Network-level Optimization for Unbalanced Power Distribution Systems

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Presentation Outline

• Motivation
• Challenges
• Proposed Approach
• Approximation and Relaxation Techniques
• Applications
• Future Research Directions
Motivation: Network-level Optimization

Modern Power Grid:
- Distributed and Non-dispatchable Generation
- Bidirectional power flow
- Controllable loads/Prosumers
- Generation Variability and Uncertainty

Key drivers for Network-level Optimization in Power Distribution Systems:
- Incorporate non-traditional resources (DERs, responsive loads, battery storage),
- Incorporate controllable loads – smart buildings/prosumers,
- Incorporate new measurements and other sources of data,
- Increased requirement for power quality and reliability,
- Ensure resilience to disasters.
Challenges: Network-level Optimization for Distribution Systems

A three-phase unbalanced Distribution System representative of a typical North American feeder

Model Details:
1. Open-loop model operated in radial configuration fed from three substations
2. 9500 single-phase nodes
3. 107 switches leading to 96 cycles (integer variables)
4. 1275 single-phase loads, 354 single-phase PVs + several utility scale DGs and storage

Added nonlinearity:
- Nonlinear power flow model must include mutual coupling;
- Nonlinear load models, voltage-dependent loads, price-responsive consumers;
- Discrete decision variables, control of voltage regulator/capacitor banks, switches

Heterogenous control:
- Time-fragmented operation (integration of local and network level control);
- Multi-stage optimization (schedule storage/price-responsive loads)

Model and Measurement uncertainty:
- Partially-known/Incorrect physical system models,
- Noisy measurements,
- Generation and load stochasticity
This Talk: Network-level Optimization for Unbalanced Distribution Systems

My Research:

• Optimal power flow algorithms for unbalanced power distribution systems.
• Proposed methods for scalable optimal power flow algorithms for large-scale distribution systems.
• Application of OPF model for distribution grid operational challenges.

Specific Applications:

• Conservation voltage reduction. (Method) Scalable three-phase optimal power flow with mixed-integer constraints.
• Network topology estimation. (Method) Estimating network topology (normal and outaged) that satisfies the measurements. Formulated as a network-level optimization problem with discrete decision variables.
• Resilient Restoration with intentional islanding. (Method) Optimal reconfiguration while meeting dynamic island feasibility considerations for improved resilience to natural disasters.
Network-level Optimization: Three-Phase Optimal Power Flow (OPF)

objective: \( \min f(x) \)

subject to: \( g(x) = b \)

For ex. Conservation voltage reduction, Loss minimization, etc.

Power flow equations, operating constraints

Branch power flow model*

\[
v_j = v_i + z_{ij} l_{ij} z_{ij}^H - Z_{ij} S_{ij} - S_{ij} Z_{ij}^H
\]

\[
diag (S_{ij} - z_{ij} l_{ij} l_{ij}^H) - s_j = diag (S_{jk})
\]

\[
\begin{bmatrix}
v_i & S_{ij} \\
S_{ij}^H & l_{ij}
\end{bmatrix} = \begin{bmatrix}
V_i \\
I_{ij}
\end{bmatrix} \begin{bmatrix}
V_i^H \\
I_{ij}^H
\end{bmatrix}
\]

\[
\begin{bmatrix}
v_i & S_{ij} \\
S_{ij}^H & l_{ij}
\end{bmatrix} \text{:- Rank } -1 \text{ PSD matrix}
\]

---

Network-level Optimization: Reducing Complexity - Three-Phase Power Flow Model

Assumptions:
1. The phase voltages are assumed to approximately 120° degree apart.

\[ V_i^a \neq V_i^b \neq V_i^c \]
\[ \frac{V_i^a}{V_i^b} \approx \frac{V_i^b}{V_i^c} \approx \frac{V_i^c}{V_i^a} \approx e^{j2\pi/3} \]

2. The phase angle difference (\( \delta_{ij}^{pq} \)) between branch currents are obtained from equivalent constant impedance model

\[ \delta_{ij} \text{ is approximated in OPF. } I_{ij} \text{ is a variable in OPF} \]
Validation of Assumptions

