



A Framework for Transmission Planning Under Uncertainty

Final Project Report

Power Systems Engineering Research Center

*Empowering Minds to Engineer
the Future Electric Energy System*



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PSERC Publication 15-03

September 2015

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Acknowledgements

This is the final report for the Power Systems Engineering Research Center (PSERC) research project titled “A framework for transmission planning under uncertainty” (project M-30). We express our appreciation for the support provided by PSERC’s industry members and by the National Science Foundation under the Industry / University Cooperative Research Center program.

We wish to thank our industry advisors, who have provided strong support and constructive feedback to our work:

- Aftab Alam, CAISO
- Bryce Bowie, SPP
- Jay Caspary, SPP
- Juan Castaneda, SCE
- Flora Flygt, ATC
- Anish Gaikwad, EPRI
- Shih-Min Hsu, Southern Co.
- Anil Jampala, Alstom
- Murali Kumbale, Southern Co.
- David Mindham, ITC Holdings
- Kip Morison, BC Hydro
- Aditya Jayam Prabhakar, MISO
- Jim Price, CAISO
- Curtis Roe, ATC
- Harvey Scribner, SPP
- Hussam Sehwal, ITC Holdings
- Robert Sherick, SCE
- Michael Swider, NYISO
- Mark Westendorf, MISO
- Lan Trinh, ABB
- Feng Zhao, ISO-NE

Executive Summary

The current transmission planning practice in the electric power industry is mainly based on the deployment of deterministic techniques. However, transmission planning, by its very nature, is faced with a wide range of sources of uncertainty, including growth in demand, renewable energy generation, fuel price, environmental requirements and legislation and new generation investment, to name just a few. In addition, the restructuring of the electric power industry, the drive for energy independence and the push for a cleaner environment have led to additional sources of uncertainty in all aspects of power system operations and planning. The competitive electricity markets, the more decentralized decision making and the new federal and state initiatives introduce additional sources of uncertainty. A particularly good example is the FERC Order No. 1000 requirements. The analytic characterization of the various sources of uncertainty is often a challenge and, typically, cannot be expressed in terms of probability distributions. Past data may allow the estimation of the ranges of values that the uncertain variables may attain so as to make possible the deployment of robust optimization approaches. We make use of such approaches to develop a decision-support system for transmission planners to allow the explicit consideration of uncertainty in the formulation of transmission plans.

We discuss our studies of two optimization criteria for the transmission planning problem with a simplified representation of load and the forecasted generation investment additions within the robust optimization paradigm. The objective is to determine either the minimum of the maximum investment requirement or the maximum regret with all sources of uncertainty explicitly represented. In this way, transmission planners can determine optimal planning decisions that are robust against all sources of uncertainty. We use a two-layer algorithm to solve the resulting tri-level optimization problems. We also construct a new robust transmission planning model that considers generation investment more realistically to improve the quantification and visualization of uncertainty and the impacts of environmental policies. With this model, we can explore the effect of uncertainty in both the size and the location of candidate generation additions. The corresponding algorithm we develop takes advantage of the structural characteristics of the model so

as to obtain a computationally efficient methodology. The two robust optimization tools provide new capabilities to transmission planners for the development of strategies that explicitly account for various sources of uncertainty.

We illustrate the application of the two optimization models and solution schemes on a set of representative case studies. These studies give a good idea of the usefulness of these tools and show their practical worth in the assessment of “what if” cases. We compare the performance of the minimax cost approach and the minimax regret approach under different characterizations of uncertain parameters. In addition, we also present extensive numerical studies on the IEEE 118-bus test system and the WECC 240-bus system to illustrate the effectiveness of the proposed decision-support system. The case study results are particularly useful to understand the impacts of each individual investment plan on the power system’s overall transmission adequacy in meeting the demand of the trade with the power output units without violation of the physical limits of the grid.

The decision-support system consisting of the two optimization models and solution approaches has wide applicability to transmission planning studies. The so-called minimax criterion models determine the set of planning decisions that result in the least-cost or the least-regret solution with the explicit consideration of uncertainty during the planning horizon. The set of planning decisions are optimal under the ranges of uncertainty given for the uncertain variables. The ability to represent the uncertainty in the investment decisions for generation addition together with the uncertainty in the retirement of units in the resource mix and the environmental regulatory requirements is a major development for the tools available to transmission planners.

