); it also provides the shadow prices (dual variables) corresponding to the constraints in the primal. Dual variables, based on linear optimization theory, can also be interpreted as the corresponding Lagrange multipliers for the constraints within the primal. From the perspective of an economist, they are interpreted as shadow prices. The constraints within the dual describe the relationships between the dual variables. Likewise, the primal variables are the corresponding shadow prices for the dual constraints.

The following example demonstrates the relationship between the primal-dual pair, where $a_i$ is a row and $A_j$ is a column from a given $A$ matrix that captures the constraint set (which consists of $M$ constraints: $M_1$ $\geq$ constraints, $M_2$ $\leq$ constraints, and $M_3$ = constraints) for the primal; each constraint has a scalar $b_i$. In addition, $c$ is the cost vector and $X$ is the vector of primal variables, where $N_1$, $N_2$, and $N_3$ denote the subset of non-negative, non-positive and unrestricted primal variables respectively. Also, $p$ denotes the penalty (or shadow) price for violating the corresponding primal constraint. This primal-dual pair presentation can be found in a variety of textbooks, including [92].

Primal:

Minimize: $c^T X$

Subject to:

$A_i^T X \geq b_i, \quad i \in M_1$

$A_i^T X \leq b_i, \quad i \in M_2$

$A_i^T X = b_i, \quad i \in M_3$

$x_j \geq 0, \quad j \in N_1$

$x_j \leq 0, \quad j \in N_2$

$x_j: \text{free}, \quad j \in N_3$

Dual:

Maximize: $p^T b$

Subject to:

$p_i \geq 0, \quad i \in M_1$

$p_i \leq 0, \quad i \in M_2$

$p_i: \text{free}, \quad i \in M_3$

$p^T A_j \leq c_j, \quad j \in N_1$

$p^T A_j \geq c_j, \quad j \in N_2$

$p^T A_j = c_j, \quad j \in N_3$.

Prior work related to optimization problems for power systems derive the properties of the prices, which come from the dual formulation, based on applying Karush-Kuhn-Tucker conditions and simplifying the equations [93]–[102]. Note that the dual formulation is derived by creating a Lagrangian dual and then simplifying it into the form presented above. Leveraging the known
properties for a primal-dual pair for linear optimization models is used in this chapter since it is more concise and straightforward [96].

4.2.2 The Dual Formulation for a Standard DCOPF Problem

This subsection provides an explicit formulation of the primal-dual pair for the DCOPF problem, which is a simplified representation of existing market formulations that generally come in the form of a security-constrained unit commitment (SCUC) or a SCED model. Most of the contemporary market models use a linearized DCOPF formulation that is based on PTDFs instead of the $B$-$\theta$ formulation that relies on the susceptance of transmission assets ($B$) and the bus voltage angles ($\theta$).

The PTDF-based formulation is easier to solve since it provides the option of ignoring the transmission assets that are inconsequential (i.e., rarely congested), thereby reducing modeling complexity. A primal problem formulation for a standard PTDF-based DCOPF is detailed below.

Minimize: $\sum_n c_n P_n$ \hfill (4.1)

Subject to:

$-P_n \geq -P_n^{max}, \forall n \in N \hfill (\alpha_n)$ \hfill (4.2)

$\sum_n PTDF_{k,n}^R (P_n - D_n) \geq -P_k^{max,a}, \forall k \in K \hfill (F_k^-)$ \hfill (4.3)

$-\sum_n PTDF_{k,n}^R (P_n - D_n) \geq -P_k^{max,a}, \forall k \in K \hfill (F_k^+)$ \hfill (4.4)

$\sum_n P_n - D_n = 0, \hfill (\delta)$ \hfill (4.5)

$D_n = \overline{D}_n, \forall n \in N \hfill (\lambda_n)$ \hfill (4.6)

$P_n \geq 0.$

The objective, (4.1), is to minimize the linear operating costs, which is equivalent to maximizing the market surplus since the demand is assumed to be perfectly inelastic. Constraint (4.2) imposes an upper bound on the real power scheduled from a generating resource. Note that, for simplicity, the minimum real power generating capacity is assumed to be zero for all generating resources. The dc power flow on a transmission asset is constrained by its normal (thermal or stability) rating, i.e., rate A, in (4.3) and (4.4) respectively. The dual variables of (4.3) and (4.4) are the flowgate marginal prices for those transmission assets or flowgates; these dual variables are used to calculate the congestion component of the LMP. Constraint (4.5) assures system-wide power balance between generation and demand; the dual variable of (4.5) captures the energy component of the LMP. Note that, in this formulation, the demand is treated as a variable following which it is fixed to equal a parameter in (4.6). The dual variable of (4.6) signifies the increase (or decrease) to the primal objective (4.1) if there is slightly more (or less) consumption by the demand at node $n$, which directly translates into the definition of the LMP. The corresponding dual problem formulation for the primal problem is given below.
Maximize : 

\[
-\sum_n P_{n, \delta_n, \lambda_n}^{\max} - \sum_k P_k^{\max, \alpha} (F_k^- + F_k^+) + \sum_n \bar{D}_n \lambda_n
\]  

(4.7)

Subject to:

\[
-\alpha_n + \sum_k \text{PTDF}_{k,n} (F_k^- - F_k^+) + \delta \leq c_n, \forall n \in N \quad (P_n)
\]

(4.8)

\[
\sum_k \text{PTDF}_{k,n} (F_k^+ - F_k^-) - \delta + \lambda_n = 0, \forall n \in N \quad (D_n)
\]

(4.9)

\[
\alpha_n \geq 0, F_k^- \geq 0, F_k^+ \geq 0, \delta \text{ free}, \lambda_n \text{ free}.
\]

At optimality, the dual objective (4.7) is equal to the primal objective (4.1) by strong duality. The first, second, and the third components of (4.7) denote the generation rent (short-term generation profit), the congestion rent, and the load payment, respectively. Since generation revenue is equal to generation cost plus generation rent, it can be proven that, at optimality, load payment is equal to generation revenue plus congestion rent. Constraints (4.8) and (4.9) represent the dual constraints corresponding to the generator production and the demand variables in the primal problem, respectively. Constraint (4.9) within the dual problem identifies \( \lambda_n \) as the LMP at node \( n \). Thus, the LMP, defined by (4.9a), is equal to the sum of the marginal energy (\( \delta \)) and the marginal congestion components. Note that, since the PTDF-based DCOPF formulation defined by (4.1)–(4.6) is assumed to be a lossless model, there is no loss component of the LMP for the work presented in this chapter. After identifying the equation that defines the LMP via (4.9), (4.8) reduces to (4.8a).

\[
\lambda_n = \delta + \sum_k \text{PTDF}_{k,n} (F_k^- - F_k^+), \forall n \in N \quad (D_n)
\]

(4.9a)

\[
-\alpha_n + \lambda_n \leq c_n, \forall n \in N. \quad (P_n)
\]

(4.8a)

The dual variable of (4.2), i.e., non-negative, signifies the marginal value of increasing a specific generator’s maximum capacity. Three cases can potentially exist in this context, while remembering that the lower bounds of all generators are assumed to be zero for this simplified DCOPF problem: 1) if a generator is producing, but not at its maximum capacity (i.e., \( \alpha_n = 0 \) by complementary slackness), then the LMP at the node of the generator is equal to its marginal cost (by complementary slackness); 2) if a generator is not producing anything (i.e., \( \alpha_n = 0 \) by complementary slackness), then the LMP at its node is less than or equal to its marginal cost; and 3) if the generator is producing at its maximum capacity (i.e., \( \alpha_n \geq 0 \)), then the LMP at its node is greater than (when \( \alpha_n > 0 \)) or equal (when \( \alpha_n = 0 \)) to its marginal cost (by complementary slackness). These results match with a simple economic interpretation of the shadow price for (4.2). Whenever you are not producing at your maximum capacity the short-term marginal benefit to increase your capacity beyond its existing capability is zero. When you are operating at your maximum capacity, the short-term marginal benefit to increase your capacity by 1 MW is equal to the difference between your LMP and your marginal cost; keep in mind that this explanation applies to the presented primal DCOPF formulation. If the presented formulation included other constraints that could restrict the generator’s output, e.g., ramp rate limits, then the description would be more complex. In general, note that if the DCOPF is formulated differently, the dual will not be the same and may result in different interpretations of that different dual.
4.3 Recent Industry Movements to Model Generator Contingencies in Market

4.3.1 Primal Formulation of the Enhanced DCOPF Problem

To meet NERC’s N-1 mandate more appropriately, recent literature suggests enhancing generator contingency modeling by ensuring post-contingency transmission security through an explicit representation of post-contingency congestion patterns for critical generator contingencies within the market auction models [86], [87], [90], [91]. The mathematical (primal) auction formulation for the enhanced DCOPF problem, motivated by the optimization problem proposed by CAISO in [90], is detailed below. Extensions to this work can be made to analyze other attempts to introduce more advanced corrective control actions.

Minimize: \( \sum_n c_n P_n \)  \( (4.10) \)

Subject to:

\[-P_n \geq -P_n^{\text{max}}, \forall n \in N \]  \( (\alpha_n) \)  \( (4.11) \)

\[\sum_n PTDF_{k,n}^R (P_n - D_n) \geq -p_k^{\text{max}}, \forall k \in K \]  \( (F^-_k) \)  \( (4.12) \)

\[-\sum_n PTDF_{k,n}^R (P_n - D_n) \geq -p_k^{\text{max}}, \forall k \in K \]  \( (F^+_k) \)  \( (4.13) \)

\[\sum_n PTDF_{k,n}^R (P_n + \text{GDF}_{n'}(c),n P_n' - D_n) \geq -p_k^{\text{max}}, \forall k \in K^{\text{crit}}, c \in C^g^{\text{crit}} \]  \( (F^-_k) \)  \( (4.14) \)

\[-\sum_n PTDF_{k,n}^R (P_n + \text{GDF}_{n'}(c),n P_n' - D_n) \geq -p_k^{\text{max}}, \forall k \in K^{\text{crit}}, c \in C^g^{\text{crit}} \]  \( (F^+_k) \)  \( (4.15) \)

\[\sum_n P_n - D_n = 0, \]  \( (\delta) \)  \( (4.16) \)

\[D_n = \overline{D}_n, \forall n \in N \]  \( (\lambda_n) \)  \( (4.17) \)

\[P_n \geq 0. \]

where:

\[\text{GDF}_{n'}(c) = \begin{cases} -1, & n = n'(c) \\ 0, & n \neq n'(c) \land n \notin S^{FR} \\ \frac{u_n P_n^{\text{max}}}{\sum_{n \in n'(c)} u_n P_n^{\text{max}}}, & n \neq n'(c) \land n \in S^{FR} \end{cases}, \forall n \in N, c \in C^g^{\text{crit}}. \]  \( (4.18) \)

In this formulation, the generation loss is distributed across the system via GDFs and is presumed to be lossless. Also, it is prorated based on the maximum online (frequency responsive) capacity, to approximate the actual system behavior, while ignoring capacity and ramp rate restrictions [90]. Equation (4.18) provides CAISO’s definition for GDFs, which is proposed to estimate the effect of generation loss and the associated system response on critical transmission assets in the post-contingency state. The post-contingency dc power flow (under a critical generator outage) on a
critical transmission asset is restricted by its emergency rating (rate C) in (4.14) and (4.15) respectively. The remainder of the formulation is consistent with the standard DCOPF formulation. Note that GDF is constructed to denote the outage of generation at a specific node, not to distinguish between the outage of a single generator at a node with multiple generators. Lastly, CAISO’s actual market model will be more complex than what is presented in (4.10)-(4.18), which does not include other modeling issues like transmission contingency modeling, reserve requirements, ramp rate limits, etc. The formulation is kept in a simpler manner to focus on the key proposed change, which is related to the inclusion of the generator contingency modeling with the use of the GDFs.

### 4.3.2 Dual Formulation for the Enhanced DCOPF Problem

The corresponding dual problem formulation is described below. Note that, while the following dual is derived based on the formulation in Section 4.3.1, other primal formulations are also possible, and the dual formulations will change as well.

\[
\text{Maximize} \quad \alpha_n \sum F_{k,n}^{R} (F_k^+ - F_k^-) + \sum_{c \in \mathcal{G}_{\text{GDF}}^c} (p_{n,k}^{\text{max},c}(F_k^c - F_k^{c+})) - \sum_{k \in \mathcal{K}_{\text{GDF}}} (p_{n,k}^{\text{max},a}(F_k^a + F_k^a)) - \sum_{k \in \mathcal{K}_{\text{GDF}}} (p_{n,k}^{\text{max},a}(F_k^a + F_k^a)) + \sum (\bar{D}_n \lambda_n) \quad (4.19)
\]

Subject to:

\[
\sum_{k \in \mathcal{K}_{\text{GDF}}} (F_k^+ - F_k^-) + \sum_{c \in \mathcal{G}_{\text{GDF}}^c} (F_k^c - F_k^{c+}) - \delta + \lambda_n = 0, \forall n \in N \quad (P_n) (4.20)
\]

\[
\sum_{k \in \mathcal{K}_{\text{GDF}}} (F_k^+ - F_k^-) + \sum_{c \in \mathcal{G}_{\text{GDF}}^c} (F_k^c - F_k^{c+}) - \delta + \lambda_n = 0, \forall n \in N \quad (D_n) (4.21)
\]

\[
\alpha_n \geq 0, F_k^+ \geq 0, F_k^- \geq 0, F_k^c \geq 0, F_k^{c+} \geq 0, \delta \text{ free, } \lambda_n \text{ free.}
\]

where:

\[
\bar{n'}(c,n) = \begin{cases} 0, & n \neq n'(c), \forall n \in N, c \in \mathcal{G}_{\text{GDF}}^c \\ 1, & n = n'(c), \forall n \in N, c \in \mathcal{G}_{\text{GDF}}^c \end{cases} \quad (4.22)
\]

\[
\lambda_n = \delta + \sum_{k \in \mathcal{K}_{\text{GDF}}} (F_k^+ - F_k^-) + \sum_{c \in \mathcal{G}_{\text{GDF}}^c} (F_k^c - F_k^{c+}), \forall n \in N. \quad (D_n) (4.21a)
\]

The dual objective now has an additional term, i.e., the third component in (4.19), which represents the post-contingency congestion rent resulting from generator contingency modeling. Constraints (4.20) and (4.21) represent the dual constraints corresponding to the generator production and the demand variables in the enhanced primal problem respectively. The primary impact that the proposed changes will have on market pricing is how it affects the LMPs. Constraint (4.21) within
the dual problem identifies $\lambda_n$ as the LMP at node $n$, which is further defined in (4.21a) and is equal to the sum of the marginal energy component, the marginal pre-contingency congestion component, and an additional marginal post-contingency congestion component that comes from the modeling of critical generator contingencies.

Note that, the enhanced primal problem defined by (4.10)-(4.18) differs from CAISO’s primal problem in [90] with respect to the following aspects. 1) The demand is first treated as a variable following which it is fixed to equal a parameter in (4.17) to enable the derivation of the LMP in a simpler manner. 2) The losses are ignored for simplification. 3) No new variables are introduced to denote the real power production from the system’s resources under a specific generator contingency. 4) Transmission contingency security constraints are ignored to allow the derivation in this study to focus on generator contingency modeling in a clear and concise manner.

4.3.3 Analyzing the Dual Formulation

To understand what is communication by the aforesaid dual problem, first, start with the objective functions of the primal and the dual. Linear optimization theory includes strong duality, which guarantees that the objective of the primal problem equals the objective of the dual problem at optimality. Achieving strong duality means there is no duality gap. Another way to interpret the strong duality relationship is through its expression of an exchange of money; payments and expenses resulting from the auction and the corresponding exchange of goods and services. There is the obvious piece from the objective of the primal problem, which is the total generation cost. The next obvious piece is the last term of the dual objective, which is the load payment, LMP times consumption. The first term of the dual objective is the short-term generator profit for a generator at node $n$, summed over all nodes, or the system-wide generation rent. The second and third terms in the dual's objective represent the system-wide congestion rent; this is to be expected as it relates the flowgate marginal price to the line flow, once complementary slackness is applied.

The discussion that follows analyzes the corresponding impact on the rent and the revenue for generators that are and are not included in the assumed set of critical generator contingencies. Recall first that GDF reflects the anticipated system response from a specific node in the network and the way it is structured in (4.18) assumes that there is only one unit at most at a node (easily modifiable). Second, note the formula to define the GDF is based purely on the generator’s capacity relative to the rest of the fleet’s capacity (for units that are frequency responsive). One obvious drawback of the proposed GDF is that it ignores the generator’s capacity, the generator’s ramp rate restrictions, and whether the ISO procured the necessary reserve product from the unit. As a result, this model assumes there is the capability to inject power at a node based on the definition of the GDF, not necessarily based on the actual ability for the generator to provide the needed reserve; for instance, a generator may be operating at its maximum capacity. The assumed GDF only accounts for capacity while not capturing the dispatch set point of the unit or whether the unit has been obligated to provide contingency reserve products (note that while the presented auction formulation does not include reserve procurement, the GDF itself does not reflect whatsoever on reserve and, as such, the impact of reserve is not captured anyway). Finally, the GDF shows up only in (4.14) and (4.15) and it is multiplied by the MW dispatch variable for the generator that is modeled to be under outage (or contingency). This translates into the post-contingency congestion for a generator outage, in the primal problem, to being directly related to the dispatch variable for the contingency generator only. For example, assume that generator 1 is
lost, which is located at bus 1. Assume that generator 2 is anticipated to completely pick up this entire loss of supply from generator 1 (so the ISO sets the GDF = 1 for generator 2) and generator 2 is at bus 2. Even though generator 2 is the unit anticipated to provide the needed injection, the functional form in (4.14) and (4.15) relate the change in the injection at bus 2 to be determined by the GDF (a fixed input parameter) and the output of generator 1. The GDF is basically masking the response that is provided by generator 2 for generator 1’s drop in supply. Thus, \( \bar{y} \) reduces to zero in (4.20) for generators that are not located at the nodes of the generators contained within the critical generator contingency list.

More importantly, the post-contingency congestion and the component of the LMPs that are reflective of this post-contingency state are driven by the cost of the generator that is lost; it is not driven by the cost that is associated to the generators that would respond. From a power engineering perspective, capturing the costs of the units responding does not fully matter in regard to ensuring a secure system; the primal problem captures the change in injection for the buses that are anticipated to have an increase in production when responding to the outage. From a cost perspective, it has an impact as the cost is only related to the generator that is lost and not the units that respond. For instance, suppose that the only generator contingency that is explicitly modeled is a large, baseload unit like a nuclear unit. Such baseload units are often cheaper in regard to their marginal cost, \$/MWh. On the other hand, the units that are likely to respond will be units that have fast ramping capabilities and are flexible; those are units that are generally more expensive. With this example it is clear that, at the very least, there is a high probability that the units that are chosen to be in the critical contingency list may be rather distinct in characteristics, and costs, than the units that are expected to respond. This is important since, again, the cost in the post-contingency state is not driven by the units responding but by the unit that is lost. At the same time, the model does not acknowledge any costs due to re-dispatch of generators in the post-contingency state. Only the pre-contingency costs are considered, which makes the impact of the post-contingency congestion a secondary influencing factor; the cost changes only by forcing a different pre-contingency dispatch set point that is secure rather than acknowledging the change in dispatch cost due to activation of reserve. Last, it is also equally important, if not more, to acknowledge the influence this has on the prices, via duality theory, that are then produced by this proposed reformulation by CAISO. This study does not dive deeper into this potential problematic issue since that is a topic that will require much more research and investigation, as identified in the future work in Section 4.4.