- The power flow is solved in OpenDSS at various loading condition for different test feeder
- \( \theta_i^{pq} \) is the phase angle difference between node voltage and \( \delta_i^{pq} \) is the phase angle difference between branch currents
- The results shows the maximum error in \( \theta_i^{pq} \) and \( \delta_i^{pq} \) for a test feeder

<table>
<thead>
<tr>
<th>Test Feeder</th>
<th>% Load</th>
<th>error in ( \delta_i^{pq} ) (degrees)</th>
<th>error in ( \theta_i^{pq} ) (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE 13 bus</td>
<td>75 %</td>
<td>1.8</td>
<td>1.8</td>
</tr>
<tr>
<td>IEEE 13 bus</td>
<td>100 %</td>
<td>2.1</td>
<td>2.2</td>
</tr>
<tr>
<td>IEEE 123 bus</td>
<td>75 %</td>
<td>0.8</td>
<td>0.9</td>
</tr>
<tr>
<td>IEEE 123 bus</td>
<td>100 %</td>
<td>1.13</td>
<td>1.3</td>
</tr>
<tr>
<td>Feeder- R3-12.47-2</td>
<td>75 %</td>
<td>0.5</td>
<td>0.55</td>
</tr>
<tr>
<td>Feeder- R3-12.47-2</td>
<td>100 %</td>
<td>0.9</td>
<td>1.05</td>
</tr>
</tbody>
</table>
Network-level Optimization: Three-Phase Optimal Power Flow of Reduced Complexity

objective: \( \min f(x) \)

subject to: \( g(x) = b \)

Nonlinear Power Flow Equations

(Linear)

\[
P_{ij}^{pp} - p_{Lj}^p = \sum_{k:j \rightarrow k} P_{jk}^{pp} - \sum_{q \in \varphi_j} l_{ij}^{pq} \left( r_{ij}^{pq} \cos(\delta_{ij}^{pq}) - x_{ij}^{pq} \sin(\delta_{ij}^{pq}) \right)
\]

\[
Q_{ij}^{pp} - q_{Lj}^p = \sum_{k:j \rightarrow k} Q_{jk}^{pp} - \sum_{q \in \varphi_j} l_{ij}^{pq} \left( x_{ij}^{pq} \cos(\delta_{ij}^{pq}) + r_{ij}^{pq} \sin(\delta_{ij}^{pq}) \right)
\]

\[
v_j^p = v_i^p - \sum_{q \in \varphi_j} 2 \Re \left[ S_{ij}^{pq} (Z_{ij}^{pq})^H \right] + \sum_{q \in \varphi_j} Z_{ij}^{pq} l_{ij}^{pq} + \sum_{q1,q2 \in \varphi_j, q1 \neq q2} 2 \Re \left[ Z_{ij}^{pq1} l_{ij}^{q1q2} (\angle \delta_{ij}^{q1q2}) (Z_{ij}^{pq2})^H \right]
\]

(Quadratic)

\[
\left( P_{ij}^{pp} \right)^2 + \left( Q_{ij}^{pp} \right)^2 = v_i^p l_{ij}^{pp}
\]

\[
\left( l_{ij}^{pq} \right)^2 = l_{ij}^{pp} l_{ij}^{qq}
\]
Validation of Power Flow

- The total number of variables in the proposed formulation are $15 \times (n - 1)$, where $n$ is the number of nodes; the original branch-flow based models had a total of $36 \times (n - 1)$ variables.

  Largest Error in Nonlinear Power Flow wrt. OpenDSS Solutions

<table>
<thead>
<tr>
<th>Test Feeder</th>
<th>% loading</th>
<th>Pflow (%)</th>
<th>Qflow (%)</th>
<th>Sflow (%)</th>
<th>V(pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE 13 Bus</td>
<td>100 %</td>
<td>0.29</td>
<td>2.03</td>
<td>0.253</td>
<td>0.003</td>
</tr>
<tr>
<td>IEEE 123 Bus</td>
<td>100 %</td>
<td>0.61</td>
<td>3.88</td>
<td>0.286</td>
<td>0.002</td>
</tr>
<tr>
<td>Feeder- R3-12.47-2</td>
<td>100 %</td>
<td>0.6</td>
<td>3.4</td>
<td>0.163</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

- Computational time to solve NLP model for conservation voltage reduction objective in MATLAB using *fmincon*:
  - IEEE 13-bus – 20 sec,
  - IEEE 123-bus (267 single-phase nodes) – 4 min,
  - Feeder- R3-12.47-2 (860 single-phase nodes) – 20 min.