Project Publications:

- Bokan Chen, Jianhui Wang, Lizhi Wang, Yanyi He, and Zhaoyu Wang, “Robust Optimization for Transmission Expansion Planning: Minimax Cost vs. Minimax Regret”, *IEEE Transactions on Power Systems*, vol. 29(6), p. 3069-3077, 2014.

- Dongbo Zhao, Sakis Meliopoulos, Zhenyu Tan, Rui Fan, "Probability State Sequence Method for Reliability Analysis of Wind Farms Considering Wake Effect," 45th North American Power Symposium (NAPS), KS, 2013.

Student Theses:

- Bokan Chen, "Three essays on the application of robust optimization to power system planning problems," Ph.D. dissertation, Iowa State University, expected September 2016.
- Dongbo Zhao, "Reliability modeling and analysis of wind turbine systems and wind farms in bulk power systems," Ph.D. dissertation, Georgia Institute of Technology, July 2015.

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1 Introduction

Transmission planning (TP) is one of the key decision processes in the power system production pipeline. Electricity transmission networks are responsible for reliably and economically delivering power from generators to consumers. Thus, a robust and resilient transmission network is essential to the operation of power systems for decades to come. Good transmission investments have many benefits, including satisfying increasing demand, promoting social welfare, improving system reliability and resource adequacy, etc. Transmission planning is also very challenging due to various sources of uncertainty that planners need to consider. Besides uncertainty sources, such as demand variations and renewable energy intermittency faced by system operators in short-term scheduling, planners also need to take into consideration uncertainty of policy changes, technological advancements and natural disasters [35]. Such sources of uncertainty cannot be characterized in terms of analytical probability distributions. For example, to cope with challenges of climate change, the power generation industry is facing increasing pressure to reduce greenhouse gas emissions. In addition, a large amount of coal plants are anticipated to retire in response to the implementation of the EPA clean power plan [8], and their replacement in part by gas-fired plants. Moreover, after the restructuring of the power system, certain behavior by generation companies introduces additional uncertainty in power system planning in general and TP in particular. However, research on the impacts of uncertain future generation developments on TP is scarce in the decentralized decision making environment in competitive market. As such, the study of this topic is warranted.

A salient need in TP is to explicitly represent the broad range of uncertainty associated with the ramifications of future generation resource investments. Such a representation is critically important in the formulation of the transmission investment decisions and leads to the additional complications in what is already a very complex decision-making process. In this report, we provide a detailed summary of the methodology we developed to address all these concerns in TP. We discuss its application to realistic power system test cases to gain new insights into the formulation of transmission investment decisions under uncertainty. We devote the remainder of

this section to present the context within which our work is developed, provide a brief survey of the state of the art in TP methodologies, and delineate the key feature of the methodology developed.

1.1 Need for robust investment strategy in transmission planning

Conventional transmission planning methodologies are typically deterministic and some make use of ad-hoc approaches to deal with uncertainty. These methodologies have served the industry relatively well in the vertically-structured industry. Present realities brought about, by the open access regime and the advent of competitive electricity markets, increased wind and solar resource outputs. Their highly time varying, uncertain and intermittent patterns, combined with the more dynamically varying and uncertain loads and a world-ranging environmental policy initiative have resulted in a more volatile utilization of transmission facilities. Critically needed is the improved planning methodologies that can effectively accommodate the realities of the new environment. Transmission planning, by its very nature, is subject to a wide range of sources of uncertainty that must be taken into account. The new regime indicates many additional sources of uncertainty and numerous complications that must be considered.

Over a 10-20 year planning horizon, major sources of uncertainty faced by transmission planning decision makers include load growth, fuel costs, climate change events, atmospheric conditions, variable generation outputs, generation expansion/retirement events, regulatory and legislative developments, technology breakthroughs and market outcomes. When the impacts of transmission plans are assessed, such studies must be carefully performed with the explicit representation of the sources of uncertainty, in light of the overarching consequences on the power system. These sources of uncertainty may be classified into two categories: aleatoric and epistemic sources of uncertainty. Sources of aleatoric uncertainty cannot be replaced by more accurate measurements but they can be statistically quantified. For instance, the coincidental uncertainty with respect to the electrical system state and network topological state can be both probabilistically and stochastically modeled and quantified if the relevant sets of input are made available. On the other hand, sources of epistemic uncertainty are due to information we may in principle know,

but do not in actual practice. Often, such sources are also called sources of systematic uncertainty. For instance, it is hard to assign meaningful probabilities to sources of uncertainty associated with government policies, technology breakthroughs, and investment in renewable energy generation.