For generators that are not included in the critical list, using the definition of \( \bar{y} \) from (4.22) and the LMP for that generator’s location \( (\lambda_n) \) from (4.21a), (4.20) can be rewritten as (4.20a). Thus, (4.8a) and (4.20a) are similar with the exception that the LMP is now capturing an added, new congestion component reflecting congestion in the post-contingency operational state with the loss of a generator. Complementary slackness is then applied to the constraint-dual variable pair in (4.20a) to create (4.23). It is noteworthy to emphasize that (4.23) turns out to be identical to what would be obtained by applying complementary slackness to (4.8a), which again allows for the determination of the generator rent. Complementary slackness can also be applied to (4.11) to form (4.24). Then, based on (4.23) and (4.24), the generator rent for a generator that is not in the assumed critical generator contingency list is given by (4.25). The short-term generator profit (or generator rent) that will be earned by the non-critical generators is equal to the generator revenue less the generator cost. This generator rent term is basically identical to what is seen from the standard DCOPF formulation excepting that the LMP has an additional congestion component.
\[-\alpha_n + \lambda_n \leq c_n \quad (P_n) \quad (4.20a)\]

\[-\alpha_n P_n + \lambda_n P_n = c_n P_n \quad (4.23)\]

\[-P_n \alpha_n = -p_n^{\text{max}} \alpha_n, \forall n \in N \quad (4.24)\]

\[p_n^{\text{max}} \alpha_n = \lambda_n P_n - c_n P_n. \quad (4.25)\]

For the critical generators, using the definition of \( \bar{\lambda} \) from (4.22) and the LMP for that generator’s location \( (\lambda_n) \) from (4.21a), (4.20) can be rewritten as (4.20b). Complementary slackness is then applied to the constraint-dual variable pair in (4.20b) to create (4.26). Then based on (4.26) and (4.23), the generator rent for a generator that is contained within the assumed critical generator contingency list is given by (4.27).

It is pertinent to note that the LMP defined in (4.21a) is consistent with CAISO’s LMP definition in [90] for the nodes that do not have critical generators. Furthermore, CAISO’s LMP definition for the nodes that do have critical generators, i.e., whose outages are modeled explicitly, is provided below in (4.21b). Note that there is a resemblance in the last term of the profit function (within square brackets) for the generators that are included in the critical contingency list in (4.27) and the last term of CAISO’s LMP definition in (4.21b). This entails a detailed investigation into the net revenue stream for the generators that are contained within the assumed critical generator contingency list.

\[-\alpha_n + \lambda_n + \sum_{k \in K^{\text{crit}}} \left[ (F_k^{c} - F_k^{c+}) \left( \sum_{s \in N} PTD F_{k,s}^{R} GDF_{n'(s),s} \right) \right] \leq c_n \quad (P_n) \quad (4.20b)\]

\[-\alpha_n P_n + \lambda_n P_n + \sum_{k \in K^{\text{crit}}} \left[ (F_k^{c} - F_k^{c+}) \left( \sum_{s \in N} PTD F_{k,s}^{R} GDF_{n'(s),s} \right) \right] P_n = c_n P_n \quad (4.26)\]

\[p_n^{\text{max}} \alpha_n = \lambda_n P_n - c_n P_n + \sum_{k \in K^{\text{crit}}} \left[ (F_k^{c} - F_k^{c+}) \left( \sum_{s \in N} PTD F_{k,s}^{R} GDF_{n'(s),s} P_s \right) \right]. \quad (4.27)\]

CAISO’s LMP definition for nodes with critical generators [90]:

\[\lambda_n = \delta + \sum_{k} PTD F_{k,n}^{R} (F_k^{c} - F_k^{c+}) + \sum_{k \in K^{\text{crit}}} PTD F_{k,n}^{R} (F_k^{c} - F_k^{c+}) + \sum_{k \in K^{\text{crit}}} \left[ (F_k^{c} - F_k^{c+}) \left( \sum_{s \in N} PTD F_{k,s}^{R} GDF_{n'(s),s} \right) \right] \quad (4.21b)\]

There are a few key issues to consider here. First, note that despite the presence of the extra term in (4.27); (4.27) describes the profit that will be earned by the critical generators. In this specific primal reformulation of the DCOPF, described by (4.10)-(4.18), only a single linear operating cost component is considered for the production from the generator at node \( n \). In addition, the generator production variable has no other restrictions other than lower and upper bound restrictions. Analogous to the dual analysis provided for the standard DCOPF problem in Section 4.2.2, three cases can potentially exist for the generators in this primal reformulation with single linear cost coefficients and only lower and upper bounds. (1) If a generator is not producing at its maximum capacity, the short-term marginal benefit (profit) to increase its capacity beyond its existing
capability is zero. (2) If it is operating at its maximum capacity, the short-term marginal benefit to increase its capacity by 1 MW is equal to the difference between what it is paid and its marginal cost. (3) If it is not producing anything, then what it is paid must be less than or equal to its marginal cost. Consequently, the penalty (shadow) price of (4.11), $\alpha$, does completely capture the $/MWh rate for the corresponding unit’s profit or the marginal benefit of increasing its maximum capacity, which makes $\alpha_n P_n$ (or $\alpha_n P_n^{max}$ by complementary slackness) its profit function. Therefore, the net revenue stream for a generator should equal the sum of its profit and cost. Equation (4.27a) defines the revenue for the generators that are contained within the assumed critical list of generators.

$$P_n^{max} \alpha_n + c_n P_n = \lambda_n P_n + \sum_{k \in C_{crit}} \left[ (F_k^c - F_k^s) \left( \sum_{S \in N} PTD_F k_S GDF n'_c S P_S \right) \right].$$  \hfill (4.27a)

Second, if the short-term generator profit for a generator at node $n$, summed over all nodes, is equal to the total generation profit, then by strong duality, at optimality (4.28) holds. In other words, at optimality, the objective of the primal reformulation must equal the objective of the dual problem, or the load payment is equal to the sum of the total generation cost, the total generation profit, and the total congestion rent.

$$\sum_n c_n P_n = -\sum_n P_n^{max} \alpha_n - \sum_k P_k^{max,r} (F_k^c + F_k^s) - \sum_{k \in C_{crit}} P_k^{max,c} (F_k^c + F_k^s) + \sum_n D_n \lambda_n.$$  \hfill (4.28)

If $\alpha_n P_n^{max}$ is equal to the short-term generator profit for a generator then it implies that (4.27) describes the profit that will be earned by the critical generators, which means that a critical unit’s revenue is not just the LMP at its location ($\lambda_n$) times the production, but its revenue also includes the added extra (last) term in (4.27). On the contrary, if $\alpha_n P_n^{max}$ is not equal to the short-term generator profit and instead a critical unit is only paid the LMP at its location ($\lambda_n$) times the production, then its profit is equal to LMP at its location ($\lambda_n$) times the production less the cost. In this case, the short-term generator profit for a generator at node $n$, summed over all nodes, will not equal the term in the dual objective that is supposed to represent the total generation profit of the entire system. In other words, this would remove the added extra term from its revenue. Furthermore, since complementary slackness dictates that (4.27) should hold, which again is the short-term generator profit for a generator at node $n$, summed over all nodes, will not equal the total generation profit of the entire system. To summarize, if $\alpha_n P_n^{max}$ does not denote the short-term generator profit for both the critical and the non-critical generators, it will result in an ISO that is not revenue neutral. The ISO will either have a revenue shortfall overall or surplus. This confirms and clarifies what the generators in the assumed critical list should be paid and explains the reasoning behind why CAISO is potentially including the added extra term in their definition of the LMP at the nodes of the critical generators in (4.21b). However, CAISO’s definition of the LMP at the nodes of the critical generators is not consistent with the traditional definition of the LMP or the LMP that is identified by the corresponding dual formulation. The critical generators should now be paid this extra term but that does not imply the extra term be included in the LMP since this will have associated implications in the FTR markets (note that this LMP is further used to settle the FTR payments in the FTR markets). Also, note that, in (4.28), the load pays the LMP identified by the dual formulation. Further extension of this work is necessary to evaluate more
advanced reformulations to enhance generator contingency modeling (an example is detailed in Section 4.4) and its corresponding effect on market prices, settlements, and revenues.

Third, it is necessary to understand the interpretation and the implications of the extra term in (4.27). If a critical generator is under an outage, the GDFs specify that the corresponding injections to compensate for the drop in its supply are defined based on its value for its locations. Essentially, the extra term in the profit function, (4.27), is what the critical generator is paid by the ISO to compensate for the loss of its production. Now, if the extra term is combined with the fact that the unit is being paid the LMP at its location for its production, what this translates into is that the corresponding critical generator is basically paying a congestion charge for the difference between injecting at its location and instead injecting at the locations identified by the GDFs. Thus, the combination of the extra term and its LMP component corresponding to the outage is basically a congestion transfer cost. Another way to interpret this is that, for that particular contingency scenario, the critical generator will inject at the locations that are identified by the GDFs instead of injecting at its own location. The LMP already compensates for the expected injection at its location. The extra term is the transfer due to injections based on the pre-defined rules of the GDF. Another way to interpret this would be that the generator that is lost has storage at each node identified by the GDF definition and is expected to compensate for its own contingency by injecting at those locations. The model still acknowledges that the generator is producing; it is just producing now magically at different locations. As such, the dual formulation suggests that the unit should be compensated exactly by that (invalid) assumption. The right way to make this work is to have the critical generator buy from the locations identified by the GDF instead or have some sort of a side contract with the generators at those locations. This study defines how this pricing structure would work if it is to follow the exact prescribed formulation proposed within the primal.

To assist in understanding the implications, it is helpful to go back to (4.14) and (4.15). Recall that the GDF shows up only in (4.14) and (4.15) and it is multiplied by the MW dispatch variable for the generator that is lost. This basically simulates the cost of a critical generator backing up its loss based on its costs at other locations. The fact that the equations are driven based on its dispatch variable and not the dispatch variables of the responding generators implies that this critical generator’s cost influences the duals for this issue and not the cost of the responding generators. Thus, the responding generators do not affect the outcome for the generator that is lost. This is an important implication because this is sensitive to which generators are chosen to be included in the list of critical generator contingencies. CAISO acknowledges this concern by stating that analogous to how transmission security constraints are selectively enforced in contemporary markets, the ISO will decide which generators are critical and need to be explicitly modeled based on engineering analysis and outage studies [90].

Finally, the definition of congestion rent is the flowgate marginal price times the flow on the line, summed over all lines. Congestion rent can be also identified by the difference in load payment and the generation revenue; it is easy to confirm that these two approaches provide the same value for the congestion rent as strong duality provides a formula where generation cost (the primal objective) is equal to the load payment minus the generation rent (short-term generation profit) minus the congestion rent at optimality. This can be more easily identified by applying strong duality to the simplified DCOPF and its dual, (4.1) and (4.7). For the more complicated primal reformulation auction model that includes security constraints associated to generator contingencies, it becomes more complex; the congestion rent can be identified by taking (4.12)
(4.15) and applying complementary slackness. Thus, the system-wide congestion rent in this case is equal to the second and third terms in the dual’s objective, (4.19); this is to be expected as it relates the flowgate marginal price to the line flow, once complementary slackness is applied.

### 4.4 Conclusions and Future Research

This study presents a comprehensive theoretical analysis of the recent industry movements, specifically, CAISO’s efforts, to model generator contingencies in market models more appropriately. The main intention is to examine (and question) and complement the movement in the industry to enhance generator contingency modeling and to analyze the market impact of the policies that are proposed in this realm of research. It is noteworthy to emphasize that if the primal (DCOPF problem in this context) is formulated differently, the dual will not be the same and may result in different interpretations of that different dual. As this study has demonstrated, this is why it is very important to perform a rigorous evaluation via duality theory to investigate the potential implications of the proposed market change.

Further extensions of the auction formulation presented in Section 4.3 is essential to evaluate more advanced reformulations to enhance generator contingency modeling and its corresponding effect on market prices, settlements, and revenues. For instance, it is pertinent to analyze an auction reformulation that enhances generator contingency modeling by incorporating an explicit representation of post-contingency power balance in addition to the previously mentioned post-contingency transmission security for critical generator contingencies. The post-contingency power balance constraints will help in assuring system-wide power balance between post-contingency generation and post-contingency demand. This essentially also provides an opportunity to model the expected post-contingency demand consumption under the different critical generator contingency states. However, the obvious setback with such an approach, i.e., the explicit inclusion of post-contingency power balance constraints, is the associated increase in the computational complexity of the corresponding problem. The anticipated impact that the corresponding change will have on market pricing is (again) how it affects the LMPs. This change will result in an LMP at a node for each of the modeled critical generator contingency states. In addition, it is also important to analyze the corresponding effect on the market settlements and revenues.

The explicit consideration of credible generator contingencies in general is expected to result in fewer ex-post OMCs (or adjustments), which is technologically and economically beneficial [86], [87], [89]. The explicit consideration of credible generator contingencies (and fewer ex-post adjustments) enable the market auction to optimize more of the market, which, in turn, results in improvements in market efficiency and improved price signals [86], [87].

Future research should include implementing and testing the effectiveness of the proposed enhancements in improving the market surplus on a large-scale test system. Furthermore, to overcome the issues identified in this chapter, future work should examine the market implications of the generator contingency modeling approach proposed by MISO in [91] and identify a means to extend MISO’s approach to include both intra-zonal and inter-zonal transmission assets in addition to modeling more than one critical generator contingencies per reserve zone. The next steps should also investigate new means to introduce corrective actions via different reformulations and the associated market impacts.
5. Conclusion

This research evaluated several essential structural changes to energy markets in three core areas: (1) flexible ramping products and multi-period pricing policies, (2) enhanced generator contingency modeling, and (3) management of stochastic resources. This research provided a comprehensive evaluation of the impacts of these changes on market outcomes. This research also provided suggestions on market reformulations to enhance market efficiency, pricing, and transparency.

First, in this work, the impacts of uncertainty modeling strategies on electricity market outcomes, pricing, and settlements were analyzed. The LMP comprises three components, i.e. energy, congestion, and loss components. A method based on the duality theory was used to shed light on the LMP calculation in a stochastic market model; this theoretical method confirmed that the value of providing N-1 reliability services can be reflected in the LMPs of such stochastic market models. This pricing scheme is then compared to the two state-of-the-art market auction models, where the corrective actions to achieve an N-1 reliable solution is postponed to OMC. Also, the market settlements of these models are calculated and compared. With these analyses, this report seeks to inform market stakeholders about the impacts of contingency modeling approaches in the DA process and their implications on pricing and settlements. Based on these analyses, it was shown that with more accurate representation of contingencies in the ESCUC, compared to SCUC models with approximation on N-1 security criteria, N-1 grid security requirements are originally captured, thereby, the value of service (contingency-based reserve) provided by generators is reflected to achieve grid security. In other words, if the market SCUC includes the reliability criterion more adequately, prices can better reflect the true marginal cost associated with the provision of the reliable electricity. Furthermore, this report analyzed the choices of objective function for the stochastic market models; a stochastic market design with an expected cost objective function was examined and compared with the base-case cost minimization objective function from two aspects: (i) realized cost during N-1 contingencies and (ii) effects of inaccurate calculation of the probabilities on market outcomes. It was shown that the stochastic market design with expected objective function does not give solutions that ensure minimum realized operating costs at the N-1 contingency states. Instead, the stochastic market design with base-case objective function had better performance compared to the aforementioned market model with respect to the base-case costs and realized N-1 costs. Moreover, the inaccuracy in estimated probability of scenarios results in larger differences in the original expected costs and the costs of scenarios of ESCUC-base and ESCUC-expected, where ESCUC-base further outperforms ESCUC-expected. It can be concluded that evidently the stochastic market model design with base-case objective function can be more efficient compared to the stochastic market design with expected objective function. It is worth noting that assessments of the effects of the OMC performed in different market models due to other the approximations (e.g., linear approximation of AC power flow, use of cutoffs for PTDFs, voltage security limits, and renewable uncertainties) on the pricing and settlements are interesting directions for future research.

Second, a DA market model was formulated analogous to the CAISO’s DA market model with addition of FRP constraints. More specifically, we have incorporated existing RT FRPs design of CAISO into a DA market model based on the hourly time interval resolution. Then, this report shed light on a subtle issue that can potentially happen in the next market processes after DA market, as a result of procurement of DA ramp capabilities only based on the hourly ramping
requirements. Then, a new FRP design was proposed to address this issue. In the proposed formulation, the DA FRP design was modified to capture the impacts of the 15-min net load variability and uncertainty on the hourly ramping capabilities in the DA market, which improves upon the existing industry models. Finally, to effectively evaluate different FRP designs from reliability and market efficiency points of view, a validation methodology was proposed similar to the RTUC process of the CAISO. It was shown that the proposed model enhances the quantity allocation of FRPs by making relatively small market changes compared to CAISO’s proposed FRP design, which leads to less expected final operating cost in the FMM, higher reliability as the power system gets close to RT operation, and less discrepancy between DA and RTUC decisions.

Third, this research investigated the causation of uplift payments due to inter-temporal ramping in real-time markets. The research shows that constant ramp rates do not cause uplift if dispatch decisions and prices in all time intervals are all financially binding. The main principle used for the proof is that convex markets do not require uplift payments. Dispatch-dependent ramp rates may result in a nonconvex market; whenever a market nonconvexity exists, that nonconvexity may cause uplift payments. Finally, whenever you have a sequential decision making process, that is, you solve a collection of intervals (say periods 1-4) and then repeat by solving the following intervals (now 2-5) in a rolling horizon process, there is no guarantee that uplifts may not occur due to the inefficiency of breaking the real world time periods into separate mathematical programs. While this situation is essentially unavoidable, the causation of uplifts is far less due to this problem whereas uplift is more so caused by: (i) nonconvexities, (ii) lost opportunity costs, and (iii) single period scheduling and pricing or multi-period scheduling with single period pricing. Our recommendation is to go with a multi-period scheduling and multi-period pricing structure. We also recommend that causation of uplift payments be appropriately identified. Ramp rates have been characterized as a cause of uplift, including in market structures where the ramp rates are not causing the uplift, which can be mathematically proven; instead, the cause is an inadequate market structure (e.g., single period scheduling and pricing). Yes, ramp rates may cause uplift payments depending on how they are modeled. Ultimately, it is critical to properly identify causation of uplift payments in order to know the true problem and fix it accordingly.

Lastly, this research presented a comprehensive theoretical analysis of the recent industry movements, specifically, CAISO’s efforts, to model generator contingencies in market models more appropriately. The main intention is to examine (and question) and complement the movement in the industry to enhance generator contingency modeling and to analyze the market impact of the policies that are proposed in this realm of research. It is noteworthy to emphasize that if the primal (DCOPF problem in this context) is formulated differently, the dual will not be the same and may result in different interpretations of that different dual. As this study has demonstrated, this is why it is very important to perform a rigorous evaluation via duality theory to investigate the potential implications of the proposed market change. The explicit consideration of credible generator contingencies in general is expected to result in fewer ex-post OMCs (or adjustments), which is technologically and economically beneficial [86], [87], [89]. The explicit consideration of credible generator contingencies (and fewer ex-post adjustments) enable the market auction to optimize more of the market, which, in turn, results in improvements in market efficiency and improved price signals [86], [87].

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References


Part II

Pricing Multi-interval Dispatch Under Uncertainty

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1. Introduction

Part II of the report considers the problem of pricing multi-interval look-ahead economic dispatch when generators are ramp-limited and demand forecasts inaccurate. This work is motivated by recent discussions among system operators on the need for ramping products in response to the “duck-curve” effect of renewable integrations [1–6]. A well-designed multi-interval look-ahead dispatch that anticipates trends of future demand can minimize the use of expensive reserves.

A standard implementation of a look-ahead dispatch is the so-called rolling-window dispatch, where the operator optimizes the dispatch over a few scheduling intervals into the future based on load forecasts. The dispatch for the immediate scheduling interval (a.k.a. the binding interval) is implemented while the dispatch for the subsequent intervals serves as an advisory signal and is updated sequentially.

In pricing rolling-window dispatch, a common practice is the rolling-window version of the multi-interval locational marginal pricing (LMP). Based on the principle of marginal cost pricing, LMP is a uniform pricing mechanism across generators and demands at the same location in the same scheduling interval. When implemented over the entire scheduling period with a perfect demand forecast, LMP has remarkable properties. LMP supports an efficient market equilibrium such that a profit-maximizing generator has no incentive to deviate from the central dispatch. LMP guarantees a nonnegative merchandising surplus for the system operator. As a uniform pricing scheme, LMP is transparent to all market participants, and the price can be computed easily as a by-product of the underlying economic dispatch.

Most of the attractive features of LMP are lost, unfortunately, when the rolling-window version of LMP (R-LMP) is used and demand forecasts inaccurate. Indeed, even if perfect forecasts are used in R-LMP, none of the nice properties are guaranteed. In particular, a missing-money scenario arises when a generator is asked to hold back its generation in order to provide ramping support for the system to meet demands in future intervals. In doing so, the generator incurs an opportunity cost and may be paid below its best offer to generate. Expecting compensations for the opportunity costs in future intervals, the generator disappoints when the anticipated higher payments do not realize due to changing demand forecasts. Therefore, a generator may not be payments above the minimum payment defined by its offer under R-LMP. Examples of such scenarios are well known and also illustrated in Example 2 in Sec 2.4. It turns out that such examples are not isolated instances; they occur to all uniform pricing schemes. (See Sec. 2.2 Theorem 2.)