**Accurate but still not scalable for large systems**
Scalability: Approximation vs. Relaxation

• The problem is non-convex due to nonlinear equality constraints

\[
(P_{ij}^{pp})^2 + (Q_{ij}^{pp})^2 = v_i^p l_{ij}^{pp} \quad \text{and} \quad (l_{ij}^{pq})^2 = l_{ij}^{pp} l_{ij}^{qq}
\]

• Ideas on making the model scalable:
  o approximating the nonlinear constraints to linear constraints
  o relaxing the nonlinear constraints to lead to a convex problem
Three-Phase Optimal Power Flow (OPF)

objective: \( \min f(x) \)

subject to: \( g(x) = b \)

For ex. Conservation voltage reduction, Loss minimization, etc.

Power flow equations, operating constraints

Branch power flow model*

\[ v_j = v_i + z_{ij} l_{ij} z_{ij}^H - Z_{ij} S_{ij} - S_{ij} Z_{ij}^H \]

\[ \text{diag} \left( S_{ij} - z_{ij} l_{ij} l_{ij}^H \right) - s_j = \text{diag} \left( S_{jk} \right) \]

\[
\begin{bmatrix}
  v_i & S_{ij} \\
  S_{ij}^H & l_{ij}
\end{bmatrix} =
\begin{bmatrix}
  V_i \\
  I_{ij}
\end{bmatrix}
\begin{bmatrix}
  V_i^H \\
  I_{ij}^H
\end{bmatrix}
\]

\[
\begin{bmatrix}
  v_i & S_{ij} \\
  S_{ij}^H & l_{ij}
\end{bmatrix} : - \text{Rank} -1 \text{ PSD matrix}
\]

Relaxation: General Model

\[ p_{ij}^{pp} - p_{L_j}^p = \sum_{k:j\rightarrow k} p_{jk}^{pp} - \sum_{q\in \varphi_j} l_{ij}^{pq} \left( r_{ij}^{pq} \cos(\delta_{ij}^{pq}) - x_{ij}^{pq} \sin(\delta_{ij}^{pq}) \right) \]

\[ q_{ij}^{pp} - q_{L_j}^p = \sum_{k:j\rightarrow k} Q_{jk}^{pp} - \sum_{q\in \varphi_j} l_{ij}^{pq} \left( x_{ij}^{pq} \cos(\delta_{ij}^{pq}) + r_{ij}^{pq} \sin(\delta_{ij}^{pq}) \right) \]

\[ v_j^p = v_i^p - \sum_{q\in \varphi_j} 2 \Re \left[ S_{ij}^{pq} (Z_{ij}^{pq})^H \right] + \sum_{q\in \varphi_j} Z_{ij}^{pq} l_{ij}^{pq} + \sum_{q1,q2\in \varphi_j,q1\neq q2} 2 \Re \left[ Z_{ij}^{pq1} l_{ij}^{q1q2}(\angle \delta_{ij}^{q1q2})(Z_{ij}^{pq2})^H \right] \]

\[ \left( p_{ij}^{pp} \right)^2 + \left( Q_{ij}^{pp} \right)^2 = v_i^p l_{ij}^{pp} \]

\[ \left( l_{ij}^{pq} \right)^2 = l_{ij}^{pp} l_{ij}^{qq} \]

Nonlinear equations in OPF model

- Can be relaxed to a second-order cone-constraint: Convex problem (SOCP)
- After relaxation

\[ v_i^p l_{ij}^{pp} \geq p_{ij}^{pp^2} + Q_{ij}^{pp^2} \]

\[ l_{ij}^{pp} \ast l_{ij}^{qq} \geq l_{ij}^{pq^2} \]
- The relaxation however was found to be not exact