Electricity demand is expected to grow continually and steadily for decades to come. In the EIA Annual Energy Outlook 2013, forecasts indicate that electricity consumption will increase by 28% from 2011 to 2040 at an average of about 1% per annum rate [11]. To accommodate such growth in electricity demand, new capacity of electricity generation must be built at a pace that meets or exceeds the growth of demand and retirement of old generators to maintain power system reliability. The additional demand and capacity require commensurate transmission capacity.

With the additional sources of uncertainty and complications facing transmission planners and the need to accommodate increasing demand as well as more variable generation capacity, there is a critical need for new TP methodologies that can address the aforementioned concerns.

1.2 Review of the literature

The most common practices to manage uncertainty in optimization include stochastic programming and robust optimization. In stochastic programming, scenarios are formulated based on an estimated probability distribution of the uncertain data. The weighted sum of the total costs under different scenarios form the typical objective function in the optimization. Stochastic programming has been successfully applied to power system capacity expansion planning problems in [13, 22, 24, 33]. Sources of uncertainty represented in the cited references include load prediction inaccuracies, transmission and generator outages, and generation capacity factors. All of those sources of uncertainty can be expressed analytically as probability distributions and may be effectively modeled in stochastic programming. However, the literature focus primarily on sources of uncertainty in system operations context and ignore sources of uncertainty in system planning. The reason is that it is difficult to obtain probability distributions of certain sources of uncertainty [7], such as policy changes and investment behavior of market players. In this project, in addition to demand uncertainty, we also take into consideration uncertain generation expansion behavior of

resource investment companies and the timing and size of coal power plant retirements.

In addition to using stochastic programming to manage operation level uncertainty in TP, some literature uses scenario analysis to address uncertain events such as policy mandate and new investment in generation capacity. In scenario analysis, multiple transmission plans are developed based on different scenarios and common investments under most scenarios are adopted [26]. However, as shown in [30], such approaches can be distinctly different than the optimal solution of stochastic optimization. The authors of [30] propose an approach to use the stochastic programming, where 3 scenarios of equal likelihood are constructed to describe the different policy mandates and fuel costs based on EIA forecasts and some educated assumptions. The optimal average costs under the three scenarios are determined. Generation investments are assumed to have the same objective as transmission investments in this paper. A similar approach is proposed in [7], where the authors construct eight scenarios. The disadvantages of this approach is that only a limited number of scenarios are explored. In addition, the constructed scenarios are more of a “big picture” rather than a detailed depiction of possible futures.

Another approach used to explore the impact of generation profile uncertainty on TP is based on game theoretic models to describe the strategic behavior of various market participants, including generation companies (GENCOs). A trilevel model is proposed in [31]. In the first level transmission investment decisions are made. In the second level, multiple GENCOs make generation investments in response to the transmission investment decisions taken. Operational decisions are made in the third level. Perfect information competition between GENCOs is assumed. A similar trilevel structure is proposed in [19] and [20], where a Cournot game between GENCOs is solved. Similar approaches are also used in [14, 29, 33]. Game theoretical models are useful in that they can provide us some insights into the behavior of different participants in the power market. However, some strong assumptions about the players are needed. For example, it is assumed that complete information is available to all the players. In addition, the high complexity of the models means that they are difficult to implement for large realistic-sized systems.

Given the limitations of the methods in the literature, there is need for practical approaches