To ensure that generators are adequately compensated, the operator provides the so-called uplift payments to generators suffering from underpayments in an out-of-the-market settlement. The roles of such uplifts have been discussed extensively in the literature [7–10]. Such settlements are typically discriminative and often subject to manipulation. They may lead to a negative merchandising
surplus that has to be socialized among consumers.

1.1 Related work

The dispatch-following incentive issues with pricing multi-interval dispatch have been widely discussed in the literature [2,6,10–14] although a formal way of analyzing such issues is lacking.

Several marginal cost pricing schemes have been proposed for the rolling-window dispatch policies. The flexible ramping product (FRP) [3] treats ramping as a product to be procured and priced uniformly as part of the real-time dispatch. FRP is a two-part tariff consisting of prices of energy and ramping. Ela and O’Malley proposed the cross-interval marginal price (CIMP) in [11] defined by the sum of marginal costs with respect to the demands in the binding and the future (advisory) intervals. Multi-settlement pricing schemes are proposed in [14,15] that generalize the existing two-settlement day-ahead and real-time markets.

Deviating from marginal cost pricing are two recent proposals aimed at minimizing the out-of-the-market payments; both employ separate pricing optimizations that are different from those used in the economic dispatch. The price-preserving multi-interval pricing (PMP), initially suggested by Hogan in [16] and formalized in [13], adds to the objective function the loss-of-opportunity cost for the generators for the realized prices and dispatch decisions. In contrast, the constraint-preserving multi-interval pricing (CMP) proposed in [13] fixes the past dispatch decisions and penalizes ramping violations. Both have shown improvements over the standard rolling-window LMP policy.

All existing pricing schemes for multi-interval economic dispatch are based on uniform pricing mechanisms. To our best knowledge, no existing pricing policies can provide dispatch-following incentives that eliminate discriminative out-of-the-market settlements.

1.2 Summary of results, contexts, and limitations

The main contribution of this work is threefold. First, we analyze dispatch-following incentives for generators. For the multi-interval dispatch model, one can treat the generation/consumption in different scheduling intervals as separate (but dependent) markets. Two types of market equilibrium models are considered. We borrow the notion of the general equilibrium model that considers all markets jointly and the partial equilibrium model that focuses on one market with fixed prices and productions in all others. Under the general equilibrium model, a generator has no incentive to deviate from its dispatch, given the equilibrium prices for all scheduling intervals. Under the partial equilibrium model, on the other hand, a generator has no incentive to deviate from the dispatch signal for the next scheduling interval, independent of prices in other scheduling intervals.

When load forecasts are perfect and the scheduling of all intervals are set simultaneously in an one-shot economic dispatch, the multi-interval economic dispatch and LMP form a general equilibrium. Consequently, no generator would deviate from the dispatch no matter how unfavorable the price in
a particular interval may appear. The dispatch and LMP in an interval, however, do not necessarily form a partial equilibrium. When demand forecasts are imperfect or when rolling-window dispatch and pricing policies are used, a generator facing unfavorable prices in a particular interval has incentives to deviate from the central dispatch signals.

Second, we introduce the notion of strong equilibrium that captures the dispatch-following incentive conditions under arbitrary load forecast errors. A dispatch and pricing policies form a strong equilibrium if they are simultaneously a general equilibrium and a partial equilibrium for all scheduling intervals. The strong equilibrium conditions guarantee that there is no incentive for a generator not to follow the dispatch even when demand forecasts are not accurate.

We begin with a negative result in Theorem 2: under mild conditions, no uniform pricing mechanism guarantees dispatch-following incentives under the rolling-window dispatch model. This result means that, if out-of-the-market uplifts are used to ensure dispatch-following incentives under uniform pricing, discriminative payments are unavoidable.

Next, we generalize LMP to nonuniform pricing and refer the generalization to as temporal locational marginal pricing (TLMP). TLMP prices the generation of a generator $i$ based on its contribution to meeting the demand in interval $t$. In doing so, TLMP encapsulates both generation and ramping-induced opportunity costs in each scheduling interval.

As shown in Proposition 2, TLMP decomposes into energy and ramping prices:

$$\pi_{t,i}^{\text{TLMP}} = \pi_{t,i}^{\text{LMP}} + \mu_{t,i} - \mu_{i(t-1)},$$

where $\pi_{t,i}^{\text{LMP}}$ is the standard LMP, and the second term is the increment of the Lagrange multipliers associated with the ramping constraints in the economic dispatch optimization, from $\mu_{i(t-1)}$ in interval $(t-1)$ to $\mu_i$ in interval $t$. The above decomposition is entirely analogous to the energy-congestion price decomposition of LMP. TLMP naturally reduces to LMP in the absence of binding ramping constraints.

Among the most important properties of TLMP is that it satisfies the strong equilibrium conditions regardless of the accuracy of the load forecast, thus eliminating completely the need of the out-of-the-market uplifts. Another significant property of TLMP (Proposition 3 of Part II) is the decomposition of the merchandising surplus of the operator into congestion and ramping surplus, which has significant implications on the revenue adequacy of ISO, incentives for truthful revelation of ramping limits, and incentives for the generators to improve their ramping capabilities.

Given that TLMP is discriminatory, one may question how different it is from other discriminative pricing schemes. Besides TLMP, the pay-as-bid (PAB) pricing is perhaps the only discriminative pricing mechanism that satisfies the strong equilibrium conditions for multi-interval economic dispatch; it trivially provides dispatch following incentives. The main short-coming of PAB is that it is vulnerable to manipulative bidding behavior.
The differences between TLMP and PAB pricing are significant, however. In the absence of binding ramping constraints, TLMP is the same as LMP; it thus inherits the same incentive compatibility advantage of LMP over PAB pricing under the perfect competition assumptions\(^*\) [18, 19]. When ramping events occur, whether a generator can benefit by inflating its generation offers depends on the ramping scenarios. Indeed, examples exist that inflating generation offers may very well lower generation profit [20].

Discriminative pricing is often criticized for the lack of transparency, which makes it difficult for the operator to provide public pricing signals to market participants. Because of the decomposition of TLMP into the standard LMP and a discriminative ramping price in (1.1), it may be argued that TLMP offers the same level of transparency as LMP when the discriminative out-of-the-market uplifts used by LMP are taken into account. TLMP simply exercises the discrimination inside rather than out of the market.

Finally, in Chapter 3 of the report, we generalize the theory of dispatch-following incentives to more general models that include network constraints and discuss a broader set of incentive and performance issues. As a generalization of LMP, we show that TLMP introduces a discriminative penalty when a generator reaches its ramping limits. Consequently, TLMP provides incentives for generators to reveal its ramping limits truthfully when all existing pricing schemes have situations that a generator is better-off to under-report its ramping limits. When comparing different pricing schemes, our results shine lights on practical tradeoffs along several dimensions: the revenue adequacy of the ISO, consumer payments, generator profits, the interplays of ramping and congestion limits, and price volatilities.

A few words are in order on the scope and limitations of this report. The models used in Chapter 2 and Chapter 3 have limitations. In particular, we do not model the strategic behavior of the generators; nor do we consider alternatives to pricing ramping explicitly in the real-time market. We discuss in Sec. 2.5 some of the implications of these omissions. We also ignore the role of unit commitment and the cost of the reserve. In Chapter 2, we illustrate the properties of LMP and TLMP with a toy example. Generalizations to systems with network constraints and more elaborate numerical examples are in Chapter 3. The proofs of the theorems can be found in the appendix section of Chapter 2 and Chapter 3.

### 1.3 Notations and nomenclature

Designated symbols are listed in Table I. Otherwise, notations used here are standard. We use \((x_1, \cdots, x_N)\) for a column vector and \([x_1, \cdots, x_N]\) a row vector. All vectors are denoted by lowercase boldface letters, nominally as columns. The transpose of vector \(x\) is denoted by \(x^\top\). Matrices are boldface capital letters. Matrix \(X = [x_{ij}]\) is a matrix with \(x_{ij}\) as its \((i, j)\)th entry. Similar to the vector notation, matrix \(X = [x_1, \cdots, x_N]\) has \(x_i\) as its \(i\)th column, and matrix \(X = (x_1^\top, \cdots, x_N^\top)\) has \(x_i^\top\) as \(i\)th row.

\(^*\) Market power under LMP may exist in practice [17].
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>0, 1:</strong></td>
<td>vector of all zeros and ones.</td>
</tr>
<tr>
<td><strong>A:</strong></td>
<td>The upper bi-diagonal matrix with (-1) as diagonals and 1 as off diagonals.</td>
</tr>
<tr>
<td><strong>d_t, d:</strong></td>
<td>demand in interval (t), (d = (d_1, \cdots, d_T)).</td>
</tr>
<tr>
<td><strong>d_t:</strong></td>
<td>demand in rolling (l) interval, (d_t = (d_t, \cdots, d_{t+W-1})).</td>
</tr>
<tr>
<td><strong>(\hat{d}_t, \hat{d}_t:</strong></td>
<td>demand forecast: (\hat{d}_t = (\hat{d}<em>t, \cdots, \hat{d}</em>{t+W-1})).</td>
</tr>
<tr>
<td><strong>f_i(\cdot):</strong></td>
<td>the bid-in cost of generator (i) in interval (t).</td>
</tr>
<tr>
<td><strong>F(\cdot):</strong></td>
<td>aggregated bid-in cost curve.</td>
</tr>
<tr>
<td><strong>(\hat{f}_i(\cdot):</strong></td>
<td>aggregated supply curve excluding generation from generator (i) in interval (t).</td>
</tr>
<tr>
<td><strong>g_i(t):</strong></td>
<td>generation/dispatch of generator (i) in interval (t).</td>
</tr>
<tr>
<td><strong>g(\cdot):</strong></td>
<td>generation/dispatch of generator (i) over several intervals, e.g., (g_i = (g_{i1}, \cdots, g_{iT})).</td>
</tr>
<tr>
<td><strong>g[t]:</strong></td>
<td>generation/dispatch vector for all generators in interval (t), (g[t] = (g_{1t}, \cdots, g_{Nt})).</td>
</tr>
<tr>
<td><strong>G:</strong></td>
<td>generation/dispatch matrix. (G = [g[1], \cdots, g[T]]).</td>
</tr>
<tr>
<td><strong>G_t, G_t:</strong></td>
<td>dispatch policy for intervals starting at (t).</td>
</tr>
<tr>
<td><strong>G^{ED}_t, G^{ED}_t:</strong></td>
<td>Multi-interval economic dispatch policies.</td>
</tr>
<tr>
<td><strong>g^{ED}_t, G^{ED}_t:</strong></td>
<td>Multi-interval economic dispatch signals.</td>
</tr>
<tr>
<td><strong>g^{R-ED}_t, G^{R-ED}_t:</strong></td>
<td>Rolling-window economic dispatch signals.</td>
</tr>
<tr>
<td><strong>H_t:</strong></td>
<td>scheduling window (H_t = {t, \cdots, t+W-1}).</td>
</tr>
<tr>
<td><strong>H:</strong></td>
<td>scheduling horizon (H = {1, \cdots, T}).</td>
</tr>
<tr>
<td><strong>LOC:</strong></td>
<td>Lost-of-opportunity cost uplift.</td>
</tr>
<tr>
<td><strong>MW:</strong></td>
<td>Make-whole uplift.</td>
</tr>
<tr>
<td><strong>P_t, P^{LMP}_t:</strong></td>
<td>multi-interval pricing policy. LMP pricing.</td>
</tr>
<tr>
<td><strong>(\pi^{LMP}, \pi^{TLMP}:</strong></td>
<td>Multi-interval LMP and TLMP.</td>
</tr>
<tr>
<td><strong>(\pi^{R-MLP}, \pi^{R-MLP}:</strong></td>
<td>Rolling-window TLMP.</td>
</tr>
<tr>
<td><strong>T:</strong></td>
<td>Total number of scheduling intervals.</td>
</tr>
<tr>
<td><strong>W:</strong></td>
<td>Scheduling window, (W \leq T).</td>
</tr>
</tbody>
</table>

**Table 1.1:** List of major symbols used in alphabetic order.
2. Dispatch-Following Incentives

2.1 Multi-interval dispatch and pricing models

We consider a bid-based real-time electricity market involving one inelastic demand, $N$ generating firms, and a system operator (SO). The scheduling period of generations involves $T$ unit-length intervals $\mathcal{H} = \{1, \ldots, T\}$, where interval $t$ covers the time interval $[t, t+1)$. Typically, $T$ is the number of intervals in a day.

We assume that each generator produces a generation offer that defines the best offer to generate over $T$ intervals. Implicitly, the generation offer expresses its generation costs (both fuel and opportunity costs) in the form of the minimum price of generation. The offer also includes generation constraints such as generation capacity and ramping limits that are used in the dispatch and pricing decisions of the operator.

The operator collects bids from all generating firms, allocates generation levels to all generators in the form of dispatch signals, and determines the prices of electricity in each scheduling interval. Because the bid of a generator represents its best offer to generate, the generator expects the total payment received over $T$ intervals to be no less than that computed from its offered prices; anything less implies the loss-of-opportunity costs to be compensated by some form of out-of-the market uplifts.

Chapter 2 assumes a single bus network, which is generalized to networks with $N$ buses subject network constraints in Chapter 3. We introduce two multi-interval scheduling and pricing models. One is the one-shot model that sets generation dispatch and prices over the entire scheduling period at once, the other the rolling-window model that sets the dispatch levels and prices sequentially with demand forecasts for several intervals into the future.

2.1.1 One-shot multi-interval dispatch and pricing policies

At $t = 1$, the operator obtains the demand forecast vector $\hat{d} = (\hat{d}_1, \ldots, \hat{d}_T)$ over the entire scheduling period, where $\hat{d}_t$ is the demand forecast for interval $t$. Let the actual demand be $d = (d_1, \ldots, d_T)$. We assume that the forecast of the first interval is accurate, i.e., $\hat{d}_1 = d_1$.

A one-shot dispatch schedules generations over the $T$-interval scheduling horizon $\mathcal{H}$ based on the initial forecast $\hat{d}$. Let $g_i$ be the dispatch of generator $i$ in interval $t$, $g_i = (g_{i1}, \ldots, g_{iT})$ the dispatch for generator $i$ over $\mathcal{H}$, $g[t] = (g_{1t}, \ldots, g_{Nt})$ the dispatch for all generators in interval $t$, and the $N \times T$ matrix $G = [g[1], \ldots, g[T]]$ the dispatch matrix with $g_i^T$ as its $i$th row.

* We make a distinction between dispatch and actual generation. When a dispatch policy sets the dispatch signal of generator $i$ in interval $t$ to $g_{it}$, the actual generation may be different.
A one-shot dispatch policy \( \mathcal{G} \) maps the demand forecast \( \hat{d} \) and the initial generation \( g[0] \) to a dispatch matrix \( G \): \[ \mathcal{G}(\hat{d}, g[0]) = G, \] where \( g[0] \) imposes the initial ramping constraints on the generations in the first interval.

Similarly, a one-shot pricing policy \( \mathcal{P} \) sets the prices in all intervals at once. A one-shot uniform price is defined by vector \( \pi = (\pi_1, \ldots, \pi_T) \) with \( \pi_t \) being the price of electricity in interval \( t \) for all generators and the demand. For a nonuniform pricing policy, \( \mathcal{P} \) sets \( \pi_0 \) the price vector for the demand and \( \pi_i = (\pi_{i1}, \ldots, \pi_{iT}) \) for generator \( i \), for \( i = 1, \ldots, N \).

### 2.1.2 One-shot economic dispatch and LMP

A special case of the one-shot dispatch is the multi-interval economic dispatch \( \mathcal{G}^{\text{ED}} \) over \( \mathcal{H} \). Let the aggregated bid-in cost function be

\[
F(G) := \sum_{i=1}^{N} \sum_{t \in \mathcal{H}} f_{it}(g_{it}),
\]

where \( f_{it}(\cdot) \) is the bid-in cost curve of generator \( i \) in interval \( t \), assumed to be convex and almost everywhere differentiable for all \( t \) and \( i \) throughout Chapter 2.

The dispatch policy \( \mathcal{G}^{\text{ED}} \) is defined by

\[
\mathcal{G}^{\text{ED}} : \begin{array}{ll}
\text{minimize} & F(G) \\
\text{subject to} & \text{for all } i \text{ and } t \in \mathcal{H} \\
& \sum_{i=1}^{N} g_{it} = \hat{d}_t, \\
& \lambda_t : \\
& (\mu_{it}, \bar{\mu}_{it}) : -\bar{L}_i \leq g_{it+1} - g_{it} \leq \bar{r}_i \\
& 0 \leq t \leq T - 1, \\
& (\rho_{it}, \bar{\rho}_{it}) : 0 \leq g_{it} \leq \bar{g}_i,
\end{array}
\]

where \( \bar{g}_i \) the generation capacity, and \( (\bar{L}_i, \bar{r}_i) \) the down and up ramp-limits, \( \lambda_t \) the dual variable for the equality constraints, and \( (\rho_{it}, \bar{\rho}_{it}, \mu_{it}, \bar{\mu}_{it}) \geq 0 \) are dual variables for inequality constraints.

The one-shot locational marginal price \( \pi^{\text{LMP}} \) (LMP for short) is a uniform price \( \pi^{\text{LMP}} = (\pi^{\text{LMP}}_1, \ldots, \pi^{\text{LMP}}_T) \) with \( \pi^{\text{LMP}}_t \) defined by the marginal cost of generation with respect to the demand in interval \( t \). In particular, we have, by the envelope theorem,

\[
\pi^{\text{LMP}}_t := \frac{\partial}{\partial \hat{d}_t} F(G^{\text{ED}}) = \lambda^{*}_t, \quad t = 1, \ldots, T,
\]

where \( G^{\text{ED}} \) and \( \lambda^{*}_t \) are part of a solution to (2.2).

\[\text{†} \] The derivative of the bid-in cost curve represents the supply curve of the generator.

\[\text{‡} \] We retain the LMP terminology even though the model considered here does not involve a network.
2.1.3 Rolling-window look-ahead dispatch model

A rolling-window dispatch policy $\mathcal{G} = (\mathcal{G}_1, \ldots, \mathcal{G}_T)$ is defined by a sequence of $W$-interval look-ahead policies that generate dispatch signals $g[1], \ldots, g[T]$ sequentially, as illustrated in Fig. 2.1. At time $t$, the policy $\mathcal{G}_t$ has a look-ahead scheduling window of $W$ intervals, denoted by $\mathcal{H}_t = \{t, \ldots, t + W - 1\}$. The interval $t$ is called the binding interval and the rest of $\mathcal{H}_t$ the advisory intervals. As time $t$ increases, $\mathcal{H}_t$ slides across the entire scheduling period $\mathcal{H}$.

At time $t$, a $W$-interval one-shot policy $\mathcal{G}_t$ maps demand forecast $\hat{d}_t = (\hat{d}_t, \ldots, \hat{d}_{t+W-1})$ and previously realized generation $g[t-1]$ to an $N \times W$ generation scheduling matrix $\hat{G}_t$ over $\mathcal{H}_t$: $[\mathcal{G}_t(\hat{d}_t, g[t-1])] = [\hat{g}_t, \ldots, \hat{g}_{t+W-1}]$. The rolling window policy $\mathcal{G}$ sets generation in interval $t$ by $g[t] := \hat{g}_t$. The rest of columns of $\hat{G}_t$ are not implemented.

![Figure 2.1: Rolling-window dispatch with window size $W = 4$ generated from one-shot dispatch policy $\mathcal{G}_t$. The same applies also to the rolling-window pricing.](image)

Similarly, a rolling-window pricing policy $\mathcal{P}$ is defined by a sequence one-shot pricing policies $(\mathcal{P}_1, \ldots, \mathcal{P}_T)$. At time $t$, $\mathcal{P}_t$ sets the prices over $\mathcal{H}_t$, and the price in the binding interval $t$ is implemented by $\mathcal{P}$.

As an example, the rolling-window economic dispatch policy $\mathcal{G}^{R-ED} = (\mathcal{G}^{ED}_1, \ldots, \mathcal{G}^{ED}_T)$ where $\mathcal{G}^{ED}_t$ is the $W$-window one-shot economic dispatch defined in (2.2) with $T = W$ and $\hat{d} = \hat{d}_t$. The rolling-window LMP policy $\mathcal{P}^{R-LMP}$ is defined by a sequence of $W$-interval LMP policies $(\mathcal{P}^{LMP}_1, \ldots, \mathcal{P}^{LMP}_T)$.