**Iterative approach to reach the feasible solution over multiple iterations of the SOCP relaxed problem**

- Additional constraint is defined to reduce feasibility gap

\[ e_1 = v_i^p l_{ij}^{pp} - p_{ij}^{pp^2} - Q_{ij}^{pp^2} \]

\[ e_2 = l_{ij}^{pp} \ast l_{ij}^{qq} - l_{ij}^{pq^2} \]

\[ v_j^p = v_i^p - \sum_{q\in \varphi_j} 2 \Re \left[ S_{ij}^{pq} (Z_{ij}^{pq})^H \right] + \sum_{q\in \varphi_j} Z_{ij}^{pq} l_{ij}^{pq} + \sum_{q1,q2\in \varphi_j,q1\neq q2} 2 \Re \left[ Z_{ij}^{pq1} l_{ij}^{q1q2}(\angle \delta_{ij}^{q1q2})(Z_{ij}^{pq2})^H \right] \]
Relaxation – Understanding Feasibility Gap

Single-phase radial distribution feeder

• The power flow will have solution on the surface of the cone.

• **Convex Relaxation** - The nonlinear equality constraints are converted into inequality constraints thus expanding the feasible space to the interior of the cone:

\[
(p_{ij}^{pp})^2 + (q_{ij}^{pp})^2 \leq v_i^p l_{ij}^{pp}
\]

• Feasibility gap is defined as

\[
error = v_i^p l_{ij}^{pp} - p_{ij}^{pp^2} - Q_{ij}^{pp^2}
\]

• For minimization problem (for ex. loss minimization) the solution for the relaxed model is known to lie at the surface of the cone under certain conditions that are usually satisfied by the single-phase network.

• Thus, the solution of relaxed problem is exact as it lies on the surface of the cone.

Relaxation – Understanding Feasibility Gap

Single-phase radial distribution feeder

- For maximization problem (for ex. maximum PV hosting), the relaxed solution was found to be inexact.
- In this case, the relaxed problem can have optimal solution inside the cone.
- Here, for the relaxed problem the solution obtained is \( c_r \) which is optimal for SOCP but infeasible for actual problem leading to feasibility gap.

\[
\text{error} = v^p_i l^p_{ij} - p^p_{ij} - Q^p_{ij}
\]

We enforce a linear inequality constraint to reduce the feasibility gap over successive iterations of the relaxed problem*.

Approach: Iterative Second-order Cone Programming for Three-phase OPF

• An iterative approach that solves multiple iterations of SOCP relaxed problems to drive the solution to feasible space.

• Feasibility gap termed as error is defined as

\[
e_1^{(k)} = v_i^{p(k)} l_{ij}^{pp(k)} - \left( p_{ij}^{pp(k)} \right)^2 - \left( Q_{ij}^{pp(k)} \right)^2
\]

\[
e_2^{(k)} = l_{ij}^{pp(k)} * l_{ij}^{qq(k)} - \left( l_{ij}^{pq(k)} \right)^2
\]

• Added linear inequality constraints that minimizes the feasibility gap in successive SOCP iterations.
Results: Iterative SOCP for Three-phase Power Distribution System

Feasibility Gap: \( \text{error} = \max \left( (v_i l_{ij} - (P_{ij}^2 + Q_{ij}^2)), \left( l_{ij}^p l_{ij}^q - \left( I_{ij}^{pq} \right)^2 \right) \right) \)

### Test feeder Loading condition Computation time (NLP) Computation time (SOCP)

<table>
<thead>
<tr>
<th>Test feeder</th>
<th>Loading condition</th>
<th>Computation time (NLP)</th>
<th>Computation time (SOCP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE 13 bus</td>
<td>minimum</td>
<td>17.95 sec</td>
<td>4.00 sec</td>
</tr>
<tr>
<td></td>
<td>maximum</td>
<td>13.51 sec</td>
<td>~20.0 sec (10 iterations)</td>
</tr>
<tr>
<td>IEEE 123 bus</td>
<td>minimum</td>
<td>~2 min</td>
<td>30.00 sec (8 iterations)</td>
</tr>
<tr>
<td></td>
<td>maximum</td>
<td>~4 min</td>
<td>40.00 sec (10 iterations)</td>
</tr>
</tbody>
</table>

The computation time required for each iteration to solve the IEEE-123 node system using CVX is approximately 4.00 sec.
Three-Phase Optimal Power Flow (OPF)

**objective:** \( \min f(x) \)

**subject to:** \( g(x) = b \)

For ex. Conservation voltage reduction, Loss minimization, etc.