to address the challenges faced by transmission planners in the current environment in the power electric industry. A promising approach to manage optimization under uncertainty is robust optimization [3,5,12]. Uncertainty parameters in robust optimization are described by parametric sets, which can contain any number of scenarios without specific knowledge of the probability distributions. Distribution-free description of uncertainty makes robust optimization more suitable to tackle sources of uncertainty whose probability distributions are difficult to obtain, including future generation expansion and retirement. Another feature of robust optimization is that it minimizes the objective value under the worst-case scenario (scenario with the largest objective value). Such a feature is desirable for a transmission planning problem with its risk-averse nature. Despite the advantages of robust optimization, its applications in TP is limited. In [16], a robust TP model with explicit representation of demand and renewable energy output uncertainty is proposed. The model is solved by a two-level Benders decomposition algorithm. In [34], the authors propose a similar model with the uncertainty in equipment outages also represented. Column and constraint generation¹ with Karush-Kuhn-Tucker (KKT) optimality conditions reformulation of the bilevel subproblem is used to solve the model. Robust optimization is also used in [28] with the explicit representation of the system with $n - k$ contingencies for transmission planning. The cited references are limited to the operational level sources of uncertainty and fail to represent the uncertainty in today's system with increasing penetrations of renewable resources, competitive conditions, and rapidly changing policies.

1.3 FERC Order No. 1000 impacts on transmission planning

The FERC decision embodied in its Order No. 1000² is viewed as a watershed event from the transmission planning and investment point of view. Prior to its issuance, transmission planning

¹With the column and constraint generation, new variables and constraints are added to the master problem to improve the objective value at each iteration

²FERC, *Final Rule, Order No. 1000, Transmission Planning and Cost Allocation by Transmission Owning and Operating Public Utilities*, FERC Stats. & Regs., Issued July 21, 2011. Available online at: <http://www.ferc.gov/whats-new/comm-meet/2011/072111/E-6.pdf>

included many sources of uncertainty³ and the process was governed by the principles laid out in Orders No. 888 and No. 890⁴. FERC Order No. 1000 puts in place a number of mandates⁵ that require—directly and indirectly—transmission planners to integrate additional sources of uncertainty into the planning process and that increase the scope and computational requirements of the planning problem. Specifically, the Order requires planners to:

- explicitly consider transmission needs driven by public policy requirements in local and regional planning;
- explicitly consider non-transmission solutions in regional transmission plans;
- cooperate with neighboring transmission regions⁶ to develop interregional plans; and
- explicitly define a transmission project cost allocation methodology for projects included in regional and interregional plans that satisfies the six allocation principles defined in the Order.

The following paragraphs describe the additional sources of uncertainty introduced by these requirements and also discusses their impacts on the computational cost to solve the transmission planning problem.

Planners must explicitly consider transmission needs driven by public policy requirements in local and regional planning

³In this section we describe emerging sources of uncertainty resulting from the implementation of FERC Order No. 1000. For examples of conventionally considered sources of uncertainty in transmission and generation planning see Ivey, M., et al. *White Paper on Accommodating Uncertainty in Planning and Operations. Consortium for Electric Reliability Technology Solutions (CERTS) Grid of the Future*, at 5-10. Available online at: <http://certs.lbl.gov/pdf/certs-uncertainty.pdf>

⁴FERC, *Final Rule, Order No. 888, Promoting Wholesale Competition Through Open Access Non-discriminatory Transmission Services by Public Utilities; Recovery of Stranded Costs by Public Utilities and Transmitting Utilities*, FERC Stats. & Regs., Issued April 24, 1996. Available online at: <http://www.ferc.gov/legal/maj-ord-reg/land-docs/order888.asp>; FERC, *Final Rule, Order No. 890, Preventing Undue Discrimination and Preference in Transmission Service*, FERC Stats. & Regs., Issued February 16, 2007. Available online at: <http://www.ferc.gov/whats-new/comm-meet/2007/021507/E-1.pdf>

⁵We focus on those elements of the order that introduce additional uncertainty into planning. For a detailed synopsis of all of the requirements of FERC Order No. 1000, see, for example, Davis, T. *FERC's Regional Transmission Policy Takes Shape*, *The Electricity Journal*, Volume 26, Issue 7, August 2013, Pages 22-32.

⁶The Order does not, however, require coordination between interconnections, though the Commission encourages such coordination.

The commission requires planners take stakeholder input as to public policy requirements that are appropriate to consider in the planning process and that drive additional transmission needs. Such policies include any that can reasonably be shown to impact the electricity supply and transmission system, such as renewable portfolio standards (RPS), carbon pricing or taxation, and more stringent pollutant emission standards.