2.2 Dispatch-following incentives and Uplifts

We say that a pricing mechanism provides dispatch-following incentives if, given the realized prices over the entire scheduling period $\mathcal{H}$, profit-maximizing generators, by themselves, would have produced generations that match the dispatch signal from the operator.

We apply the market equilibrium models for dispatch-following incentives. To this end, we consider two types incentives: (i) the ex-post incentive that applies to the entire scheduling period $\mathcal{H}$ after all generations have been realized; (ii) the ex-ante incentive that applies to only the current (binding) scheduling interval. The former guarantees dispatch-following incentives when a gen-
erator considers the total profit over the entire scheduling period. The latter guarantees dispatch-following incentives only for the binding interval.

### 2.2.1 Ex-post dispatch incentive and general equilibrium

For a multi-interval dispatch and pricing problem, generation and consumption in each interval can be treated as a separate market; we thus have a set of \( T \) inter-dependent markets over \( \mathcal{H} \). For purposes of analyzing dispatch-following incentives, we borrow the notion of general equilibrium [21, p. 547] for the multi-interval pricing problem.

**Definition 1** (General equilibrium). Let \( \mathbf{d} \) be the actual demand, \( \mathbf{g}_i \) the dispatch for generator \( i \) and \( \pi \) the vector of electricity prices over the entire scheduling period \( \mathcal{H} \). Let the \( N \times T \) matrix \( \mathbf{G} = (\mathbf{g}_1^T, \cdots, \mathbf{g}_N^T) \) be the realized generation matrix for all generators. We say \((\mathbf{G}, \pi)\) forms a general equilibrium if the following market clearing and individual rationality conditions are satisfied:

1. Market clearing condition: \[ \sum_{i=1}^{N} g_{it} = d_t \text{ for all } t \in \mathcal{H}. \]

2. Individual rationality condition: for all \( i \), the dispatch \( \mathbf{g}^{(i)} = (g_{i1}, \cdots, g_{iT}) \) is the solution of the individual profit maximization:

\[
\begin{align*}
\text{maximize} & \quad \sum_{t=1}^{T} (\pi_t g_{it} - f_{it}(g_{it})) \\
\text{subject to} & \quad \text{for all } t = 1, \cdots, T - 1, \\
& \quad -\bar{r}_i \leq g_{t+1} - g_{t} \leq \bar{r}_i, \\
& \quad 0 \leq g_{t} \leq \bar{g}_i, \forall t \in \mathcal{H}. \\
\end{align*}
\]

We call \( \pi \) an equilibrium price supporting generation \( \mathbf{G} \).

In the context of analyzing dispatch-following incentives, we are interested in whether price signal \( \pi \) and dispatch \( \mathbf{G} \) satisfy the general equilibrium condition. Because \( \mathbf{G} \) is computed centrally, the market-clearing condition is already met; only individual rationality is needed.

It turns out that, in the absence of forecasting error, the one-shot LMP supports the one-shot economic dispatch as stated in Theorem 1. This result is entirely analogous to the well-known property of LMP for the single period economic dispatch with network constraints [22].

**Theorem 1** (LMP as a General Equilibrium Price). *When there is no forecast error, \( \mathbf{d} = \mathbf{d} \), the one-shot economic dispatch matrix \( \mathbf{G}^{\text{ED}} \) and the one-shot LMP \( \pi^{\text{LMP}} \) form a general equilibrium.*

As a general equilibrium price, \( \pi^{\text{LMP}} \) does not guarantee that \( \pi^{\text{LMP}}_i g_{it} \geq f_{it}(g_{it}) \) for all \((i, t)\). In other words, a generator may be underpaid in some intervals despite that the generator is maximally compensated under \( \pi^{\text{LMP}} \) over the entire scheduling period. See Example 1 in Sec. 2.4.
2.2.2 Ex-ante dispatch incentives and partial equilibrium

When rolling-window dispatch is used, the forecasts in the look-ahead window (hence the dispatch over the window) change, which creates the missing payment problem even when the forecast over the look-ahead window is perfect.

Consider the example of rolling-window economic dispatch $G^{R-ED}$ and LMP $\pi^{R-LMP}$ policies. Suppose that a generator $i$ is underpaid in interval $t$, i.e., $f_{it}(g_{it}^{R-LMP}) \geq \pi_{it}^{R-LMP} g_{it}^{R-ED}$. Because $g_{it}^{R-ED}$ is generated by the $W$-window economic dispatch based on forecast $\hat{d}_t$, generator $i$ expects the underpayment in interval $t$ be compensated later in $t' \in \mathcal{H}_t$. At time $t'$, however, a different forecast $\hat{d}_{t'}$ is used to generate dispatch $g_{it'}^{R-ED}$. There is no guarantee that $\pi_{it'}^{R-LMP}$ is high enough to compensate for the loss incurred in the interval $t$, hence the missing payment problem.

To provide dispatch-following incentives under forecasting uncertainty, we need stronger equilibrium conditions.

**Definition 2** (Partial equilibrium and strong equilibrium). Consider price vector $\pi = (\pi_1, \cdots, \pi_T)$ and generation matrix $G$ over the entire scheduling horizon $\mathcal{H}$. The dispatch-price pair $(g[t], \pi_t)$ in interval $t$ is a partial equilibrium if it satisfies the market clearing and individual rationality conditions in interval $t$:

1. **Market clearing condition:** $\sum_{i=1}^{N} g_{it} = d_t$;

2. **Individual rationality condition:** for all $i$, the dispatch of signal $g_{it}$ is the solution to the individual profit maximization:

$$\begin{align*}
\text{maximize} & \quad (\pi_t g - f_{it}(g)) \\
\text{subject to} & \quad 0 \leq g \leq \bar{g}_i \\
& \quad -L_i \leq g - g_{i(t-1)} \leq \bar{L}_i.
\end{align*}$$

(2.4)

The dispatch-price pair $(G, \pi)$ is a **strong equilibrium** and $\pi$ a **strong equilibrium price supporting $G$** if $(G, \pi)$ is a general equilibrium and $(g[t], \pi_t)$ a partial equilibrium for all $t$.

The notion of partial equilibrium used here is slightly different from the standard because of the sequential nature of multi-interval dispatch and pricing problems. At time $t$, the dispatch in the interval $t$ is necessarily constrained by the past dispatch. The dispatch in the future intervals is advisory and subject to change, which is the reason that only the ramping constraints from the previous interval are imposed.

The strong equilibrium conditions impose stricter constraints than that required by the general and partial equilibrium definitions; strong equilibrium implies general equilibrium. Unlike the case of a general equilibrium price that only needs to satisfy the rationality condition at the end of the scheduling horizon, a strong equilibrium price must provide a dispatch-following incentive in every
interval independent of future realized dispatches. Consequently, even if schedules and prices may change, for the binding interval, there is no incentive for the generator to deviate from the dispatch signal.

An immediate corollary of Theorem 1 is that, in absence of ramping constraints, \((G^{ED}, \pi_{LMP})\) forms a strong equilibrium. However, we also know from Example 1 in Sec. 2.4 that, when ramping constraints are binding, \((G^{ED}, \pi_{LMP})\) may not be a strong equilibrium. Does there exist a uniform price \(\pi\) such that \((G^{ED}, \pi_{LMP})\) satisfies the strong equilibrium conditions?

**Theorem 2** (Strong equilibrium and uniform price). Let \(\hat{d} = d\) be the actual demand and \(G^{ED}\) the one-shot economic dispatch over \(\mathcal{H}\). If there exists a generator \(i\) and an interval \(t\) such that, under the one-shot economic dispatch,

1. generator \(i\) is marginal, i.e., \(0 < g^{ED}_{it} < \bar{g}_{it}\), and

2. the ramping constraint of \(g^{ED}_{it}\) between interval \(t - 1\) and \(t\) is not binding and that between interval \(t\) and \(t + 1\) binding with all multipliers being positive,

then there does not exist uniform prices \(\pi\) for which \((G^{ED}, \pi)\) is a strong equilibrium.

The conditions above on the one-shot economic dispatch are mild; they are easily satisfied for stochastic demands over a sufficiently large \(T\). As is shown in the appendix section of Chapter 2, empirical studies based on a practical network and demand model shows that the above conditions are satisfied by overwhelming majority of cases.

The significance of Theorem 2 is that all uniform prices suffer from the lack of dispatch-following incentives. As a result, out-of-the-market uplifts are necessary to ensure that generators follow the operator’s dispatch signal.

### 2.2.3 Out-of-the-market settlements

The out-of-the-market settlement, also known as uplift, is a process for the operator to compensate market participants for inadequate payments due to inaccurate, incomplete, or non-convex models. Out-of-the-market settlements are in general discriminative and determined in ex-post over the entire scheduling horizon \(\mathcal{H}\) [7,9,23]. Two popular schemes are the make-whole (MW) settlement used in most SOs in the U.S. and the lost-of-opportunity-cost (LOC) settlement implemented in ISO-NE.

Let \(\pi\) be the price vector over \(\mathcal{H}\) and \(g_{i} = (g_{i1}, \ldots, g_{iT})\) the generation of generator \(i\). The make-whole (MW) payment \(\text{MW}(\pi, g_{i})\) and the lost-of-opportunity cost (LOC) payment \(\text{LOC}(\pi, g_{i})\) for
generator $i$ are defined by, respectively,

$$\text{MW}(\pi, g_i) = \max \{0, \sum_{t=1}^{T} (f_{it}(g_{it}) - \pi_t g_{it})\},$$  \hfill (2.5)$$

$$\text{LOC}(\pi, g_i) = Q_i(\pi) - \sum_{t=1}^{T} (\pi_t g_{it} - f_{it}(g_{it})), $$ \hfill (2.6)$$

where $Q_i(\pi)$ is the maximum profit the generator would have received if the generator self-schedule for the given price $\pi$:

$$Q_i(\pi) = \maximize_{p=(p_1, \ldots, p_T)} \sum_{t=1}^{T} (\pi_t p_t - f_{it}(p_t))$$

subject to

$$0 \leq p_t \leq \bar{g}_i$$

$$-\bar{r}_i \leq p_{t+1} - p_t \leq \bar{r}_i.$$  \hfill (2.7)$$

It turns out that, when $Q_i(\pi) \geq 0$, we always have $\text{LOC}(\pi, g_i) \geq \text{MW}(\pi, g_i)$. See the appendix of this chapter for detailed proofs.

The following proposition, an immediate consequence of the general equilibrium conditions, shows that the LOC uplift is a measure of the disincentives of generators to follow the dispatch.

**Proposition 1 (LOC and general equilibrium).** A dispatch matrix-price pair $(G = [g_1, \ldots, g_N]^T, \pi)$ satisfies the general equilibrium condition if and only if the LOC uplifts for all generators are zero.

### 2.3 Temporal Locational Marginal Price

Because uniform pricing cannot provide dispatch-following incentives in general, we now consider nonuniform pricing mechanisms. To this end, we extend LMP to the **temporal locational marginal price (TLMP)** and establish that TLMP is a strong equilibrium price.

#### 2.3.1 TLMP: a generalization of LMP

We first consider the one-shot TLMP defined over $\mathcal{H}$; the rolling-window TLMP follows the same way as the rolling-window LMP.

As in LMP, TLMP prices a load by the marginal cost of satisfying its demand. Unlike LMP, TLMP prices the generation from generator $i$ by its contribution to meeting the system load. In particular, we treat generator $i$ as a negative demand and pay generator $i$ at the marginal benefit of its generation. Roughly speaking, generator $i$ is paid at the marginal cost to the system when generator $i$ reduces 1 MW of its generation.

Define a **parameterized economic dispatch** by treating $g_{it}$ as a parameter rather than a decision variable in (2.2). Let the partial cost be $[F_{it}(G) := F(G) - f_{it}(g_{it})]$, which excludes the cost of generator $i$ in interval $t$. The parameterized economic dispatch is defined by (2.2) with $F_{it}(G)$ as the cost function and $\{g_{i't'}, (i', t') \neq (i, t)\}$ as its decision variables.
**Definition 3 (TLMP).** The TLMP for the demand in interval $t$ is defined by the marginal cost of meeting the demand: \[ \pi_{0t}^{\text{TLMP}} := \frac{\partial}{\partial q_{it}} F(G^{\text{ED}}). \] The TLMP for generator $i$ in interval $t$ is defined by the marginal benefit of generator $i$ at $g_{it} = g^{\text{ED}}$ : \[ \pi_{it}^{\text{TLMP}} := - \frac{\partial}{\partial g_{it}} F_{it}(G^{\text{ED}}). \]

Proposition 2 gives an explicit expression for TLMP.

**Proposition 2.** Let $G^{\text{ED}}$ be the solution of the multi-interval economic dispatch in (2.2) and $(\lambda^{*}, \mu^{*}, \bar{\mu}^{*}, \rho^{*}, \bar{\rho}^{*})$ the dual variables associated with the constraints. The TLMP for the demand in interval $t$ is given by \[ \pi_{0t}^{\text{TLMP}} = \lambda^{*}. \] The TLMP for the generator $i$ in interval $t$ is given by \[ \pi_{it}^{\text{TLMP}} = \lambda^{*} + \Delta^{*}, \] where $\Delta^{*} = \Delta \mu^{*}_{it} - \Delta \mu^{*}_{i(t-1)}$, and $\Delta \mu^{*}_{it} := \bar{\mu}^{*}_{it} - \mu^{*}_{it}$.

Proposition 2 reveals the structure of TLMP as a natural extension of LMP; it adds to the uniform pricing of LMP with a discriminative ramping price $\Delta^{*}$. The LMP portion of TLMP is public as it represents the system-wide energy price whereas the private ramping price accounts for the individual ramping capabilities. Note also that TLMP incurs no additional computation costs beyond that in LMP.

Four interpretations of the ramping price $\Delta^{*}_{it}$ in TLMP are in order. First, note that TLMP expression above is consistent with that in (1.1); both expressions give the interpretation that the ramping price in TLMP is the increment of the shadow prices associated with the ramping constraints.

Second, the ramping price $\Delta^{*}_{it}$ can be positive or negative. When the ramping price $\Delta^{*}_{it} > 0$, it can be interpreted as an upfront payment for the ramping-induced lost-of-opportunity cost, which ensures that the generator under TLMP is never under-paid below its generation cost. When it is negative, it has the interpretation of a penalty for its inability to ramp for greater welfare. See discussions on Example I in Sec. 2.4.

Third, the ramping price $\Delta^{*}_{it}$ can be viewed as the price adjustments around LMP such that, if each generator self schedules, the economic dispatch $G^{\text{ED}}$ is the profit maximizing schedule.

Finally, the ramping price $\Delta^{*}_{it}$ serves to penalize generators with limited ramping capabilities. For simplicity, ignore the down-ramp limits (i.e., $\mu^{*}_{it} = 0$) and compute the revenue $R_{i}^{\text{TLMP}}$ of generator $i$:

\[ R_{i}^{\text{TLMP}} = \sum_t \pi_{it}^{\text{TLMP}} g_{it}^{\text{ED}} = R_{i}^{\text{MP}} - \sum_t \bar{\mu}^{*}_{it} f_{i} \leq R_{i}^{\text{LMP}}. \]

This implies that, comparing with LMP, TLMP penalizes generators with limited ramping capability, which incentivizes the generator to report truthfully its ramping limits. See Proposition 4 and related discussions in Chapter 3 [24].
2.3.2 Dispatch-Following Incentives of TLMP

We now consider the equilibrium and dispatch-following properties. Because TLMP is nonuniform pricing, the general and partial equilibrium definitions are given in the previous section need to be generalized slightly.

- Instead of having a single price vector for all generator, we now have an individualized price vector $\pi_i$ for each generator $i$.

- The individual rationality conditions extend naturally by replacing $\pi_t$ in (2.3-2.4) by $\pi_{it}$.

Theorem 3 establishes the strong equilibrium property for the one-shot TLMP.

**Theorem 3** (One-shot TLMP as a strong equilibrium price). When there is no forecasting error, i.e., $\hat{d} = d$, the one-shot multi-interval economic dispatch policy $G^{ED}$ and the TLMP policy $G^{TLMP}$ form a strong equilibrium, thus there is no incentive for any generator to deviate from the economic dispatch signal.

In addition, the one-shot TLMP guarantees revenue adequacy for the operator with total merchandising surplus equal to the ramping charge:

$$MS := \sum_t \pi_{it}^{TLMP} d_t - \sum_{t>0,t} \pi_{it}^{TLMP} g_{it}^{ED}$$

$$= \sum_{t,f} (\bar{\mu}_{it}^* f_i + \mu_{it}^* r_i).$$

(2.9)

The intuition behind the above theorem is evident from a dual perspective of the economic dispatch. Specifically, the Lagrangian of the one-shot economic dispatch (2.2) with the optimal multipliers can be written as

$$\mathcal{L} = \sum_{i,t} \left( f_{it}(g_{it}) - (\lambda_{it}^* + \Delta_{it}^*) g_{it} + (\bar{p}_{it}^* - p_{it}^*) g_{it} \right) + \cdots$$

where the rest of the terms above are independent of $g_{it}$. It is evident that, with TLMP $\pi_{it}^{TLMP} := \lambda_{it}^* + \Delta_{it}^*$, the optimal dispatch $g_{it}^*$ should always satisfy the individual rationality condition for all $i$ and $t$.

The positive merchandising surplus and (2.9) are also not surprising; they are entirely analogous to the same property for LMP when network congestions occur.

What happens when the load forecasts are not accurate? More importantly, is the rolling-window TLMP a strong equilibrium price for the rolling-window dispatch?
Theorem 4 (R-TLMP as a strong equilibrium price). Let $g^R_{ED}$ be the rolling-window dispatch for generator $i$ and $\pi^R_{TLMP}$ its rolling-window TLMP. Then, for all $i$ and under arbitrary demand forecast error, $(g^R_{ED}, \pi^R_{TLMP})$ forms a strong equilibrium, and
\[
\text{LOC}(\pi^R_{TLMP}, g^R_{ED}) = 0.
\] (2.10)

Note that, when a generator has a positive final profit (with LOC uplift), then the make-whole payment for the generator is also zero.

The above theorem highlights the most significant property of TLMP for practical situations when the load forecasts used in the rolling-window dispatch are not perfect. There is no uniform pricing policy that can achieve the same.

2.4 Illustrative Examples

We consider two toy examples involving $T = 3$ intervals, one for the one-shot dispatch and pricing policies with perfect load forecasts, the other for the rolling-window policies with inaccurate forecasts. The parameters of the generators are the same for both examples and shown in Table II.

<table>
<thead>
<tr>
<th>Capacity $\bar{g}_i$</th>
<th>Marginal cost $c_i$</th>
<th>Ramp limit $\bar{r}_i$</th>
<th>$t = 1$ $g^LMP_t$, $\pi^{LMP}_t$</th>
<th>$t = 2$ $g^LMP_t$, $\pi^{LMP}_t$</th>
<th>$t = 3$ $g^LMP_t$, $\pi^{LMP}_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1 500</td>
<td>25</td>
<td>500</td>
<td>(380, 25, 25)</td>
<td>(500, 35, 35)</td>
<td>(500, 30, 30)</td>
</tr>
<tr>
<td>G2 500</td>
<td>30</td>
<td>50</td>
<td>(40, 25, 30)</td>
<td>(90, 35, 30)</td>
<td>(90, 30, 30)</td>
</tr>
<tr>
<td>$d_t$</td>
<td>—</td>
<td>—</td>
<td>420</td>
<td>590</td>
<td>590</td>
</tr>
</tbody>
</table>

Table 2.1: One-shot economic dispatch, LMP, and TLMP. Initial generation $g[0] = 0$. The price for demand $d_t$ is $\pi^{LMP}_t$.

2.4.1 Example I: one-shot dispatch and pricing

The economic dispatch, LMP, and TLMP over three intervals are given in the right part of Table II. We make four observations.

First, G1’s ramping limits are not binding over the three intervals. The LMP and TLMP are the same for G1.

Second, the ramping constraint for G2 is binding between the first and second intervals, making the price of generation under TLMP different from its LMP. Note that in interval $t = 1$, G2 is scheduled to generate at the LMP of $25/MW$, $5$ below its marginal cost of $30/MW$. As a result, G2 incurs
an opportunity cost of $200 so that it can ramp up to the maximum to the next interval and be paid at $5 above its marginal cost. Despite the loss in the first interval, the total surplus over the three intervals is maximized. By the general equilibrium property of LMP, there is no incentive for G2 to deviate from the dispatch.