Power flow equations, operating constraints

**Branch power flow model**

\[
\begin{align*}
  v_j &= v_i + z_{ij} l_{ij} z_{ij}^H - Z_{ij} S_{ij}^H - S_{ij} Z_{ij}^H \\
  \text{diag} \left( S_{ij} - z_{ij} l_{ij} l_{ij}^H \right) - s_j &= \text{diag} \left( S_{jk} \right) \\
  \begin{bmatrix} v_i & S_{ij} \\ S_{ij}^H & l_{ij} \end{bmatrix} &= \begin{bmatrix} V_i \\ I_{ij} \end{bmatrix} \begin{bmatrix} V_i \\ I_{ij} \end{bmatrix}^H \\
  \begin{bmatrix} v_i & S_{ij} \\ S_{ij}^H & l_{ij} \end{bmatrix} &\text{:- Rank -1 PSD matrix}
\end{align*}
\]

Approximation: General Model

Nonlinear program general form

\[
\begin{align*}
\min & \quad f(x) \\
\text{st} & \quad g(x) = b
\end{align*}
\]

Penalty Successive Linear Programming:

\[
\begin{align*}
\min & \quad f(x^k) + \nabla f \Delta x + w \sum_i p_i + n_i \\
\text{st} & \quad g_i(x^k) + \nabla g_i \Delta x - b_i = p_i - n_i \\
& \quad -s^k \leq \Delta x \leq s^k \\
& \quad l \leq x^k + \Delta x \leq u \\
& \quad p_i \geq 0, \quad n_i \geq 0
\end{align*}
\]

Nonlinear equations in OPF model

\[
\left(p_{ij}^{pp}\right)^2 + \left(Q_{ij}^{pp}\right)^2 = v_i^p l_{ij}^{pp} \\
\left(l_{ij}^{pq}\right)^2 = l_{ij}^{pp} l_{ij}^{qq}
\]

- Nonlinear quadratic equations
- Linearized around optimal operating point obtained from linearized three-phase OPF model
- Solves within 3-4 iterations of linear programming
Approximation: Three-phase OPF and Flowchart

- Solves within 3-4 iterations of linear programming
# Results: Approximate Optimal Power Flow Model

- Optimization for CVR objective (Conservation voltage reduction)
- Solved for three-phase example test feeders: IEEE 13 bus, IEEE 123 bus (267 single-phase nodes), Feeder- R3-12.47-2 (860 single-phase nodes)
- Validated against NLP solver - KNITRO

<table>
<thead>
<tr>
<th>Test feeder</th>
<th>Loading condition</th>
<th>NLP (pu)</th>
<th>PSLP (pu)</th>
<th>Computation time (NLP)</th>
<th>Computation time (PSLP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE 13 bus</td>
<td>minimum</td>
<td>0.5376</td>
<td>0.5376</td>
<td>17.95 sec</td>
<td>1.75 sec</td>
</tr>
<tr>
<td></td>
<td>maximum</td>
<td>2.054</td>
<td>2.045</td>
<td>13.51 sec</td>
<td>1.46 sec</td>
</tr>
<tr>
<td>IEEE 123 bus</td>
<td>minimum</td>
<td>0.58</td>
<td>0.55</td>
<td>~2-min</td>
<td>10.97 sec</td>
</tr>
<tr>
<td></td>
<td>maximum</td>
<td>3.43</td>
<td>3.28</td>
<td>~2-min</td>
<td>11.94 sec</td>
</tr>
<tr>
<td>Feeder- R3-12.47-2</td>
<td>minimum</td>
<td>0.653</td>
<td>0.679</td>
<td>~15-min</td>
<td>15.66 sec</td>
</tr>
<tr>
<td></td>
<td>maximum</td>
<td>4.531</td>
<td>4.461</td>
<td>~15-min</td>
<td>16.96 sec</td>
</tr>
</tbody>
</table>
Applications

Application 1 - Conservation voltage reduction
Method: Scalable three-phase optimal power flow with mixed-integer constraints.

Application 2 - Topology estimation: during normal and outage condition
Method: Estimation for topology that satisfies power flow measurements. Formulated as a network-level optimization problem.