The inclusion of public policy requirements introduces a significant source of uncertainty into the planning process. For example, the introduction of a national RPS may seriously increase the need for large-scale transmission to bring wind generated electricity and other renewables generation from areas of abundant renewable resources to load centers. However, the amount of renewable generation required by the RPS and the timeline for its implementation will have a large impact on the magnitude, locations, and timelines for development of potential transmission projects needed to meet the RPS requirements. The impacts of uncertainty associated with policy design and implementation translate into uncertainty in the planning process. Such a statement is equally true for carbon pricing or more stringent emission requirements that may impact the construction and retirement of coal-fired generation and therefore add to the uncertainty of future power flows in the transmission system.

Planners must explicitly consider non-transmission solutions in regional transmission plans

Non-transmission solutions include generation, demand response, energy efficiency, energy storage, or other resource that can contribute to ensure that the supply-demand balance is met economically and reliably within the physical constraints of the transmission system. Non-transmission solutions may substitute for transmission line additions/modifications by provision of congestion relief or counter flows so as to increase the grid's available transfer capability. Non-transmission solutions introduce additional sources of uncertainty, such as the expected level of activity by demand response resources, and increase the impacts of the conventional sources of uncertainty, such as future fuel prices and the timelines for generator interconnection and retirement/decommissioning, as well as the investment costs, which must be taken into account. Furthermore, the inclusion of non-transmission alternatives in transmission planning increases the scope of the transmission

planning problem and as a result increases the computational burden of solving the problem.

Planners must cooperate with neighboring transmission regions to develop interregional plans

Many public utility transmission providers currently undertake planning limited to the geographic scope of their jurisdiction. FERC Order No. 1000 puts in place the requirement that public utility transmission providers form planning regions, consisting of multiple transmission providers. The Order extends the nine principles of planning enumerated in Order No. 890 to both regional and interregional planning activities⁷.

Interregional planning will drive the need for a standard set of metrics to assess the regional benefits of transmission projects. The assessment of any specified set of metrics requires treatment of conventionally considered sources of uncertainty and those introduced by the Order. Furthermore, an increase in the geographic scope of transmission studies brings with it increased computational requirements.

Planners must explicitly define a transmission project cost allocation methodology for projects included in regional and interregional plans that satisfies the six allocation principles defined in the Order

FERC Order No. 1000 requires cost allocation methodologies adhere to the following six principles:

- the costs are allocated in a way that is roughly commensurate with the benefits;
- there is no involuntary allocation of costs to non-beneficiaries;
- projects must adhere to a specified benefit to cost threshold ratio;
- allocation must be solely within transmission planning region(s) unless those outside entities voluntarily assume costs;
- there must be a transparent method to determine benefits and to identify beneficiaries;
- different allocation methods may apply for different types of facilities.

⁷The nine principles are coordination, openness, transparency, information exchange, comparability, dispute resolution, regional coordination, economic planning studies, and cost allocation.

The introduction of an explicit requirement to connect the allocation of project costs with the distribution of the benefits derived from the project drives the need for more specific definitions of the benefits of transmission investments⁸ and methods to quantify those benefits and the associated uncertainties. For example, whether or not a project will be eligible for regional cost allocation depends on its ability to meet the cost to benefit ratio principle. The satisfaction of this threshold depends on which benefits can be and are quantified and the range of these benefits given the level of uncertainty in their quantification.

The explicit representation of the additional sources of uncertainty and associated computational requirements in transmission planning processes poses a key challenge to planners in the implementation of the Order.

Under FERC Order 1000, transmission planners are forced to take a broader view than ever before of the transmission needs of the system and how those needs can and are met. Specifically, planners must consider system transmission needs driven not simply by reliability and economic considerations, but also by public policy requirements. This new requirement represents a significant break from conventional planning practices and are likely to have significant ramifications on the integration of renewables and on the system resource mix in the implementation of the Order. Further, the Order impact transmission planners as they update and expand planning processes and tools to include the new requirements. The development of computationally efficient approaches to integrate the sources of uncertainty introduced by the Order into existing planning tools is a key requirement for the successful implementation of the Order. Similar statements hold for generation planners for the EPA Clean Power Plant rules as the coal plant retirements have to proceed on an earlier time line than anticipated.