Third, in contrast to LMP, TLMP pays G2 up-front the opportunity cost by adding $5/MW to the energy price of $25/MW. The up-front payment removes the incentive for G2 to deviate not knowing future demands. For this reason, the discriminative part of TLMP in (3.6) has an interpretation as the premium for the ramping-induced opportunity cost. Note also that, the opportunity cost premium paid to G2 in interval 1 is removed in interval 2.

Fourth, consider the case when the true ramping limit of G2 is 100 MW. Had G2 reported the ramping limit truthfully, G2 would have been dispatched to generate 0 MW in interval 1 and 90 in interval 2 at $30 MW/h with total profit of zero dollar. But if G2 falsely declares that it has ramp limit of 50 MW as shown in Table II, we see that G2 under LMP would have made $250 profit. This shows that under LMP, there is an incentive for G2 to under-declare its ramp limit. Under TLMP, on the other hand, there is no incentive for G2 to lie about its ramp limit. See more examples in Chapter 3 [24].

2.4.2 Example II: rolling-window dispatch and pricing

Table III shows the rolling-window economic dispatch and rolling-window prices with window size $W = 2$. The load forecasts $\hat{d}_t = (\hat{d}_t, \hat{d}_{t+1})$ are listed and $\hat{d}_t = \tilde{d}_t$ being the actual load. Note that $\hat{d}_t$ contains forecast errors.

<table>
<thead>
<tr>
<th>Capacity</th>
<th>Marginal cost $c_i$</th>
<th>Ramp limit $L_i = \bar{r}_i$</th>
<th>$(g_{it}^{R-ED}, \pi_{it}^{R-LMP}, \pi_{it}^{R-TLMP})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>500</td>
<td>25</td>
<td>(370, 25, 25)</td>
</tr>
<tr>
<td>G2</td>
<td>500</td>
<td>30</td>
<td>(50, 25, 30)</td>
</tr>
<tr>
<td>$\hat{d}_t$</td>
<td>—</td>
<td>—</td>
<td>(420,600)</td>
</tr>
</tbody>
</table>

Table 2.2: Rolling-window economic dispatch, LMP, and TLMP. Initial generation $g[0] = 0$. Load is settled at the LMP $\pi_{t}^{LMP}$ for all $t$.

We again make four observations. First, the missing money scenario happens in this example. G2 is underpaid by $\pi_{1}^{R-LMP}$ in the interval $t = 1$. Unlike the one-shot LMP case, the underpayment is never compensated under R-LMP. The underpayment is compensated out of the market. The LOC and MW uplifts to G2 are both $250$.

Second, from Table I, the dispatch of G2 satisfies the conditions in Theorem 2. There is no uniform
price can be a strong equilibrium price for the economic dispatch. For this example, the argument becomes trivial. Consider interval \( t = 1 \), for any price greater than $25, G1 self-scheduling would have generated more than 380 (MW). If the price is $25, G2 self-scheduling would have generated zero (MW).

Third, for G2 in interval \( t = 1 \), given the inaccurate load forecast of 600 for interval \( t = 2 \), the rolling-window dispatch for interval \( t = 2 \) is 100, which makes the ramping constraints from \( t = 0 \) to \( t = 1 \) and from \( t = 1 \) to \( t = 2 \) both binding. The Lagrange multipliers associated with these two binding constraints are zero and five\(^{\S} \), respectively. The TLMP for G2 is, therefore, $5 above the LMP, which compensates the underpayment of LMP to the level of marginal cost. In intervals of \( t = 2, 3 \), there are no binding ramping constraints for G2. G2 is paid at the LMP. No missing money for TLMP.

Fourth, there is again no incentive for G2 to declare its ramp limit untruthfully under TLMP; it will be paid at its marginal costs. Under LMP, however, there is an incentive for G2 to declare that it has high ramping limits, say 100 MW, and avoid the opportunity cost in the first interval.

We should caution that the toy examples considered in this section are designed to gain insights into the behavior of these pricing mechanisms. The observations drawn from the examples may not hold in general. See Chapter 3 for more elaborate Monte Carlo simulations.

2.5 Discussions

We discuss in this section aspects of pricing multi-interval dispatch that are not covered in Chapter 2 and Chapter 3. The purpose is to provide a broader perspective and contexts beyond the scope of this report. We focus on two issues: one is the general market model used in developing multi-interval pricing, the other the impacts of strategic bidding.

2.5.1 Pricing model

A rigorous study of pricing can be treated as a mechanism design problem, for which a complete model includes a model for the auctioneer (the system operator) and models for the bidders (the generating firms). An excellent example of using the compete market model to analyze pricing of electricity is the work of Hobbs [25] where a Nash-Cournot competition is formulated in analyzing decentralized (bilateral) and centralized (poolco) power markets. Another example is the work of Philpott, Ferris, and Wets [26] on the equilibrium, uncertainty, and risk in hydro-thermal systems, which is relevant to the current work for its modeling of inter-temporal constraints and uncertainty.

The bid-based market model assumed here includes only the model for the operator who collects the bids (generation offers) and makes two decisions: one is the allocation of the production levels

\(^{\S} \) Primal degeneracy occurs in this example. Shadow prices associated with ramping and power balance constraints are used to compute TLMP.
of the goods (the dispatch over multiple intervals). The other is the purchasing prices of the goods in each interval. We do not have models for the $N$ generating firms in this chapter, assuming only that each firm produces a bid that represents its best offer to generate. Behind such bids, however, are $N$ optimizations that take into account (fuel and opportunity) costs as well as generation constraints. Some of these constraints such as generation capacities and ramping limits are revealed as part of the bids and used explicitly by the operator in its allocation and pricing decisions. Other constraints such as fuel contracts or reservoir constraints of a hydro plant are kept private and not used by the operator. The costs of meeting those unrevealed constraints are internalized as part of their bids or revealed to the operator for possible opportunity cost adjustments. As an example, see the computation of opportunity cost adder for the use-limited resources in CAISO’s proposal [27].

By neglecting the underlying optimization of individual firms, we lose the ability to characterize fully strategic behavior of the firms. We gain, on the other hand, the analytical tractability that isolates those factors most pertinent to ramping-induced incentives. In practice, the model assumed here captures the pricing problem of the operator who rarely has access to the optimization models of the market participants.

Relevant to ramping constraints, there is a fundamental question on who should be responsible for pricing ramping costs. Should the cost of ramping be reflected entirely in the bids of the generating firms through their individual optimizations, or should it be captured explicitly by the operator in its pricing model as assumed in recent proposals of ramping products and in this chapter? This is a highly complex question and is outside the scope of this chapter. See discussions and comments in [28].

2.5.2 Strategic bidding under discriminative pricing

There is a general concern that discriminative pricing may be vulnerable to market manipulation from firms with market power. The gold standard for incentive compatibility is the Vickrey-Clark-Groves (VCG) mechanism [29] which happens to be a discriminative pricing mechanism when it is applied to the electricity market [30]. VCG mechanism, however, has various undesirable properties and vulnerabilities in practice as shown in [31, 32].

At the outset, note that, as a generalization of LMP, TLMP inherits the vulnerabilities and some advantages of LMP against strategic manipulation. For instance, LMP is known to be vulnerable to strategic manipulations by firms with market power. So is TLMP. Under LMP, the so-called hockey-stick bidding can cause significant loss of social welfare in scarcity scenarios. The same can happen to TLMP. Mitigation strategies against hockey-stick bidding such as CSM [33] has similar effects on both LMP and TLMP.

PAB is a discriminative pricing scheme that also eliminates the need of out-of-the-market uplifts. PAB is known to be vulnerable to manipulations by simply raise its bids beyond the marginal cost. Depending on scenarios of network-wide binding ramping constraints, a bidding strategy that works well under PAB can lead to lower revenue when it is applied to TLMP. See examples in [20].
To develop an optimal bidding strategy for multi-interval problem requires that the firm has the ability to forecast not only future demand but also binding ramping conditions. Such conditions depend on varying hyper-parameters of the overall system that are difficult to estimate.

2.6 Conclusion

We have developed a theory for dispatch-following incentives based on concepts of equilibrium prices in competitive markets. Given that there is no uniform pricing mechanism that can guarantee dispatch-following incentives without discriminative out-of-the-market uplifts, a non-uniform pricing mechanism such as TLMP serves as an alternative. As an extension of LMP, TLMP prices generation based on both the energy and the ramping-induced opportunity costs. As a strong equilibrium pricing mechanism, TLMP guarantees dispatch-following incentives under arbitrary forecast errors.

Evaluating pricing schemes in practice must take into account many factors. In [24], we conduct more careful simulation studies under relevant performance metrics to compare several benchmark pricing schemes.

2.7 Appendix

2.7.1 Preliminaries

We derive a more compact vector-matrix representation of LMP, TLMP and associated representations. For convenience, we focus on scheduling window \( \mathcal{H} = \{1, \ldots, W\} \). Let the demand (or forecasted demand) be \( \mathbf{d} = (d_1, \ldots, d_W) \) be the demand in \( \mathcal{H} \), \( \mathbf{g}_i = (g_{i1}, \ldots, g_{iW}) \) the generation of generator \( i \), and \( \mathbf{G}^T = [\mathbf{g}_1, \ldots, \mathbf{g}_N] \) the generation matrix. The \( W \)-interval economic dispatch in the vector-matrix form is defined by

\[
\mathcal{G}^{ED} : \quad \text{minimize} \quad F(G) = \sum_i f_i(g_i) \\
\text{subject to} \quad \text{for all } 1 \leq i \leq N \\
\lambda : \quad G^T \mathbf{1} = \mathbf{d} \\
(\rho_i, \bar{\rho}_i) : \quad 0 \leq g_i \leq \mathbf{g}_i, \\
(\mu_i, \bar{\mu}_i) : \quad -\mathbf{r}_i \leq A g_i \leq \tilde{r}_i, 
\]

(2.11)

where \( f_i(g_i) = \sum_t f_{it}(g_{it}) \) is the total cost for generator \( i \), \( \lambda = (\lambda_1, \ldots, \lambda_W) \), the vector of dual variables for the equality constraints and \( (\rho_i, \bar{\rho}_i, \mu_i, \bar{\mu}_i) \) vectors of dual variables for the inequalities associated generator \( i \), and \( A \) an upper triangular matrix with \(-1\) as diagonals, \( 1 \) as the first off-diagonals, and zero elsewhere.
Let the Lagrangian of \( G^{ED} \) be
\[
L = \sum_i f_i(g_i) + \lambda^T (d - G^T 1) \\
+ \sum_i \left( \tilde{\mu}_i^T (Ag_i - \bar{r}_i) - \mu_i^T (Ag_i + \bar{e}_i) \right) \\
+ \sum_i \left( \tilde{\rho}_i^T (g_i - \bar{g}_i) - \rho_i^T g_i \right) .
\] (2.12)

Let \((G^{ED}, \lambda^*, \rho^*_i, \tilde{\rho}_i^*, \mu^*_i, \bar{\mu}_i^*, \bar{\rho}_i^*)\) be the solution of \( G^{ED} \). The KKT condition gives
\[
\nabla f_i(g^*_i) - \lambda^* + A^T \Delta \mu^*_i + \Delta \rho^*_i = 0
\] (2.13)
where \( \Delta \mu^*_i = \tilde{\mu}_i^* - \mu^*_i \) and \( \Delta \rho^*_i = \tilde{\rho}_i^* - \rho^*_i \).

The vector form of the multi-interval LMP and TLMP of generator \( i \) are given by, respectively,
\[
\pi^{LMP} = \lambda^* , \quad \pi^{TLMP}_i = \lambda^* - A^T \Delta \mu^*_i .
\] (2.14)

For the individual rationality condition, for generator \( i \), we have the following profit maximization problem for given price \( \pi \):
\[
G_i : \text{minimize } f_i(g) - g^T \pi \\
\text{subject to } (\eta, \bar{\eta}) : \ -\bar{r}_i \leq Ag \leq \bar{r}_i , \\
(\zeta, \bar{\zeta}) : \ 0 \leq g \leq \bar{g}_i .
\] (2.15)
By the KKT condition, the solution of the above must satisfy
\[
\nabla f_i(g) - \pi + A^T \Delta \eta + \Delta \zeta = 0 ,
\] (2.16)
where \( \Delta \eta = \bar{\eta} - \eta \) and \( \Delta \zeta = \bar{\zeta} - \zeta \).

### 2.7.2 Proof of Theorem 1

Let \( G^{ED} \) be the one-shot economic dispatch and \( \pi^{LMP} \) the LMP. The market clearing condition is already satisfied by \( G^{ED} \). The individual rationality condition (2.16) holds by setting \((g = g^{ED}_i, \Delta \eta_i = \Delta \mu^*_i, \Delta \zeta_i = \Delta \rho^*_i)\). \( \square \)

### 2.7.3 Proof of Theorem 2

Suppose that \((G^{ED}, \pi)\) satisfies the strong equilibrium conditions. Let \( g^{ED}_i \) be the economic dispatch for generator \( i \). From the general equilibrium condition, \((g^{ED}_i, \pi)\) satisfies the individual rationality
condition for every \( i \). Let \( \eta_i, \bar{\eta}_i, \xi_i, \bar{\xi}_i \) be a set of Lagrange multipliers associated with \( g_{it}^{\text{ED}} \) of (2.15). By the KKT condition, \([\nabla f_i(g_{it}^{\text{ED}}) - \pi + A^T \Delta \eta_i + \Delta \xi_i = 0_i]\) where \( \Delta \eta_i = \bar{\eta}_i - \eta_i \) and \( \Delta \xi_i = \bar{\xi}_i - \xi_i \).

By the KKT condition of (2.11), \((\pi, \eta_i, \bar{\eta}_i, \xi_i, \bar{\xi}_i)\) must also be a solution of the optimal multipliers associated with (2.11).

Suppose that \( g_{it}^{\text{ED}} \) satisfies the conditions in Theorem 2. From the KKT condition of (2.11),
\[
\frac{d}{dg} f_{it}(g_{it}^{\text{ED}}) = \pi_t - \Delta \eta_{it}^*,
\]
which contradicts (2.17). By the envelope theorem, at the optimal solution \( \Delta \eta_{it}^* \neq 0 \). But for \((G^{\text{ED}}, \pi)\) to be a strong equilibrium, \( g_{it}^{\text{ED}} \) must be a solution of
\[
\min_{g \in \mathcal{G}_i} (f_{it}(g) - \pi_t g) \Rightarrow \frac{d}{dg} f_{it}(g_{it}^{\text{ED}}) = \pi_t, \ \forall t.
\]
which contradicts (2.17). \(\square\)

### 2.7.4 Proof of Proposition 2

TLMP for demand \( \hat{d}_t \) is same as LMP; it is defined by the marginal cost of serving \( \hat{d}_t \):
\[
\pi_{it}^{\text{TLMP}} := \frac{\partial}{\partial \hat{d}_t} F(G^{\text{ED}}) = \lambda_t^*.
\]

To compute TLMP for generator \( i \) in interval \( t \), consider the modified multi-interval economic dispatch with generator \( i \) in interval \( t \) fixed at the optimal economic dispatch level, \( g_{it} = g_{it}^{\text{ED}} \):

\[
\mathcal{G}' : \begin{align*}
\text{minimize} & \quad F_{it}(G) \\
\text{subject to} & \quad \text{for all } j \neq i \text{ and } t' \in \mathcal{H} \setminus \{t\} \\
& \quad \lambda_{it'} : \sum_{j \neq i} g_{jt'} = d_{t'} \\
& \quad (\bar{\gamma}_{jt'}, \bar{\eta}_{jt'}) : \quad 0 \leq g_{jt'} \leq \bar{g}_j; \\
& \quad (\eta_{jt'}, \bar{\eta}_{jt'}) : \quad -L_i \leq g_{jt'} - g_{jt} \leq \bar{r}_j, \\
& \quad \lambda_{it} : \sum_{j \neq i} g_{jt} = d_t - g_{it}^{\text{ED}} \\
& \quad (\eta_{it}, \bar{\eta}_{it}) : \quad -L_i \leq g_{it+1} - g_{it}^{\text{ED}} \leq \bar{r}_i, \\
& \quad (\eta_{it-1}, \bar{\eta}_{it-1}) : \quad -L_i \leq g_{it-1} - g_{it}^{\text{ED}} \leq \bar{r}_i.
\end{align*}
\]

By the envelope theorem, at the optimal solution \( G^* = [g_{it}^*] \) and \((\gamma_{it}^*, \bar{\gamma}_{it}^*, \eta_{it}^*, \bar{\eta}_{it}^*)\) of \( \mathcal{G}' \), we have
\[
-\frac{\partial}{\partial g_{it}^*} F_{it}(G^* ) = \bar{\lambda}_{it}^* + \Delta \eta_{it}^* - \Delta \eta_{it(t-1)}^* = \lambda_t^* + \Delta \eta_{it}^* ,
\]
where, for the last equality, we have \( \lambda_t^* = \lambda_t^*, \eta_{it}^* = \mu_{it}^* \) at the optimal dispatch defined in (2.2). \(\square\)
2.7.5 Proof of Theorem 3

We first show that \((G^{\text{ED}}, (\pi_i^{\text{TLMP}}))\) satisfies the general equilibrium conditions. Again, we only need to check the individual rationality condition since the economic dispatch \(G^{\text{ED}}\) already satisfies the market clearing condition as well as all the ramping constraints.

For the individual rationality condition, we consider the optimization \(\tilde{G}_i (2.15)\) with \(\pi = \pi_{\text{TLMP}}\). Setting \(\eta = \tilde{\eta} = 0\) and \(\Delta \zeta = \Delta \rho_i^*\), by the KKT condition, \(g_i^{\text{ED}}\) is a solution of \(\tilde{G}_i\). Thus \((\pi_i^{\text{TLMP}}, g_i^{\text{ED}})\) satisfies the individual rationality condition for all \(i\). \(\Box\)

To show that \((G^{\text{ED}}, (\pi_i^{\text{TLMP}}))\) also satisfies the strong equilibrium condition, we note that \((G^{\text{ED}}, \tilde{\eta}_i = \eta_i = 0, \rho_i^*, \rho_i^*)\) is a solution of (2.15). Because the dual variables for ramping constraints are all zero, the multi-interval optimization decouples in time under \(\pi_i^{\text{TLMP}}\). We have \(q_i^{\text{ED}}\) as a solution of (2.4) for individual rationality.

To show the revenue adequacy for the operator, we compute the merchandising surplus under TLMP. From (2.14),

\[
MS = d^T \lambda^{\text{LMP}} - \sum_i \left(\lambda_i^{\text{LMP}} - A^T \Delta \mu_i^*\right)^T g_i^{\text{ED}}
\]

\[
= \sum_i (\Delta \mu_i^*)^T A g_i^{\text{ED}}
\]

\[
= \sum_i \tilde{r}_i^T \tilde{\mu}_i^* + \tilde{r}_i^T \mu_i^* \geq 0,
\]

where the last equality comes from the complementary slackness condition. \(\Box\)

2.7.6 Proof of Theorem 4

Within this proof, we will focus on a particular generator, say generator \(i\). For brevity, we drop the subscript \(i\) of all variables associated with generator \(i\).

Let \(g^\text{R-ED} = (g^\text{R-ED}_1, \cdots, g^\text{R-ED}_T)\) be the rolling-window economic dispatch over \(\mathcal{H}\) and \(\pi^\text{R-TLMP} = (\pi^\text{R-TLMP}_1, \cdots, \pi^\text{R-TLMP}_T)\) the rolling-window TLMP vector.

Let \(g_t^{\text{ED}}\) be the \(W\)-window economic dispatch at time \(t\) over \(\mathcal{H}_t\) from (2.11) based on \(d_t = (d_{t1}, \cdots, d_{tW})\). Note that \(d_{t1} = d_t\), the actual demand for interval \(t\), and the rest of entries of \(d_t\) are forecasts with errors. Let \(\pi_t^{\text{TLMP}}\) be the corresponding TLMP vector given in (3.6).