Application 3 - Resilient Restoration with intentional islanding
Method: Optimal reconfiguration while meeting dynamic island feasibility considerations for improved resilience to natural disasters.
Application 1: Conservation Voltage Reduction/Volt-VAR Optimization

Coordinated control grid’s legacy devices (voltage regulator, capacitor banks) and new devices (smart inverters) to reduce feeder voltages and thereby demand from feeder’s voltage-dependent loads.
Application 1: Volt-VAR Optimization

**Approach** - Using network-level optimization to coordinate legacy devices and smart inverters and meet objectives of conservation voltage reduction.

- Mathematically problem is formulated as mixed integer nonlinear program (MINLP)
- Mixed integer due to discrete and continuous variables and nonlinear due to three-phase nonlinear power flow.

**Level 1: Mixed integer linear programming (MILP)**

Objective function: \( \text{min} \sum_{p \in \phi_s} P_s^p(t) \)

Subject to: linear power flow, voltage regulator control, capacitor bank control, voltage limits, and reactive power limits on DGs

*Control variables:* Regulator tap \((A_p(t))\), capacitor \((u_i(t))\) and DG reactive power \((Q_{DG})\)

Fix the status of Regulator tap \((A_p(t))\), capacitor switch \((u_i(t))\)

**Level 2: Nonlinear programming (NLP)**

Objective function: \( \text{min} \sum_{p \in \phi_s} P_s^p(t) \)

Subject to: non-linear power flow, voltage limits, and reactive power limits on DGs

*Control variables:* reactive power from DG \((Q_{DG})\)

Application 1: Volt-VAR Optimization: Results

IEEE-123 bus system

- Total number of single-phase nodes in this unbalanced system = 267
- Control Devices: 4 Voltage regulators, 4 capacitor banks, and 9 distributed generators

- Without CVR control
  - VR and capacitor banks are working autonomously and PVs operating at unity power factor
- With CVR Control
  - Reduction in power consumption from substation
  - Higher reduction in power demand at minimum loading condition
Application 1: Volt-VAR Optimization: Results

PNNL-329 bus system

- Total number of single-phase nodes in this unbalanced system = 860
- Control Devices: 1 Voltage regulator, 4 capacitor banks, and 9 distributed generators
Application 2: Topology Estimation - Problem Statement

- To estimate the operational topology given planning model
- Identify topology: normal & outaged
- Outaged topology: not fault location

Planning model

Tree-1

Tree-2

Sub-tree

Application 2: Topology Estimation

**Approach** - Use network-level optimization to estimate the switch statuses using erroneous measurements.

- Formulated as an estimation problem
- Measurements used flow, load, and smart meter ping measurements

\[
\begin{align*}
\text{Minimize} & \quad \sum_{\varphi \in \{a,b,c\}} \left( \sum_{j \in I} \left| \hat{s}_{Lj}^{\varphi} - s_{Lj}^{\varphi} \right| \sigma_{s_{Lj}^{\varphi}} + \sum_{ij \in B} \left| \hat{s}_{ij}^{\varphi} - s_{ij}^{\varphi} \right| \sigma_{s_{ij}^{\varphi}} \right) \\
\end{align*}
\]

- Error in load variables and load measurements
- Error in flow variable and flow measurements

**Measurement Error Model**

- Errors in continuous measurements (flow and load):
  \[
e \left( \hat{p}_{ij}^{\varphi} \right) \sim N \left( 0, \sigma_{p_{ij}^{\varphi}}^2 \right) \quad \text{and} \quad e \left( \hat{q}_{ij}^{\varphi} \right) \sim N \left( 0, \sigma_{q_{ij}^{\varphi}}^2 \right)
\]
- Errors in Smart Meter Ping Measurements (discrete):
  \[
e(\hat{y}_j) = (\hat{y}_j - y_l) \sim \text{Bernoulli}(q)
\]
  \[
q = P \left( (\hat{y}_j - y_l) = 1 \right)
\]
- Sum of Errors in Smart Meter Ping Measurements (Gaussian Approximation)
  \[
S_{np} \sim B \left( n_p, q \right) \rightarrow S_{np} \sim N \left( \mu_e, \sigma_e \right)
\]
  \[
\mu_e = n_p q \quad ; \quad \sigma_e = \sqrt{n_p q (1 - q)}
\]
Application 2: Topology Estimation - Results

Model Characteristics:
- Four-feeder 1069 nodes
- Each feeder:
  - 40 sectionalizing switches
  - Three $1-\phi$ and one $3-\phi$ Capacitor
- Seven tie-switches
- Large number of possible operating topologies