Moreover, the Order broadens the scope of the potential portfolios to meet the system's transmission needs to include non-transmission solutions, such as generation and demand response. The consideration of a wider range of resources to meet the identified system transmission needs

⁸The definition of the various components of the benefits of building additional transmission are described in detail in Chang, J., et al. *The Benefits of Electric Transmission: Identifying and Analyzing the Value of Investments, Prepared for WIRES by The Brattle Group, July 2013*. Available online at: <http://cleanenergytransmission.org/uploads/WIRES%20Brattle%20Rpt%20Benefits%20Transmission%20July%202013.pdf>

increases the complexity of the planning problem but also provides additional degrees of freedom to transmission planners to ensure the system has adequate transmission transfer capability. The transmission planning process envisioned by the Order is similar in nature to the conventional process of integrated resource planning (IRP) practiced by some planning entities today⁹. Planners can look to the successes and challenges of IRP to inform the effective implementation of Order No 1000.

The identification of the transmission needs of the system and a broad range of resources to meet those needs is accompanied by the requirement to select a portfolio for development based on demonstrated benefits. Indeed, FERC Order No. 1000-A requires planners to be “definite” about the benefits and beneficiaries of transmission projects so as to provide the impetus for the development of methodologies and tools to quantify the various components of transmission investment benefits¹⁰.

Furthermore, the regional cooperation requirements in the Order emphasize the importance of data management and data sharing between transmission planning entities. With expanded geographic scope and integration of additional sources of uncertainty, the data needs of transmission planning increase considerably. Successful interregional planning can be accomplished only if accompanied by extensive interregional data coordination.

In summary, transmission planning has undergone a significant shift with the issuance of FERC Order No. 1000. As transmission planners adapt their processes and develop, where necessary, new processes to meet the Order’s new requirements, there is a need and ample opportunity to apply scenario-based and probabilistic approaches for integrating conventionally considered and emerging sources of uncertainty.

⁹For an overview of IRP and some recent examples of utility implementation see, *Wilson, R. and Biewald, B. Best Practices in Electric Utility Integrated Resource Planning: Examples of State Regulations and Recent Utility Plans. Prepared for The Regulatory Assistance Project by Synapse Energy Economics, June 2013.* Available online at: <http://www.synapse-energy.com/Downloads/SynapseReport.2013-06.RAP.Best-Practices-in-IRP.13-038.pdf>

¹⁰*FERC, Final Rule, Order No. 1000-A, Transmission Planning and Cost Allocation by Transmission Owning and Operating Public Utilities, FERC Stats. & Regs., Issued May 17, 2012.* Available online at: <http://www.ferc.gov/whats-new/comm-meet/2012/051712/E-1.pdf>

1.4 Motivation

We propose to develop a new decision-support system for transmission planning with explicit consideration of various sources of uncertainty. As a starting point, we will specify a set of representative requirements. The salient features of the decision-support system are its two interrelated elements: (I) a set of appropriate performance metrics to quantify the economic costs, benefits, reliability and environmental attributes of a transmission plan and (II) a set of models and analytical techniques to formulate optimal transmission planning decisions under the explicit representation of various sources of uncertainty and their impact.

In this project, we make detailed applications in robust optimization to construct models to solve the transmission planning problem considering generation investment uncertainty. Firstly, we compare two optimization criteria – minimax cost and minimax regret, under the robust optimization paradigm. Demand and generation expansion uncertainty is considered, where we assume that generation capacity at specific locations is in a polyhedral set. The models are applied to an IEEE 118-bus system. Then, we modify the robust transmission planning model to consider generation expansion uncertainty in more detail. The structure of our new model is similar to the models in Section 2.1. However, a few key aspects are different, including uncertainty modeling, number of decision time points and solution algorithm. Table 1 summarizes the comparison between our new model and some previous works.

1.5 Contributions of this project

In this project, we develop a decision-support system for transmission planning under uncertainty. In this system, we propose two models under the robust optimization paradigm to take into consideration multiple sources of uncertainty facing transmission investors. We provide two optimization criteria, minimax regret and minimax cost, for Model A. Planners have the freedom and flexibility to choose the appropriate criterion based on their beliefs on the levels of uncertainty. Uncertainty in both the loads and the future generation investment is considered. Effective algorithms are then proposed to solve the resulting trilevel optimization problems. We then applied the models and