From the proof of Theorem 3 (with \(T = W\)), the profit maximization,

\[
\tilde{G}_i : \begin{array}{ll}
\text{minimize} & f_i(g) - g^T \pi_t^{\text{TLMP}} \\
\text{subject to} & (\eta, \tilde{\eta}) : \quad -\tilde{r} \leq Ag \leq \tilde{r}, \\
& (\zeta, \tilde{\zeta}) : \quad 0 \leq g \leq \tilde{g}_t,
\end{array}
\]

(2.19)
has a solution $g_{ED}^t$ with $\eta = \bar{\eta} = 0$, where $f_t(g)$ is the generation cost over $\mathcal{H}_t$. This means that $g_{ED}^t$ is a solution of the ramp-unconstrained optimization

$$g_{ED}^t = \arg \min_{0 \leq g \leq \bar{g}} (f_t(g) - g^\top \pi_t^{R-TLMP}).$$

By the rolling-window dispatch and pricing policies, the first entry of $g_{ED}^t$ is $g_{R-ED}^t$ — the dispatch that is implemented in interval $t$ — and the first entry of $\pi_t^{R-TLMP}$ is the the rolling-window price $\pi_t^{R-TLMP}$ in interval $t$. We thus have

$$g_{R-ED}^t = \arg \min_{0 \leq g \leq \bar{g}} (f_t(g) - g^\top \pi_t^{R-TLMP}),$$

which implies that $g_{R-ED}$ is the solution of the ramp-unconstrained optimization

$$g_{R-ED} = \arg \min_{0 \leq g \leq \bar{g}} (f(g) - g^\top \pi_{R-TLMP}).$$

Let $g^*$ be the solution of the (ramp-constrained) LOC optimization (2.15) with $\pi = \pi^{R-TLMP}$, we must have

$$f(g_{R-ED}^t) - (g_{R-ED})^\top \pi_{R-TLMP} \leq f(g^*) - (g^*)^\top \pi_{R-TLMP}.$$  

Note, however, that $g_{R-ED}$ satisfies all the constraints in (2.15), the above inequality holds with equality, and $g_{R-ED}^t$ is a solution of (2.15). Therefore, $\text{LOC}(g_{R-ED}^t, \pi_{R-TLMP}) = 0$.

By Proposition 1, $(G_{R-ED}^t, \Pi_{R-TLMP})$ is a general equilibrium. From (2.20), we conclude that $(G_{R-ED}^t, \Pi_{R-TLMP})$ also satisfies the strong equilibrium conditions.

### 2.7.7 On the validity of assumptions in Theorem 2

We present empirical test results on how frequently assumptions in Theorem 2 of Chapter 2 hold. To this end, we used the ISO-NE’s 8-zone systems, shown in the left panel of Fig. 2.2 [34]. Parameters of transmission lines and the 76 benchmark generators in ISO-NE 8-zone systems came from [35], including generation capacities, generation costs, ramping limits, network topologies, transmission line capacities, and transmission line reactances.

Based on ISONE load data profile in [34] [35], 5000 load scenarios over 24 hours were generated with normalized standard deviation 4% in this Monte Carlo simulation, which is shown in the right panel of Fig. 2.2. Because these 76 generators had a combined installed generation capacity account for 72% of the actual ISO-NE capacity [34], we decreased the load demand to 72% based on the actual demand data. Among the 5000 test scenarios, 4927 (98.54%) were found to satisfied both assumptions of Theorem 2.
Figure 2.2: Top: ISO-NE system. Bottom: Load scenarios at NE Mass & Boston.
3. Generalization and Performance Evaluation

3.1 Introduction

Chapter 2 and Chapter 3 address some of the open problems in pricing multi-interval dispatch subject to ramping constraints and forecasting uncertainty. Chapter 2 focuses on theoretical issues surrounding dispatch-following incentives with two major conclusions. One is that, under the rolling-window dispatch model, uniform-pricing schemes cannot provide dispatch-following incentives that avoid out-of-the-market uplifts. Because such uplifts are discriminative, price-discrimination is necessary.

Another conclusion is that the temporal locational marginal pricing (TLMP)—a generalization of locational marginal pricing (LMP)—provides full dispatch-following incentives that eliminate the need for any out-the-market settlement under the rolling-window economic dispatch model and arbitrary demand forecast accuracy.

Providing dispatch-following incentive is only one of the many measures that pricing mechanisms need to be evaluated for adoption. This chapter presents a study on a broader range of issues relating to pricing multi-interval dispatch under a more general network model.

We focus on two categories of performance measures. The first is on incentive compatibilities specific to multi-interval dispatch. One type of incentive is the degree for which a particular pricing mechanism provides the necessary dispatch-following incentives for generators. Here we measure the lack of dispatch incentives by the size of the (ex-post) lost-of-opportunity (LOC) payment. The higher the LOC payment, the greater the incentive for a generator to deviate from the dispatch signal. The other type is the incentive for a generator to reveal its ramping limits truthfully. If a generator receives higher profit under a pricing mechanism by under-reporting ramping capability, then the pricing mechanism not only distorts the actual ramping ability of the system but also discourages a generator from improving its ramping capacity.

The second category of performance measures is on the revenue adequacy and social welfare distribution. We are particularly interested in whether a pricing mechanism ensures the revenue adequacy of the independent system operator (ISO). For multi-interval dispatch over a network with power flow constraints, the revenue of the operator needs to cover the generation cost, the cost of ramping-induced out-of-the-market uplifts, and congestion rent. Since the operator is regulated to be revenue-neutral, a revenue reconciliation process typically redistributes the surplus or shortfall of the system operator to its consumers. Thus the revenue adequacy of the operator affects costs to consumers.

Different pricing mechanisms result in different allocations of social welfare. A pricing scheme that
yields higher generator profits may be more costly to consumers. In general, no pricing mechanism dominates all others across a wide range of performance measures. A regulator of public utility typically favors a pricing policy that guarantees the generators’ revenue adequacy while minimizing the cost to consumers.

Transparency and volatility are also relevant metrics for evaluating pricing mechanisms. Uniform pricing schemes are transparent and effective pricing signals for market participants. The use of out-of-the-market uplifts, however, affects the transparency of uniform pricing. Nonuniform pricing, in general, lacks transparency.

3.1.1 Summary of results and related work

The main contribution of Chapter 3 is twofold. First, we extend key theoretical results in Chapter 2 to a network setting in Proposition 3-7. Whereas most theoretical results such as the strong equilibrium property of TLMP generalize naturally to systems with network constraints, we obtain new results that demonstrate succinctly the spatial-temporal decomposition of TLMP.

Proposition 4 gives an explicit decomposition of TLMP into energy, congestion, and ramping prices, which shows that TLMP is the sum of a public price in the form of locational marginal price (LMP) and a private ramping price; the former is transparent to all participants, and the latter plays the role of in-the-market discrimination among generators with different ramping capabilities.

We show in Proposition 5 that, under the one-shot economic dispatch model with perfect demand forecast, the merchandising surplus under TLMP is positive and is equal to the sum of the congestion surplus (congestion rent) and ramping surplus defined by the surplus due to binding ramping constraints. In contrast, Proposition 3 shows the merchandising surplus of LMP covers only the congestion rent. This result explains partially that the revenue of the operator under LMP is often inadequate to cover the out-of-the-market uplifts due to binding ramping constraints.

Proposition 5 also shows that payments to a generator under LMP is higher than that to TLMP, implying that TLMP imposes a penalty on generators for their inabilities to ramp sufficiently. From the individual generator’s perspective, Proposition 6 shows that, under TLMP, a generator with higher ramping limits receives higher payments than an identical generator with limited ramping capability. This result partially explains that TLMP discourages under-reporting ramping limits and encourages generators to improve ramping capability.

The second part of our contribution is the empirical simulation studies on incentives, the revenue adequacy of the ISO, consumer payments, and generator profits. We are interested in particular in the effects of forecasting error and congestions on these performance measures. We compared several benchmark pricing schemes in the literature under the rolling-window dispatch model: the classical multi-interval LMP, TLMP, price preserving multi-interval pricing (PMP) [13, 16], constraints-preserving multi-interval pricing (CMP) [13], and multi-settlement LMP (MLMP) [14].
There is a fairly extensive literature on pricing multi-period dispatch. See a summary of related work in Chapter 2 [36] and references therein. The impact of multi-interval dispatch on LMP was considered in [37]. The work most relevant to this chapter is the recent work of Hua et al. [13] and Zhao, Zheng, and Litvinov [14] that articulate some of the critical issues and set forth formal statements of investigation.

Proofs, some detailed derivations, and additional simulations involving network constraints can be found in the appendix at the end of this chapter.

3.2 System and operation models

3.2.1 Generation, demand, and network models

We consider a power system with \( M \) buses under the direct-current (DC) power flow model with line-flow constraints. We follow the same notations used in Chapter 2, adding bus indices as superscripts to relevant variables.

Without loss of generality, we assume that every bus has \( N \) generators*. Let \( g_{it}^m \) be the dispatch of generator \( i \) at bus \( m \) in interval \( t \), \( g^{m}[t] = (g_{1t}^m, \ldots, g_{Nt}^m) \) the dispatch vector at bus \( m \), and \( g^{t} = (g^1[t], \ldots, g^M[t]) \) the dispatch vector in interval \( t \) from all generators.

We assume that there is one aggregated inelastic demand at each bus. For the demand at bus \( m \), let \( d_{it}^m \) be the actual demand in interval \( t \), \( \hat{d}_{it}^m \) the forecasted demand, \( d^{t} = (d^1_t, \ldots, d^M_t) \) the demand vector from all buses in interval \( t \), and \( \hat{d}^{t} \) the forecast of \( d^{t} \).

The spatial property of the power flow is governed by the DC power flow model where the branch power flow vector is a linear function of the net power injection \( (q^{t} - d^{t}) \) where \( q^{t} = (q^1_t, \ldots, q^M_t) \) is the vector of bus generations, and \( q^m_t = \sum_{i} g_{it}^m \) the total generation from bus \( m \) in \( t \).

For a network with total \( B \) branches, the \( 2B \)-dimensional vector \( z^{t} \) of branch power flows\(^\ddagger\) satisfies \( z^{t} = S(q^{t} - d^{t}) \) where \( S \) is the \( 2B \times (M - 1) \) shift-factor matrix\(^\ddagger\).

3.2.2 The rolling-window dispatch model

The rolling-window economic dispatch (R-ED) policy \( G_{R-ED} \) is defined by a sequence of \( W \)-interval look-ahead economic dispatch policies \( (G_{R-ED}^{t}, t = 1, \ldots, T) \).

At time \( t \), \( G_{R-ED}^{t} \) solves the following \( W \)-interval economic dispatch optimization using (i) the re-* One use non-generating generators to make up total \( N \) generators by setting the generation capacities to zero of such generators.

\(^\ddagger\) Each branch has two directional power flows.

\(^\ddagger\) Matrix \( S \) can be made time varying without affecting the results.
alized dispatch \(g^{R-ED}[t-1]\) in interval \(t-1\) and (ii) the load forecast \((\hat{d}[t], \ldots, \hat{d}[t+W-1])\) in the next \(W\) intervals, assuming that the forecast in the binding interval \(t\) is perfect, i.e., \(\hat{d}[t] = d[t]\).

\[
\begin{align*}
\mathcal{G}_t^{R-ED} & : \text{ at time } t, \\
& \text{ minimize } \{ \mathcal{G} = [g^m_i] \} \quad F_t(\mathcal{G}) \\
& \text{ subject to: } \text{Network constraints:} \\
& \quad \lambda'_{t'} : \sum_{m=1}^{M} \sum_{t=1}^{N} g^m_{it'} = \sum_{m=1}^{M} \hat{d}^m_t, \\
& \quad \phi[t'] : S(q[t'] - \hat{d}[t']) \leq c, \\
& \quad \text{ for all } t \leq t' < t+W. \\
& \quad \text{Generation constraints:} \\
& \quad (\mu^m_{it'}, \tilde{\mu}^m_{it'}) : -\ell^m_i \leq g^m_{it'} - \tilde{g}^m_{it'} \leq \bar{\ell}^m_i, \\
& \quad (\rho^m_{it'}, \tilde{\rho}^m_{it'}) : 0 \leq g^m_{it'} \leq \bar{g}^m_i, \\
& \quad \text{ for all } m, t \leq t' < t+W. \\
& \quad \text{Boundary ramping constraints:} \\
& \quad \tilde{\mu}^m_{it(t-1)} : g^m_{it(t-1)} - g^{R-ED}_{mit(t-1)} \leq \tilde{\rho}^m_{it}, \\
& \quad \tilde{\mu}^m_{it(t-1)} : g^{R-ED}_{mit(t-1)} - g^m_{it(t-1)} \leq \tilde{\lambda}_i, \\
\end{align*}
\]

where \(\mathcal{G} = [g[t], \ldots, g[t+W-1]]\) is the matrix of all generation variables in the \(W\)-interval look-ahead window, and \(F_t(\mathcal{G})\) is the total bid-in costs

\[
F_t(\mathcal{G}) := \sum_{i=1}^{N} \sum_{m=1}^{M} \sum_{t'=t}^{t+W-1} f^m_{it'}(g^m_{it'}). 
\]

Here \(f^m_{it'}(\cdot)\) is the bid-in cost of generator \(i\) at bus \(m\) in interval \(t\), assumed to be convex and piecewise linear (or quadratic). Vector \(c \geq 0\) is the vector of line-flow constraints.

Dual variables in (3.1) play a prominent role in multi-interval pricing, where \(\lambda_{t'}\) is the dual variable associated with the power balance equation in interval \(t'\), \(\phi[t']\) the dual variables associated with line constraints, and \((\mu^m_{it'}, \tilde{\mu}^m_{it'}, \rho^m_{it'}, \tilde{\rho}^m_{it'})\) the dual variables for the lower and upper limits for ramping and generation, respectively.

Let \((g^m_{it^*})\) be the solution of the above optimization, and \(\mu^m_{it^*}, \tilde{\mu}^m_{it^*}, \rho^m_{it^*}, \tilde{\rho}^m_{it^*}\) the optimal dual variables. Under R-ED policy \(\mathcal{G}_t^{R-ED}\), the dispatch in the binding interval \(t\) is set at

\[
\begin{align*}
& g^{R-ED}_{mit} := g^m_{it^*}. \\
& \text{(3.2)}
\end{align*}
\]

Also relevant are the (shadow) ramping prices \((\mu^m_{it^*}, \tilde{\mu}^m_{it^*}, \rho^m_{it^*}, \tilde{\rho}^m_{it^*})\) that capture the interdependencies of decisions across sliding windows. For later references, define the boundary ramping prices as

\[
\begin{align*}
& \mu^{R-ED}_{mit} := \mu^m_{it^*}, \quad \tilde{\mu}^{R-ED}_{mit} := \tilde{\mu}^m_{it^*}, \\
& \rho^{R-ED}_{mit} := \rho^m_{it^*}, \quad \tilde{\rho}^{R-ED}_{mit} := \tilde{\rho}^m_{it^*}. \\
& \text{(3.3)}
\end{align*}
\]

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In contrast to rolling-window dispatch, the *one-shot economic dispatch* $G^{\text{1-ED}}$ produces the dispatch of the entire scheduling period at once using the solution $G^*$ of (3.1) at $t = 1$ and window size $W = T$.

### 3.3 Rolling-window LMP and TLMP

A rolling-window pricing policy $\mathcal{P} = (\mathcal{P}_1, \cdots, \mathcal{P}_T)$ follows the same structure as the rolling-window economic dispatch. At time $t$, $\mathcal{P}_t$ sets prices at all $M$ buses for the binding interval $t$. It may also provide advisory prices for the future intervals within the pricing window $\mathcal{H}_t = \{t, \cdots, t + W - 1\}$.

Here we generalize the standard rolling-window LMP (R-LMP) policy $\mathcal{P}^{\text{R-LMP}}_t$ and the rolling-window TLMP $\mathcal{P}^{\text{R-TLMP}}_t$ derived in Chapter 2 for systems with power flow constraints. Both R-LMP and R-TLMP are marginal cost pricing mechanisms derived from the R-ED optimization (3.1); they are by-products of the R-ED policy.

#### 3.3.1 Rolling-window LMP (R-LMP) and Properties

Let the realized price vector in the binding interval $t$ set by R-LMP be $\pi^{\text{R-LMP}}[t] = (\pi^{\text{R-LMP}}_{1t}, \cdots, \pi^{\text{R-LMP}}_{Mt})$ where $\pi^{\text{R-LMP}}_{mt}$ is the uniform price for all generators and demand at bus $m$.

The R-LMP $\pi^{\text{R-LMP}}_{mt}$ is defined by the marginal cost of meeting demand $d^m_t$ at bus $m$ in interval $t$. From (3.1) and by the envelope theorem, we have

\[
\pi^{\text{R-LMP}}[t] = \nabla_{d[t]} F_t(G) = \lambda^{\text{R-LMP}}_t - S^T \phi^{\text{R-LMP}}[t],
\]

where $1$ is a vector of 1’s, $\lambda^{\text{R-LMP}}_t$ and $\phi^{\text{R-LMP}}[t]$ the shadow prices§ from (3.1) for the power balance and congestion constraints in interval $t$, respectively.

We summarize next main properties of R-LMP. Even though R-LMP is computed based on the current and future demand forecasts subject to ramping constraints, properties of the single-period LMP hold for the multi-interval R-LMP.

- **Energy-congestion price decomposition**
  
  The R-LMP expression (3.4) shows an explicit energy-congestion price decomposition, where the first term $\lambda^{\text{R-LMP}}_t$ is the system-wide uniform-price of energy for all generators and demands. The second term $S^T \phi^{\text{R-LMP}}[t]$ is the congestion-induced price discrimination at different locations. Note that there are no ramping prices explicitly shown in R-LMP; the R-LMP expression is identical to that in the standard single-interval LMP. The inter-temporal effects of ramping on R-LMP are hidden in the sequence of R-LMP prices $\pi^{\text{R-LMP}}[t]$.

§ When defining prices with Lagrange multipliers, we implicitly assume that the solutions to the dual optimization are unique.
• **Equilibrium properties**

We have shown in Chapter 2 that, for the single-bus network and under the perfect load forecast assumption, the one-shot economic dispatch \( G_{\text{ED}} \) and LMP \( \pi_{\text{LMP}} \) form a general equilibrium. This property holds for systems with network constraints. Unfortunately, the rolling-window version of economic dispatch and LMP \( (g_{\text{R-ED}}, \pi_{\text{R-LMP}}) \) do not satisfy the general equilibrium condition in general, even when the load forecasts are accurate; out-of-the-market uplifts are necessary.

• **ISO’s revenue adequacy**

The classical LMP theory for the single-interval LMP policy [22] states that the ISO has a non-negative merchandising surplus that covers the system congestion rent. This result extends to R-LMP under arbitrary forecast errors when there are ramping constraints.

**Proposition 3** (ISO revenue adequacy under R-LMP). For all \( (g_{\text{R-ED}}[t], \pi_{\text{R-LMP}}[t]) \) generated by the R-ED and R-LMP policies under arbitrary forecasting errors, the ISO has non-negative merchandising surplus

\[
MS_{\text{R-LMP}} = \sum_{t=1}^{T} c^\top \phi_{\text{R-LMP}}[t] \geq 0.
\]

Proposition 3 shows that the merchandising surplus from R-LMP covers and only covers the congestion rent designated to pay transmission-line owners and financial transmission right (FTR) holders. There is no extra surplus within the market settlement to cover the out-of-the-market uplifts designed to ensure dispatch-following incentives. Thus the ISO is likely to be revenue inadequate under R-LMP when the ISO has to pay out-of-the-market uplifts.

### 3.3.2 Rolling-window TLMP (R-TLMP) and Properties

As a generalization of R-LMP to a nonuniform marginal-cost pricing, R-TLMP allows individualized prices for generators and demands. Specifically, the R-TLMP at bus \( m \) in interval \( t \) is a set of prices

\[
\pi_{\text{R-TLMP}} m t = (\pi_{m0t}^\text{R-TLMP}, \pi_{m1t}^\text{R-TLMP}, \ldots, \pi_{mNt}^\text{R-TLMP}),
\]

where \( \pi_{m0t}^\text{R-TLMP} \) is the price for the demand and \( \pi_{mit}^\text{R-TLMP} \) the price for generator \( i \) at bus \( m \).

For the demand at bus \( m \) in interval \( t \), its R-TLMP \( \pi_{m0t}^\text{R-TLMP} \) is defined as the marginal cost to the system to satisfy the demand \( d_{mit} \) — the same definition used in LMP:

\[
\pi_{m0t}^\text{R-TLMP} := \frac{\partial}{\partial d_{mit}} F_{t}(G) = \pi_{mit}^\text{R-LMP}.
\]
The R-TLMP for generator \( i \) at bus \( m \), on the other hand, is defined by the marginal benefit of generator producing power \( g^m_{it} \). In other words, generator \( i \) is treated as an inelastic negative-demand set at the R-ED solution to (3.1), i.e., \( g^m_{it} = g^m_{it}^* \). As defined in Chapter 2,

\[
\pi_{mit}^{R-TLMP} := -\frac{\partial}{\partial g^m_{it}} F^m_{it}(G^*),
\]

where \( F^m_{it}(G) = F_i(G) - f^m_{it}(g^m_{it}) \) is the total generation cost excluding that from generator \( i \) at bus \( m \) in interval \( t \). When the partial derivative above does not exist, a subgradient is used.