Given planning model

Solve power flow (OpenDSS)

Add $N(0, \sigma)$ & $B(n_p, q)$ errors

\[ \%err = \frac{3\sigma}{\mu} \times 100 \]

Get real-time measurements

Estimate operational topology (MATLAB)

Calculate Metrics
Application 2: Topology Estimation - Results

%MDR, %MMS and %MMO for tested topologies with outages

<table>
<thead>
<tr>
<th>% error in $\gamma_i^0$</th>
<th>% error in load measurements</th>
<th>%MDR</th>
<th>%MMS</th>
<th>%MMO</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>0.53</td>
<td>2.80</td>
<td>7.10</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>0.91</td>
<td>4.35</td>
<td>7.23</td>
</tr>
</tbody>
</table>

MDR - Missed Detection Ratio (MDR)
MMS - Mean Missed Switches (MMS)
MMO - Mean Missed Outages (MMO)

Most of the misdetections are only due to one switch pair wrongly detected

Frequently misdetected load sections: frequently repeated and supplied by same upstream meter
Application 3: Resilient Restoration (Problem)

Service Restoration
- Transmission and distribution network remain intact
- Single fault due to component failure
- No stochastic feature involved in general analysis
- DERs and backup feeders can be utilized for supplying the outaged loads
- Quickly repair and restore

Restoration during Extreme Events
- Distribution feeder disconnected from main grid
- Distribution system itself under stress
- Restore critical loads as soon as possible
- DERs can be utilized for supplying the CLs
- Combinatorial problem – large number of options to restore the network using all available resources

Power distribution system restoration using backup feeders and available DER. The out-of-service area is restored with suitable switching scheme after the fault has been isolated.
Application 3: Resilient Restoration - Approach

**Approach** - Use network-level optimization to maximize the total load restored using all available resources: feeders, DGs (intentional islanding), smart switches, legacy voltage control devices.

- **Effective Restoration unavailability**
- **Maximize critical load restoration**

1. **Graphical Constraints**
   - Nodal
   - Radial topology
   - Line and switch unavailability

2. **Operational Constraints**
   - Power balance
   - Voltage
   - DER Capacity

Application 3: Resilient Restoration - Results

- DGs and normally-open switches in coordination help to restore additional loads during faults

- How to effectively use them in extreme events to restore critical loads??

Without DGs

Load shedding of 654 KW

With DGs

All loads restored
**Application 3: Resilient Restoration - Results**

### Extreme event case:
- Main grid not available and several faults within distribution network
- Four utility-owned DGs (one in each feeder)
  - Grid-forming technology
- Several customer-owned DGs
  - Grid-following technology
  - Help offset generation from utility-owned DGs

### Customer-owned DGs in improving restorability

<table>
<thead>
<tr>
<th>Penetration level</th>
<th>0%</th>
<th>10%</th>
<th>20%</th>
<th>50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total buses with DG</td>
<td>0</td>
<td>24</td>
<td>48</td>
<td>124</td>
</tr>
<tr>
<td>Total peak load of buses with DG</td>
<td>-</td>
<td>1442.7</td>
<td>2860.0</td>
<td>7077.8</td>
</tr>
<tr>
<td>Total DG capacity (PV+batt.)</td>
<td>-</td>
<td>721.35</td>
<td>1430.2</td>
<td>3538.9</td>
</tr>
<tr>
<td>Average 95th percentile</td>
<td>-</td>
<td>1269.6</td>
<td>2516.8</td>
<td>6228.5</td>
</tr>
</tbody>
</table>

**Grid-forming DGs increasing their zone of service to neighboring feeders with help of tie switches**

![Diagram showing load restoration results for different scenarios and DG penetration levels.](image_url)
Demonstration using GridAPPS-D Platform

- GridAPPS-D - an open-source, standards based ADMS application development platform
  - Developed for the U.S. Department of Energy’s Advanced Distribution Management System (ADMS) Program by the Grid Modernization Laboratory Consortium project GM0063
  - Provides a method for developers to run their new applications on a real-time simulator with extensive modeling and tool support.