Table 1: Comparison of the proposed model with literature

Reference	Objective	Generation investment	Uncertainty	Model	Solution Method	Case Study	# periods
[16]	minimax cost	none	load, renewable intermittency	RO	BD	IEEE 118-bus	1
[30]	average cost	centralized	regulatory, market conditions	SP	MIP	WECC 240-bus	3
[31]	costs	player	load	game	MIP,KKT	SIC 33-bus	1
[34]	minimax cost	none	load, renewable intermittency, equipment failure	RO	C&CG, KKT	RTS 24-bus	1
[28]	minimax cost	none	$n-k$ contingency	RO	C&CG, BD	IEEE 300-bus	1
[9]	minimax cost, minimax regret	uncertainty	Load, generation capacity	RO	C&CG, KKT	IEEE 118-bus	1
Our model	minimax cost	uncertainty	generation investment	RO	C&CG, dual	WECC 240-bus	4

RO: robust optimization. BD: Benders decomposition. SP: stochastic programming. MIP: mixed-integer programming. C&CG: column and constraint generation

algorithms to an IEEE 118-bus test system to illustrate the effectiveness of the decision-support system. The results are analyzed to compare the performances of the transmission plans devised by our model with different optimization criteria under different scenarios.

To provide a modeling framework that enables more detailed representation of generation expansion and retirement uncertainty, we also provide another model in our decision-support system. With this new model, the uncertainty set of our robust transmission planning model can describe uncertainty in not only the capacity of generators but also in their locations. In addition, because of the special structure of our new model, we are able to devise a solution method for the trilevel problem without the need to directly solve the KKT conditions, making the application to larger-sized systems more practical. We applied the model and algorithm on a WECC 240-bus test system and compare the performance of the transmission plans yielded by our decision-support system with the performance of plans devised by heuristic rules under multiple scenarios with different natural gas price and renewable energy policy mandate. The results show that the investment plans our decision-support system yields not only have superior performance across different scenarios, but also are very robust under natural gas price and renewable energy penetration data variations.

The remaining sections of this report are the outline of the report. We provide a detailed

description of the modeling framework in Sections 2.1 and 2.2, including the model assumptions and the formulations. Section 3 reports on the construction of the solution schemes customized for our specific models. Section 4 reports the results of our representative case studies on the IEEE 118-bus and the WECC 240-bus test systems. The case studies illustrate the ability of the project to devise robust candidate investment plans under different sensitivity cases. We provide summary remarks in Section 5 and restate the key findings and conclusions. We also discuss directions for future work in the same section.

2 Model formulation

Transmission planning problems are usually modeled as two-stage problems to account for the long planning horizon, where the transmission planning decisions are made in the first stage when there is limited information on uncertain parameters and the operational decisions are made in the second stage after uncertainty realizations are observed. In this section, we first present the model with the simplified load and generation uncertainty representation. Then we introduce the model with the more realistic generation uncertainty representation.

2.1 Model A: Simplified load and generation uncertainty

2.1.1 Nomenclature

Sets and indices

- U The polyhedron uncertainty set of demand and new generation capacity profile
- V Set of nodes
- L Set of existing transmission lines
- N Set of candidate transmission lines
- T Set of years in the planning horizon
- M Set of load blocks
- K Set of technology types

Parameters

- $P_{i,k,t}$ Capacity of existing generator of technology k at node i at time t
- $c_{ij,t}^T$ Cost of building the new transmission line ij at time t
- $c_{i,t}^L$ Cost of load curtailment at node i at time t
- $c_{i,k,t}^P$ Cost of power production of technology k at node i

$f_{k,t,m}^C$	Average capacity factor of generation technology k at year t load block m
F_{ij}^{\max}	The maximum power flow on transmission line ij
B_{ij}	Susceptance of transmission line ij
M	A big constant used to linearize the power flow constraint
$\bar{d}_{i,t,m}$	The average amount of demand at year t load block m at node i
$\bar{p}_{i,k,t}^N$	The average amount of generation expansion of technology k at node i at time t
q_{\min}	The lower bound of voltage angles
q_{\max}	The upper bound of voltage angles
I	Market interest rate (Inflation included)
$p_{i,k,t}^{N,\min}$	Minimum amount of new generation at a node
$p_{i,k,t}^{N,\max}$	Maximum amount of new generation at a node
$p^{N,\min}$	Lower bound on the total amount of generation at all the nodes
$p^{N,\max}$	Upper bound on the total amount of generation at all the nodes