The following proposition generalizes the TLMP expression in Chapter 2.

**Proposition 4** (Price decomposition of R-TLMP). Let \( (\lambda_t^*, \Phi^*_t[t], \bar{u}^{m*}_{i(t-1)}, \bar{u}^{m*}_{i(t-1)}, \underline{u}^{m*}_i, \underline{u}^{m*}_i) \) be the optimal values of the dual variables associated with the constraints in (3.1).

The R-TLMP for the demand \( \hat{d}^m_t \) at bus \( m \) in interval \( t \) is given by

\[
\pi_{m\hat{d}}^{R-TLMP} = \lambda_t^* - s^\top_m \Phi^*_t[t] = \pi_{mt}^{R-LMP},
\]

where \( s_m \) is the \( m \)-th column of the shift-factor matrix \( S \) corresponding to bus \( m \).

The R-TLMP for generator \( i \) at bus \( m \) in interval \( t \) is given by

\[
\pi_{mit}^{R-TLMP} = \lambda_t^* - s^\top_m \Phi^*_t[t] + \Delta^{m*}_{it} = \pi_{mi}^{R-LMP} + \Delta^{m*}_{it},
\]

where \( \Delta^{m*}_{it} = \Delta u^m_{it} - \Delta u^m_{i(t-1)} \), and \( \Delta u^m_{it} := \bar{u}^{m*}_i - \underline{u}^{m*}_i \).

Properties of R-TLMP for power systems with network constraints are summarized next.

- **Energy-congestion-ramping decomposition**
  The specific form of R-TLMP in (3.6) reveals an explicit space-time decomposition of payment to generators: a system-wide uniform energy price in \( \lambda_t^* \) applies to all generators and demands everywhere, a spatial discriminative price in the form of location-specific congestion prices in \( s^\top_m \Phi^*_t[t] \) applying to all generators and demands at bus \( m \), and a generator-specific temporal ramping prices in \( \Delta^{m*}_{it} \) that serves as a “penalty” to the generator for its limited ramping capability. The penalty interpretation of \( \Delta^{m*}_{it} \) is especially important for the incentives of the truthful revelation of ramping limits, as discussed next.

- **Public-private price decomposition and transparency**
  The structure of R-TLMP shown in (3.7) shows a public and private price decomposition: the R-LMP part of R-TLMP captures the standard uniform pricing for the energy and congestion costs that are transparent to all market participants. By revealing the R-LMP part of the TLMP, the system operator can provide the necessary system-wide pricing signal effectively for market participants.
On the other hand, the ramping price $\Delta m^\ast$ of R-TLMP is private; it pertains to the ramping conditions of individual generators. It is neither necessary nor practical to make this part of the price transparent. Another interpretation of $\Delta m^\ast$ is that it plays the role of uplift payments for uniform prices that ensures dispatch following incentives for the generator, except that it is computed within the real-time market. It is in this interpretation that R-TLMP has the same level of transparency of all uniform pricing schemes that require out-of-the-market uplifts.

- ISO’s revenue adequacy

The space-time decomposition of R-TLMP provides insights into sources of ISO’s surplus. To this end, we consider the ideal case of one-shot TLMP with a perfect load forecast.

**Proposition 5 (ISO revenue adequacy under TLMP).** Consider the one-shot economic dispatch $\mathcal{G}^{1-ED}$ defined in (3.1) with $t = 1, W = T$ and perfect demand forecast. Let the solution of the dual variables associated with the constraints be $(\lambda^\ast, \phi^\ast[t], \mu^m\ast, \bar{\mu}^m\ast)$. The total ISO merchandising surplus decomposes into ramping and congestion surpluses:

$$MS_{\text{TLMP}} = MS_{\text{ramp}} + MS_{\text{con}}, \quad (3.8)$$

where

$$MS_{\text{ramp}} = \sum_{m, i, t} (\bar{\mu}^m_i \bar{r}^m_i + \mu^m_i \bar{r}^m_i \bar{L}^m_i) \geq 0, \quad (3.9)$$

$$MS_{\text{con}} = \sum_t c^T \phi^\ast[t] \geq 0. \quad (3.10)$$

The above proposition does not generalize to the rolling-window TLMP policy, unfortunately. There are indeed cases when TLMP does not guarantee revenue adequacy (after the congestion surplus is removed). Nonetheless, simulations show that the shortfall in TLMP is considerably smaller than those of its alternatives.

- Ramping price as a penalty for inadequate ramping

Note that the TLMP and LMP have the same demand price (thus the same revenue) and the same congestion surplus. From (3.8) and the fact that $MS_{\text{LMP}} = MS_{\text{con}}$, the total generator payment under TLMP must be less than that under LMP. The following proposition suggests that the ramping price $\Delta m^\ast$ of TLMP plays the role of penalty for inadequate ramping.

**Proposition 6 (Revenue gap under LMP and TLMP).** Consider the one-shot economic dispatch $\mathcal{G}^{1-ED}$ defined in (3.1), and let $(g^m_{it}, \mu^m_{it}, \bar{\mu}^m_{it})$ be the solution of the primal and dual variables associated with generator $i$ at bus $m$ and interval $t$. If $\mu^m_{i0} = \mu^m_{iT} = 0$, then the revenue difference for delivering $(g^m_{it}, t = 1, \ldots, T)$ under LMP and TLMP is nonnegative and

$$R_{\text{LMP}}_{mi} - R_{\text{TLMP}}_{mi} = \bar{r}_i \sum_t \bar{\mu}^m_i + \bar{r}_i \sum_t \mu^m_i \geq 0.$$

The assumption $\mu^m_{i0} = \mu^m_{iT} = 0$ has minimum impact for large $T$ and becomes innocuous if the initial and final ramping constraints can be relaxed.
For an interpretation, consider two generators at the same bus with the same generation level. One generator has high ramping limits so that there are no binding ramping constraints; the other has binding ramping constraints. Under LMP, the two generators receive the same payment. Proposition 6 shows that, under TLMP, however, the one with high ramping limits receives a higher payment than the one having binding ramping constraints. This suggests that it is to the generator’s benefit not to under-report its ramping limit, and the generator is incentivized to improve its ramping capability. This insight is validated in simulations in Sec 3.5.

- **Equilibrium properties**
  The strong equilibrium property of R-TLMP shown in Chapter 2 holds when network constraints are imposed. Under TLMP, there is no incentive for any generator to deviate from the dispatch signal regardless of the accuracy of demand forecast and no need for out-of-the-market uplifts.

**Proposition 7** (Strong equilibrium property of TLMP). For every load forecast, let $G_{R-ED}$ and $\pi_{R-TLMP}$ be the rolling-window economic dispatch and the rolling-window TLMP, respectively. Then $(G_{R-ED}, \pi_{R-TLMP})$ satisfies the strong equilibrium conditions that result in zero LOC uplifts.

### 3.4 Related Benchmark Pricing Policies

We present here several benchmark pricing policies that also use the same rolling-window dispatch model. Missing in the discussion is the flexible ramping product (FRP) that has been implemented in CAISO because FRP uses a different optimization procedure that produces different dispatch signals. The development here follows [6, 13, 14].

#### 3.4.1 Price-Preserving Multi-interval Pricing (PMP)

Unlike LMP and TLMP that derive prices from R-ED, PMP [13, 16] employs a separate pricing optimization aimed at minimizing the uplift payment.

The rolling-window PMP policy $G_{R-PMP}$ at time $t$ sets uniform prices $\pi_{R-PMP}[t]$ in the binding interval $t$ using (i) the past rolling-window PMP prices $\pi_{R-PMP}[t-1], \ldots, \pi_{R-PMP}[1]$ and (ii) the demand forecasts $(\hat{d}[t], \ldots, \hat{d}[t+W-1])$ in the look-ahead window.

At time $t$, let $G = [g[1], \ldots, g[t+W-1]]$ be all the generation variables involved in the past, current, and look-ahead intervals. The rolling-window PMP policy $G_{R-PMP}$ solves the following

---

1 In practical implementation, one may include only a few past decision intervals.
optimization:

\[ G^{\text{R-PMP}}_t : \text{ at time } t, \]

\[ \text{minimize } \quad F(G) - \sum_{t'=1}^{t'-1} q^t q^{t'} \pi^{\text{R-PMP}}_t \]

subject to: for all \( t \leq t' < t+W \)

\[ q[t'] = (\sum_i g_{it'}, \ldots, \sum_i g_{im}), \]

\[ \lambda_{t'} : \quad 1^t q[t'] = 1^t \hat{d}[t'] \]

\[ \phi[t'] : \quad S(q[t'] - \hat{d}[t']) \leq c, \]  

(3.11)

where \( G^{\text{R-PMP}} \) represents the set of individual generation constraints such as ramp and generation limits. See appendix of this chapter.

The rolling-window PMP sets the price for generation in interval \( t \) by

\[ \pi^{\text{R-PMP}}_t = \lambda^{\text{R-PMP}}_t - S^t \phi^{\text{R-PMP}}_t, \]  

(3.12)

where \( \lambda^{\text{R-PMP}}_t \) and \( \phi^{\text{R-PMP}}_t \) are the multipliers associated with power balance and line-flow constraints in (3.11).

Note that the objective function can be written as

\[ \left[ \sum_{t'=1}^{t'-1} \sum_m i f_{it'}(g_{it'}, \ldots, g_{im}) - \sum_{t'=1}^{t'-1} \sum_m i \pi^{\text{R-PMP}}_t g_{it'} - f_{it'}(g_{it'}) \right] \]

where the first term is the (bid-in) generation cost in the look-ahead window. Ignoring the first term, the second term (without the negative sign) represent the estimate of the total surplus (including the LOC uplifts) up to time \( t-1 \).

3.4.2 Constraint-Preserving Multi-interval Pricing (CMP)

CMP [13] is another policy that generates uniform prices in a separate optimization different from the rolling-window economic dispatch. Instead of involving past settled prices in PMP, CMP enforces the ramping constraints between the rolling-window economic dispatch and the dispatch variables used in the pricing models.

The rolling-window CMP policy \( G^{\text{R-CMP}}_t \) at time \( t \) sets prices \( \pi^{\text{R-CMP}}_t \) in the binding interval \( t \) using (i) the past rolling-window economic dispatch \( g^{\text{R-ED}}_t \), (ii) shadow prices from (3.1) \( (\mu^{\text{R-ED}}, \bar{\mu}^{\text{R-ED}}) \) that tie generation between intervals \( t-1 \) and \( t \), and (iii) load forecasts \( (\hat{d}[t], \ldots, \hat{d}[t+W-1]) \) in the look-ahead window.

Let \( G = [g[t], \ldots, g[t+W-1]] \) be the generation variables within the \( W \)-interval look-ahead window, and \( F_t(G) \) the total cost of generation. The rolling-window CMP policy \( G^{\text{R-CMP}}_t \) solves the
following optimization:

$$G^*_t \in G^{R-CMP} : \text{at time } t,$$

minimize $F_t(G) + \sum_{m,i}(\hat{g}_{mit}^{R-ED} - \mu_{mit}^{R-ED})g_{it}^m$

subject to: for all $t \leq t' < t + W$

$$q[t'] = (\sum_i g_{it}^1, \ldots, \sum_i g_{it}^M),$$

$$\lambda_{t'} : 1^\top q[t'] = 1^\top \hat{d}[t'],$$

$$\phi[t'] : S(q[t'] - \hat{d}[t']) \leq c,$$

(3.13)

where $G^{R-CMP}$ represents the set of individual generation constraints. See appendix of this chapter.

Let $\lambda_t^{R-CMP}, \phi_t^{R-CMP}[t]$ be the dual variable solution to the above optimization associated with the power balance equation and line flow constraints, respectively. The rolling-window CMP set the price at bus $m$ and interval $t$ by

$$\pi_t^{R-CMP}[t] = \lambda_t^{R-CMP} - S^\top \phi_t^{R-CMP}[t].$$

(3.14)

3.4.3 Multi-settlement LMP (MLMP)

The multi-settlement LMP extends the two-settlement LMP used in the day-ahead and real-time markets to the rolling-window dispatch setting.

<table>
<thead>
<tr>
<th>$t^* - 2$</th>
<th>$t^* - 1$</th>
<th>$t^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Settlement 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_{it}^{m,1}$</td>
<td>$\mathcal{H}_{t^*-2}$</td>
<td></td>
</tr>
<tr>
<td>Settlement 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_{it}^{m,2}$</td>
<td>$\mathcal{H}_{t^*-1}$</td>
<td></td>
</tr>
<tr>
<td>Settlement $W$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_{it}^{m,W}$</td>
<td>$\mathcal{H}_t$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3.1: Rolling-window dispatch with window size $W = 3$. The final generation and payments are determined in $W = 3$ settlements, each produces the generation quantities and prices for deviation from the quantity in the previous settlement.

We use Fig. 3.1 to illustrate the settlement process for generation and demand in interval $t^*$. When pricing the $W$-interval rolling-window dispatch $g_{mit}^{R-ED}$, for generator $i$ at bus $m$, we consider $W$ settlements from $W$ sequential “markets”, one for each sliding window that includes interval $t^*$ as shown in Fig. 3.1.

The first settlement occurs at time $t = t^* - W + 1$ with scheduling window $\mathcal{H}_{t^*-W+1} = \{t^* - W + 1, \ldots, t^*\}$. Let $\hat{g}_{it}^{m,1}$ be the advisory dispatch for generator $i$ at bus $m$ in interval $t^*$ computed by the $W-$interval economic dispatch (3.1) and $\hat{\pi}_t^{m,1}$ its LMP. Here the superscript “1” indicates that this
is the first market that the dispatch in interval $t^*$ is settled financially. The first financially binding settlement for generator $i$ at bus $m$ is $\hat{\pi}_{i,t^*}^{m,1} \times \hat{g}_{it^*}^{m,1}$ ($) for the advisory dispatch $\hat{g}_{it^*}^{m,1}$ in interval $t^*$. (This settlement is analogous to the day-ahead settlement in the two-settlement process.)

The second settlement for generator $i$ at bus $m$ occurs at time $t^* - W + 2$ using the rolling-window dispatch over scheduling window $\mathcal{H}_{t^* - W + 1}$. Let $(\hat{g}_{it^*}^{m,2}, \hat{\pi}_{i,t^*}^{m,2})$ be the dispatch-LMP pair computed by the economic dispatch over $\mathcal{H}_{t^* - W + 1}$. The second financially binding settlement for generator $i$ at bus $m$ is $\hat{\pi}_{i,t^*}^{m,2} \times (\hat{g}_{it^*}^{m,2} - \hat{g}_{it^*}^{m,1})$ ($) for the advisory dispatch of $\hat{g}_{it^*}^{m,2}$ in interval $t^*$.

As the window slides forward one interval at a time, the process generates a sequence of $W$ dispatch-LMP pairs $(\hat{g}_{it^*}^{m,1}, \hat{\pi}_{i,t^*}^{m,1}), \ldots, (\hat{g}_{it^*}^{m,W}, \hat{\pi}_{i,t^*}^{m,W})$ for generator $i$ at bus $m$. In the last settlement occurs at time $t = t^*$ when generator $i$ at bus $m$ physically delivers $\hat{g}_{it^*}^{m,W} = g_{mit^*}^{R-ED}$ and receives the final settlement $\hat{\pi}_{i,t^*}^{m,W} \times (\hat{g}_{it^*}^{m,W} - \hat{g}_{it^*}^{m,W-1})$ ($\pi_{it^*}^{R-ED}$). Note that $\hat{\pi}_{i,t^*}^{m,W} = \pi_{m,t^*}^{R-ED}$.

Under the multi-settlement LMP, the total revenue $R_{mit^*}^{M-LMP}$ for generator $i$ at bus $m$ for delivering power $g_{mit^*}^{R-ED}$ is

$$R_{mit^*}^{M-LMP} = \hat{\pi}^{m,1}_{i,t^*} (\hat{g}^{m,1}_{it^*}) + \sum_{k=2}^{W} \hat{\pi}^{m,k}_{i,t^*} (\hat{g}^{m,k}_{it^*} - \hat{g}^{m,k-1}_{it^*}) \tag{3.15}.$$  

Note that, although $R_{mit^*}^{M-LMP}$ is a linear function with respect to $(\hat{g}^{m,1}_{it^*}, \ldots, \hat{g}^{m,W}_{it^*})$, it is not linear with respect to the power delivered $g_{mit^*}^{R-ED}$ in interval $t^*$.

### 3.5 Performance I: Single Bus Cases

We present here simulation results involving three generators at a single bus. Simulations for larger networks including one involving an ISO-NE 8-zone 76 generators can be found in the appendix of this chapter. As concept demonstrations, these small setups, although not realistic in practice, are sufficiently complex to reveal non-trivial characteristics of multi-interval dispatch and pricing.

#### 3.5.1 Simulation settings

The top part of Fig 3.2 shows the parameters of the generators and a ramping path used in the simulations. Specifically, we evaluated the performance of benchmark schemes by varying ramping limits of G2 and G3 along the path from scenario A to H while fixing the ramping limits of generator G1 to 25 MW/h. Scenario A had the most stringent ramping constraints and H the most relaxed.

The bottom part of Fig 3.2 shows the 300 realizations and average demand over 24 hour period generated from a CAISO load profile and a standard deviation of 4% of the mean value. We used a standard forecasting error model** where the demand forecast $\hat{d}_{(t+k)|t}$ of $d_{t+k}$ at time $t$ had error variance $k\sigma^2$ increasing linearly with $k$.

** The forecast $\hat{d}_{(t+k)|t}$ at $t$ of demand $d_{t+k}$ is $d_{t+k} = d_{t+k} + \sum_{i=1}^{k} \epsilon_k$ where $\epsilon_k$ is i.i.d. Gaussian with zero mean and variance $\sigma^2$.
<table>
<thead>
<tr>
<th>Capacity g</th>
<th>Marginal cost C</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1 100 MW</td>
<td>28 $/MW</td>
</tr>
<tr>
<td>G2 100 MW</td>
<td>30 $/MW</td>
</tr>
<tr>
<td>G3 100 MW</td>
<td>40 $/MW</td>
</tr>
</tbody>
</table>

Figure 3.2: Top left: generator parameters. The ramp limit for G1 is fixed at 25 (MW/h). Top right: a path of ramping events. Bottom left: average demand. Bottom right: demand traces.

All simulations were conducted with rolling-window optimization over the 24-hour scheduling period, represented by 24 time intervals. And the window size is four intervals in each rolling window optimization.

### 3.5.2 Dispatch-following and ramping-revelation incentives

- **LOC and dispatch-following incentives**
  
  We first considered dispatch incentives measured by the LOC payment; the greater the LOC payment, the higher the incentive to deviate the dispatch signal (in the absence of LOC payment). The computation of LOC for the pricing models followed that defined in Chapter 2 and given in detail in the appendix of this chapter.

  Fig. 3.3 shows the total LOC payment from the ISO to generators at different ramping rates along the ramping trajectory in Fig. 3.2. Notice the general trend that all schemes converged to zero as scenarios of binding ramping constraints diminished at scenario H.

  As predicted by the equilibrium property, the LOC for TLMP was strictly zero, and all other pricing schemes had positive LOC payments. PMP designed to minimize the LOC appeared to have the least LOC among the rest of the uniform pricing schemes. The same conclusion held for the larger scale simulations considered in the appendix. Shown also in Fig. 3.3 is
that LOC increased with the forecasting error variance, as expected.

- **Truthful revelation of ramping limits**
  This simulation aimed at illustrating incentives of the truthful revelation of ramping limits under various pricing schemes. We varied the *revealed* ramping limit of one generator and kept the others fixed at the true ramp limits.

Fig. 3.4 shows the generator profit as a function of its *revealed ramping limits* for the ramping scenario H with true ramp limits as 25 MW/h for G1, 60 MW/h for G2 and G3. Under TLMP, profits of all generators grew as the revealed ramping limits grew to their true values. The implication was that the generators had incentives to reveal their ramp limits truthfully and to improve their ramping capabilities. For the rest of uniform pricing schemes, the profits of generators G2 and G3 increased as the revealed ramp limits deviate from their true values, implying that generators had incentives to under-report their ramp limits.
3.5.3 Revenue adequacy of ISO

Fig. 3.5 shows the ISO’s merchandising surplus that included the LOC payments. The results validated the fact that uniform pricing schemes, in general, have positive LOC, resulting in a deficit for the ISO. As a regulated utility, any deficit (and surplus) was redistributed to the consumers in a revenue reconciliation process [38].