Future Research Directions

**Challenges not addressed in this work**

*Large-scale engineered systems with time and space fragmented control and an increased level of variability and uncertainty from DERs*

- Challenging to solve operational problem for a large-scale optimization problem (mostly non-convex and with mixed-integer) in a stochastic setting for a three-phase unbalanced system.

*Poor situational awareness due to limited measurement and sensing devices and inaccurate or unknown physical system planning and operational model along with noisy and compromised heterogeneous measurements:*

- Simultaneously handling measurement and model uncertainty and noisy and compromised measurements.
Future Research Directions

• For optimal distribution system operations, mostly model-based methods have also emerged in past few years.

  Why not simply learn optimal decision from measurements?

• Completely model-free methods have also been proposed for estimation, control, and optimization.

• However, these do not generalize well.

Future Research Directions

A combination of data-driven and physics-based model that can learn probabilistic relationships within observed and unobserved variables using measurement set and physics-based models.
Further Information

• Website: https://eecs.wsu.edu/~adubey/index.html
• Research Projects: https://eecs.wsu.edu/~adubey/research.html
• Publications: https://eecs.wsu.edu/~adubey/papers.html

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Thank you

Questions?

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Additional Slides
Device Models

Voltage regulator

A 32-step voltage regulator:

\[ V_j^p = V_i^p = a^p V_i^p, \quad I_{i'i'^j}^p = a^p I_{i'i'^j}^p \]

\[ a^p = \sum_{j=1}^{32} b_j x_j \quad A_p = a_p^2 \]

\[ v_j^p = A_p v_i^p \quad (v_i^p = V_j^p^2) \]

\[ A_p = \sum_{j=1}^{32} b_j^2 x_j \quad (x_j \in \{0,1\}) \]

\[ \sum_{j=1}^{32} x_j = 1 \text{ (Voltage Regulator tap position)} \]

where, \( b_j \in \{0.9, 0.92, \ldots \ldots \ldots 1.1\} \)

Load model

- Voltage dependent load model is derived using the CVR factor

\[ CVR = \frac{\% \text{ reduction of } P/Q}{\% \text{ voltage reduction}} \]

\[ p_{Li}^p = p_{i0}^p + CVR_p \frac{p_{i0}^p}{2}(v_i^p - V_0^2) \]

\[ q_{Li}^p = q_{i0}^p + CVR_q \frac{q_{i0}^p}{2}(v_i^p - V_0^2) \]

CVR factor can be obtained using the ZIP model

Capacitor banks

- Reactive power support is constant

\[ Q_i = u_i v_i Q_C \]

where, \( u_i \in \{0,1\} \), status of capacitor bank at node \( i \)

DERs/Smart Inverters

- Reactive power support depends on rating of DGs

Mathematically:

taking, \( S_i = 1.15 * P_{i,rated} \)

\[ Q_i = \pm \sqrt{S_i^2 - P_i^2} \]

\( Q_i \) is the control variable for the optimization
Power Flow with Switch Model

- Linear real and reactive power flow function of switch status

\[ \sum_{i \rightarrow j \in E} \delta_{ij} P_{ij} = s_j P_{Lj} + \sum_{(j \rightarrow c) \in E, i \neq c} \delta_{jc} P_{jc} \]

\[ \sum_{i \rightarrow j \in E} \delta_{ij} Q_{ij} = s_j Q_{Lj} + \sum_{(j \rightarrow c) \in E, i \neq c} \delta_{jc} Q_{jc} \]

\[ \delta_{ij} (U_i - U_j) = 2(\bar{r}_{ij} P_{ij} + \bar{x}_{ij} Q_{ij}) \]

- Switch open or close status decides which radial configuration to operate
  - Power flow in open switch = 0
  - Voltage drop equations along the closed switch only

**Tree 1**

\[ P_{47} = Q_{47} = 0 \]

\[ U_7 = \mathcal{F}(U_6, P_{67}, Q_{67}, r_{67}, x_{67}) \]

**Tree 2**

\[ P_{67} = Q_{67} = 0 \]

\[ U_7 = \mathcal{F}(U_4, P_{47}, Q_{47}, r_{47}, x_{47}) \]
Integration with PNNL’s GridAPPS-D Platform:
Restoration of Power Distribution Network

IEEE 123-bus test case

Fault at line-72

Integration in Platform

Fault isolation and restoration

IEEE 123-bus test case simulated in Platform