For TLMP, the ramping charge on generators led to a positive merchandising surplus, as shown in Proposition 5. The simulations involving a a larger network in the appendix also showed that the rolling-window TLMP had a merchandising surplus from both ramping and congestion. Coupled with the fact TLMP always had zero LOC, TLMP showed a positive merchandising surplus.

![Figure 3.5: ISO surplus vs. ramp limits. Left panel: ISO surplus evaluated at σ = 6%. Right panel: Ramping scenario A.](image)

The ISO surpluses for all pricing schemes converged to the congestion rent (which was zero in the single-node case) as ramping events diminished with increasing ramping limits. For TLMP, the ISO surplus decreased from the positive because ISO collected less penalty charges to generators. The ISO surpluses for all other pricing schemes increased from the negative because of the decreasing LOC payments.

3.5.4 Consumer payments and generator profits

We assumed that ISO was financially neutral; when the ISO had a positive surplus (after excluding the congestion surplus), the consumers received a price reduction as a rebate. When the ISO had a deficit, the consumers paid additional to cover the deficit.

Fig. 3.6 shows the consumer payments under the assumption that the demand is credited (or charged) for any ISO surplus (or deficit). TLMP was the least expensive for the consumer and PMP the least expensive among uniform pricing schemes. The decreasing trend of consumer payments with less ramping constraints under uniform pricing schemes was due to the decreasing costs of LOC payments to the generators. The initial increasing trend of consumer payment under TLMP was due to the less surplus of ISO passed to the consumers for collecting penalties from generators.
Again, the consumer payments increased with the forecasting error.

The total generator profit figures have identical trends as those of consumer payments because the operator has zero surplus. TLMP had the least generator profits, and PMP had the least generator profits among uniformly priced schemes. Note that the forecasting errors resulted in higher generator profits for LMP, CMP, and MLMP because of high LOC payments to generators.

![Graph showing consumer payment vs. ramp and price volatility](image)

**Figure 3.6:** Consumer payment vs. ramp. Left panel: consumer payment evaluated at $\sigma = 6\%$. Right panel: Ramping scenario A.

### 3.5.5 Price volatility

The volatility of a random price in an hour can be measured by the standard deviation of the price normalized by the average of the price in the hour. A highly volatile price makes LMP forecasting difficult.

Fig. 3.7 includes a table of price volatility averaged over all hours. Among the compared pricing mechanisms, TLMP†† showed consistently lower volatility. We also noticed that price volatility increased with stricter ramping limits and increasing demand forecasting errors. The same trend was also observed in simulations involving larger networks in the appendix of this chapter.

### 3.6 Performance II: Systems with network constraints

#### 3.6.1 Simulations on a three-bus network

We considered a three-generator two-transmission line network abstracted from the 8-zone ISO New England (ISO-NE) case shown in Fig. 3.8. Base on an ISONE load data profile, 300 load scenarios were generated as in the single bus case. The standard deviation of the one-step forecasting error was set at $\sigma = 0.6\%$. Other settings in these simulations are similar to the one-node case unless otherwise specified.

Pricing performance was evaluated with varying ramping limits. Line capacity of Line 1 (L1) was

†† The normalized standard deviation of TLMP is averaged over for all the demand and generators.
Ramp scenario | A | E | H
---|---|---|---
Forecast error | σ=0% | σ=6% | σ=0% | σ=6% | σ=0% | σ=6%
TLMP | 0.0652 | 0.0795 | 0.0286 | 0.0311 | 0.0202 | 0.0202
LMP/MLMP | 0.1074 | 0.1362 | 0.0335 | 0.0364 | 0.0202 | 0.0202
PMP | 0.0992 | 0.1453 | 0.0383 | 0.0419 | 0.0202 | 0.0202
CMP | 0.1142 | 0.1638 | 0.0383 | 0.0440 | 0.0202 | 0.0202

Figure 3.7: Average ratio of normalized standard deviation of hourly prices. Top: normalized standard deviation under different ramping scenarios and standard deviation of forecasting errors. Bottom: normalized standard deviation at different hours with \( \sigma = 6\% \) (left) and \( \sigma = 0 \) (right).

1000MW, and Line 2 (L2) was always congestion-free. The ramp limits of generations G2 and G3 varied along the path from scenario A to E with A having the most stringent ramping constraints and E the most relaxed.

Conclusions drawn for the single-node cases held mostly in simulations involving a network with power flow constraints. We provide here additional comments, especially related to congestions.

- **Dispatch-following and ramping-revelation incentives**

  Fig. 3.9 shows measures of incentives. The top left panel of Fig. A2 shows the total LOC payment at different ramping rates along the ramping trajectory in Fig. 3.9. As shown in Proposition 7, the strong equilibrium property of TLMP dictated that its LOC strictly zero. All other pricing schemes had LOC with PMP having the least amount. The LOC converged when ramping constraints vanished at scenario E.

  In evaluating incentives for generators to reveal its ramping limits truthfully, we fixed all but one generator at their true ramp limits and varied the revealed ramping limits of a single generator. The three figures in Fig. 3.9 show that only TLMP received the highest profit when the revealed ramp limit matched with the actual value. There are incentives for G2 and G3 to under-report ramping limits for all other pricing schemes.
Figure 3.8: Left panel: the 3-node-2-transmission line case. Right panel: average load demand and ramping scenarios (The ramping limit for G1 is fixed at 300MW/h).

- **Revenue adequacy of ISO**

  The revenue adequacy ISO when the network is in congestion needs to take into account congestion rent. The total ISO surplus is thus given by

  \[
  \text{ISO surplus} = \text{Payment from demand} - \text{Congestion rent} - \text{Payment to generators (inc LOC)}.
  \]

  Fig. 3.10 shows the surplus of ISO without excluding (left) congestion rent and after excluding (right) congestion rent. Before the congestion rent was excluded, all pricing mechanisms showed a positive surplus for ISO, indicating that demand payments include substantial congestion-related payments. Note also that all pricing schemes converged to the congestion rent as ramping limits diminished under scenario E.

  After congestion rent was removed, uniform pricing schemes had a noticeable negative ISO surplus. Under TLMP, ISO remained to be revenue adequate for all cases except under scenario C where there was a small deficit. Note that this is not in contradiction to Proposition 5 where the single-shot dispatch and perfect forecasting are assumed.

- **Consumer payments and generator profits**

  Fig 3.11 shows the consumer payments (left) and generator profits (right) under the assumption that the operator charges its shortfall (and returns its profit) to consumers. Note that consumer payments and generator profits were strongly dependent; the lower the consumer payment was, the lower generator payments were.

  Consumer payments under TLMP were shown to be the lowest. Unlike the single-node case, LMP had the least consumer payment among uniform pricing schemes. Note also that,
except for LMP, all uniform pricing schemes resulted in decreasing consumer payments when ramping constraints were relaxed from A to E. The consumer payment for TLMP increased along the same path of ramping scenarios. These trends were consistent with the single-node case. The trend for LMP, however, did not follow that in the single-node case due to a complicated interaction of congestion and ramping constraints.

• **Price volatility**

The price volatility was evaluated under both ramping and congestion constraints. Fig. 3.12 shows that the average price standard deviation table under strict (A), relaxed (E), and unconstrained (H) ramping limit scenarios. The price standard deviation in the table was averaged over the 24-hour period whereas the two figures below are the hourly average price standard deviation.

The same conclusions as in the single node case held. For the most part, TLMP appeared to be the least volatile among pricing schemes evaluated in this chapter.
3.6.2 Simulation results for the 76 generator ISO-NE system

We considered the 8-zone ISO New England (ISO-NE) case with 76-generator and 12-transmission line shown in Fig. 3.13 with parameters from [34, 35]. All line capacities were set at 1000MW. Monte Carlo simulations of 200 load scenarios were conducted with normalized standard deviation of the one-step forecasting error set at $\sigma = 0.6\%$. Other settings in these simulations were similar to the one-node case unless otherwise specified.

Pricing performance was evaluated with varying ramping limits. In the ISO-NE data set, there were two groups of generators with different ramping limits. The ramping limit of Group 1 varied from 30MW/h to 120MW/h and that of Group 2 varied from 100MW/h to 700MW/h. From Fig. 3.13, the ramp limits of generators in two groups varied along the path from scenario A to F with A having the most stringent ramping constraints and F the most relaxed.

Conclusions drawn from previous single-node cases and three-generator two-transmission line cases mostly held for the ISO-NE simulations. Here we highlight some of the differences in the ISO-NE test cases from the previous smaller-scale simulations.
• Dispatch-following and ramping-revelation incentives

As is shown on the left panel of Fig. 3.14, only TLMP supported dispatch-following incentives with strictly zero LOC, as expected from Proposition 7. This simulation also showed that the average LOC decreased dramatically from ramping setting A to B where ramp limits of generators in Group 1 increased from 30MW to 60MW. See also the performance in the magnified LOC plot for the ramping path from B to F.

For the truthful revelation of ramping limits, the right panel of Fig. 3.14 shows the scatter plot of the profit change of a generator when its ramp limits was increased by 10% while others were truthful. When the profit change was negative, it meant that the the generator had incentive not to reveal the true ramp limit. Among the five benchmark pricing schemes, TLMP had the smallest number of non-incentive compatible cases.

• Revenue adequacy of ISO

Fig. 3.15 shows the merchandising surplus of ISO with (left) and without (right) congestion rent. Once congestion rent is removed, TLMP was the only one with positive surplus over the entire ramping path with the highest surplus when ramp limits were tight (scenario A and B). The order of the rest of benchmarks in this larger network was not entirely consistent with the three generator case. Most noticeably was that PMP had the least surplus for the tight ramp case (A) and the highest for the less ramp-limited scenarios (C and D).
• **Consumer payments and generator profits**

Fig 3.16 shows the consumer payments (left) and generator profits (right) under the assumption that the operator charges its shortfall (and returns its profit) to consumers.

As expected, the consumer payment and generator revenue were strongly correlated. Unlike previous small case studies, consumer payments and generator profit under PMP for the 76 generator case were the lowest for scenario A and the highest for scenarios C and D. MLMP had the lowest total consumer payment and generator profit for scenarios B, C, D, and E.

• **Price volatility**

The price volatility was evaluated under different ramping constraints and load forecast errors with network congestion considered. Fig. 3.17 shows that the average price standard deviation table under strict (A), relaxed (D), and unconstrained (F) ramping limit scenarios. The conclusion here was consistent with previous two small-scale case studies. For the most part, TLMP appeared to be the least volatile among pricing schemes evaluated in this chapter.

### 3.7 Conclusion

Chapter 2 and Chapter 3 consider the pricing of multi-interval dispatch under demand forecast uncertainty. We establish that, to provide dispatch-following incentives, discrimination in the form of uniform pricing with out-of-the-market uplifts or nonuniform pricing becomes necessary. In particular, we show that, as a generalization of LMP, the nonuniform TLMP eliminates the need for the out-of-the-market uplifts under arbitrary forecasting uncertainty. We also consider incentives of the truthful revelation of ramping limits. We show that, by penalizing the ramping limits, TLMP
Figure 3.14: Left: LOC vs. ramping scenarios from A to F. Right: under scenario A, the profit change of one generator when its ramp limits was changed by 10%.

Figure 3.15: Left: ISO surplus without excluding congestion rent. Right: ISO surplus after excluding congestion rent.

provides incentives for generators to improve its ramping capability and reveal the actual ramping limits. Unfortunately, such incentives are lacking in the existing pricing schemes.

Under the rolling-window dispatch, different pricing schemes differ in the distribution of the overall social welfare among generators and consumers. Among the pricing mechanisms considered in this chapter, TLMP leads to the least consumer payment but also the lowest generator profit. Likewise, among uniform pricing schemes, PMP leads to the least consumer payment and the lowest generator profits.
3.8 Appendix

3.8.1 Proof of Proposition 1

Let $q^{R-ED}[t]$ be the vector of injections in interval $t$ from the R-ED policy. From (3.4), the ISO surplus is given by

$$MS^{R-LMP} := \sum_{t=1}^{T} (d[t] - q^{R-ED}[t])^T (\lambda^{R-LMP} - S^T \phi^{R-LMP}[t])$$

$$= \sum_{t=1}^{T} (q^{R-ED}[t] - d[t])^T S^T \phi^{R-LMP}[t]$$

$$= \sum_{t=1}^{T} e^T \phi^{R-LMP}[t],$$

where the last equality uses the complementary slackness condition of (3.1). \[\square\]

3.8.2 Proof of Proposition 2

The proof of Proposition 2 follows directly from the Proof of Proposition 3 in Chapter 2. See [36].

3.8.3 Proof of Proposition 3

The proof of Proposition 3 follows that of Proposition 1.
### 3.8.4 Proof of Proposition 4

Because Proposition 4 focuses on the payment to generator $i$ at bus $m$, we drop the subscripts $i$ and superscript $m$ in the relevant notations.

\[
\Delta R := R^{LMP} - R^{TLMP} = \sum_{t=1}^{T} (\pi_{t}^{LMP} - \pi_{t}^{TLMP}) g_{t}^* \\
= \sum_{t=1}^{T} \Delta \mu_{t-1}^* g_{t}^* - \sum_{t=1}^{T} \Delta \mu_{t}^* g_{t}^* \\
= \sum_{t=1}^{T} \Delta \mu_{t-1}^* (g_{t}^* - g_{t-1}^*) + \Delta \mu_{0}^* g_{0}^* - \Delta \mu_{T}^* g_{T}^* \\
= \sum_{t=1}^{T} \Delta \mu_{t-1}^* (g_{t}^* - g_{t-1}^*).
\]

By the complementary slackness condition,

\[
\Delta \mu_{t}^* (g_{t+1}^* - g_{t}^*) = \begin{cases} 
\bar{\mu}_{t}^* \bar{r}_{i} & g_{t+1}^* - g_{t}^* = \bar{r} \\
\bar{\mu}_{t}^* \bar{l}_{i} & g_{t+1}^* - g_{t}^* = \bar{l} \\
0 & \text{otherwise}
\end{cases} \\
= \bar{\mu}_{t}^* \bar{r} + \mu_{t}^* \bar{r} \geq 0.
\]
3.8.5 Proof of Proposition 5

The proof of Proposition 5 follows exactly the proof of Theorem 4 given in Appendix F of [36].

3.8.6 Pricing optimization of PMP and CMP

- Optimization in PMP
  The rolling-window PMP policy $\mathcal{G}_t^{\text{R-PMP}}$ solves the following optimization:

$$
\mathcal{G}_t^{\text{R-PMP}} : \text{at time } t,
\begin{align*}
\min_{\{G\}} & \quad F(G) - \sum_{t'=1}^{t-1} q^T[t'] \pi^{\text{R-PMP}}[t'] \\
\text{subject to:} & \quad \text{Network constraints:} \\
& \quad q[t'] = (\sum_i g^1_{it'}, \ldots, \sum_i g^M_{it'}), \\
& \quad \lambda_{q[t']} = 1^T q[t'] = 1^T \hat{d}[t'] \\
& \quad \phi[t'] : S(q[t'] - \hat{d}[t']) \leq c \\
& \quad \text{for all } t \leq t' < t + W. \\
& \quad \text{Generation constraints:} \\
& \quad (\mu^m_{it'}, \bar{\mu}^m_{it'}) : -\bar{L}^m \leq g^m_{i(t'+1)} - \bar{g}^m_{it'} \leq \bar{L}^m, \\
& \quad (\rho^m_{it'}, \bar{\rho}^m_{it'}) : 0 \leq g^m_{it'} \leq \bar{g}^m_i \\
& \quad \text{for all } m, 0 < t' < t + W.
\end{align*}
$$

(3.16)

The rolling-window PMP sets the price for generation in interval $t$ by

$$
\pi^{\text{R-PMP}}[t] = \lambda_{\pi^{\text{R-PMP}}} - S^T \phi^{\text{R-PMP}}[t],
$$

(3.17)

where $\lambda_{\pi^{\text{R-PMP}}}$ and $\phi^{\text{R-PMP}}[t]$ are the multipliers associated with power balance and line-flow constraints in (3.16).

- Optimization in CMP
  Let $G = [g[t], \ldots, g[t+W-1]]$ be the generation variables within the $W$-interval lookahead window, and $F_t(G)$ the total cost of generation. The rolling-window CMP policy $\mathcal{G}_t^{\text{R-CMP}}$
solves the following optimization:

\[
G_{t}^{\text{R-CMP}} \quad \text{at time } t,
\]

\[
\min_{\{G\}} F(G) + \sum_{m,i} (\bar{\mu}_{mit} - \mu_{mit}^{\text{R-ED}}) g_{it}^m
\]

subject to:

Network constraints:

\[
q[t'] = (\sum_{i} g_{it}^1, \cdots, \sum_{i} g_{it}^M),
\]

\[
\lambda_{t'} : \quad \lambda_{t'}^\top q[t'] = \lambda_{t'}^\top \hat{d}[t']
\]

\[
\phi[t'] : \quad S(q[t'] - \hat{d}[t']) \leq c
\]

for all \( t \leq t' < t + W \).

(3.18)

Generation constraints:

\[
-\bar{r}_m \leq g_{it}' - g_{it}' - g_{it}' \leq \bar{r}_m,
\]

\[
0 \leq g_{it}' \leq \bar{g}_i,
\]

for all \( m, t \leq t' < t + W \).

boundary ramping constraints:

\[
-\bar{r}_i \leq g_{it}' - g_{it}' \leq \bar{r}_i.
\]

3.8.7 Computation of LOC

- Single settlement prices

For the single settlement pricing schemes, the computation of LOC is with respect to a specific generator under dispatch vector \( g = (g_1, \cdots, g_T) \) and price vector \( \pi = (\pi_1, \cdots, \pi_T) \). The LOC associated with \( \pi \) is given by

\[
\text{LOC}(\pi) = Q(\pi) - (\pi^\top g - F(g))
\]

(3.19)

where \( F(g) = \sum_t f_t(g_t) \) is the total (true) cost of generating \( g \), and \( Q(\pi) \) is the post-uplift generator-surplus defined by

\[
\begin{align*}
Q(\pi) &= \max_{p=(p_1, \cdots, p_T)} \sum_{t=1}^T (\pi_t p_t - f_t(p_t)) \\
\text{subject to} & \quad g \leq p_t \leq \bar{g} \\
& \quad -\bar{r} \leq p_{t+1} - p_t \leq \bar{r}.
\end{align*}
\]

(3.20)

Using (3.19-3.20), the computation of LOC under the rolling-window version of LMP, TLMP, PMP, and CMP are made by substituting \( g \) by the rolling-window dispatch \( g_{\text{R-ED}} \) and \( \pi \) by the rolling-window of the corresponding prices.

- Multi-settlement prices

For the multi-settlement pricing such as MLMP, note that the generator can only affect the revenue by setting the dispatch in the binding interval. Therefore, in calculating \( \text{LOC}^{\text{M-LMP}} \), the only decision variables are the realized generations.
Let $\tilde{g}_t$ be the pre-binding dispatch of interval $t$, i.e., $\tilde{g}_t$ is the advisory dispatch level $\hat{g}_m^{W-1}$ for generator $i$ at bus $m$ in the $(W-1)$th settlement as shown in Fig. 3.1. The generator profit maximization problem is given by

$$Q(\pi^{M-LMP}) = \max_{\mathbf{p} = (p_1, \ldots, p_T)} \sum_{t=1}^{T} (\pi_t^{R-LMP} (p_t - \tilde{g}_t) - f_t(p_t))$$

subject to

$$g \leq p_t \leq \bar{g}$$

$$-L \leq p_{t+1} - p_t \leq \bar{r}.$$ 

Since $\tilde{g}_t$ is given, the above optimization has the same solution as $\mathbf{p}^*$ as that of (3.20) for R-LMP using $\pi^{R-LMP}$ (although the values of the two optimization are different).

The LOC for MLMP is therefore

$$\text{LOC}^{M-LMP} = \sum_{t=1}^{T} \left( \pi_t^{R-LMP} (p_t^* - g_{t(t-1)}) - f_t(p_t^*) \right)$$

$$- \left( \sum_{t=1}^{T} \pi_t^{R-LMP} (\hat{g}_t^{R-ED} - g_{t(t-1)}) - F(\hat{g}_t^{R-ED}) \right)$$

$$= \text{LOC}^{R-LMP} \geq 0.$$
References