Decision Variables

x_{ij}	Binary variables indicating whether a transmission line is built
$p_{i,k,t}^N$	The amount of new generation capacity of technology k at node i at time t . $p_{i,k,t}^N$ is negative in the case of power plant retirement
$f_{ij,t,m}$	Power flow from node i to node j at year t load block m
$p_{i,k,t,m}$	Power production of technology k at node i at year t load block m
$r_{i,t,m}$	The amount of load shedding at node i at year t load block m
$q_{i,t,m}$	Voltage angle at node i at year t load block m
$d_{i,t,m}$	Demand at year t load block m at node i

2.1.2 Overview

In this section, we propose a robust optimization approach to address two main sources of uncertainty: load and generation expansion behavior of generating companies. Two criteria, minimax cost (MMC) and minimax regret (MMR), are used as the objective of our models. The MMC

criterion has been used widely in robust optimization applications [4]. The MMR criterion is considered in [18] for the unit commitment problem. In comparison with the MMC criterion, it is concluded that MMR outperforms MMC for certain unit commitment problems. However, the same conclusion may not apply to transmission planning problems due to the different structures of such problems. In [25], regret is considered as one of the objectives in a multi-objective optimization framework. It is applied to handle non-random sources of uncertainty in [2, 10]. Both criteria use the performance of a decision under the worst possible scenario as the objective for optimization, but their main difference is how the “worst scenario” is defined. The MMC criterion focuses on the cost associated with a decision under a scenario, so the scenario that results in the highest cost is identified as the worst scenario. On the other hand, the MMR criterion defines the worst scenario as the one that leads to the highest regret for the decision maker. For a given decision d^0 and a given scenario s^0 , the regret is the highest potential cost savings had the decision maker known that scenario s^0 would occur and made a decision accordingly. More rigorously,

$$R(d^0, s^0) = C(d^0, s^0) - \min_{d \in D} C(d, s^0),$$

where $C(d^0, s^0)$ is the cost associated with d^0 and s^0 , D is the set of all feasible decisions, and $R(d^0, s^0)$ is the regret associated with d^0 and s^0 . Using these notations, the MMC and MMR criteria can be respectively formulated as

$$\begin{aligned} & \min_{d \in D} \max_{s \in U} C(d, s) \quad \text{and} \\ & \min_{d \in D} \max_{s \in U} R(d, s) = \min_{d \in D} \max_{s \in U} C(d, s) - \min_{d^0 \in D} C(d^0, s) \quad . \end{aligned}$$

We use two simple examples in Tables 2 and 3 to demonstrate the differences between MMC and MMR. In the first example, under MMC, D_2 is the optimal decision because its worst scenario

cost, \$8, is lower than that of D_1 , \$9. Under MMR, D_1 is the optimal decision because its worst scenario regret, \$1, is lower than that of D_2 , \$5. The argument for MMR is that since scenario S_1 is a “bad” scenario anyway because D_1 and D_2 both lead to higher costs in S_1 than S_2 , the difference between the costs associated with the two decisions, which is the regret, may provide more information for decision making than the absolute value of the cost itself. In the second

Table 2: Motivating example for the minimax regret model

Cost / Regret	Decision D_1	Decision D_2
Scenario S_1	\$9 / \$1	\$8 / \$0
Scenario S_2	\$2 / \$0	\$7 / \$5

example, decision D_3 is obviously a bad choice because of its high cost in scenario S_4 . Decision D_5 will be selected under MMC because its worst cost, \$18, is lower than that of D_3 , \$40, and D_4 , \$19. Under MMR, decision D_4 will be selected since its worst-case regret is \$13 while the regret of D_5 is \$14. We argue the MMC solution D_5 is better in this example because it is only slightly worse than the MMR decision D_4 in terms of regret in scenario S_3 only because of the existence of decision D_3 , which cannot be selected anyways, but has a much lower cost in scenario S_4 .

Table 3: Motivating example for the minimax cost model

Cost / Regret	Decision D_3	Decision D_4	Decision D_5
Scenario S_3	\$4 / \$0	\$16 / \$12	\$18 / \$14
Scenario S_4	\$40 / \$34	\$19 / \$13	\$6 / \$0

From the previous two examples, we can see that there are no clear cut answers as to which criterion is superior. Each of them has advantages and disadvantages. The examples above can shed some light on which criterion may perform better in what situations. In the first example, both decisions perform better in one scenario and worse in the other. In this case, it makes sense to use MMR as the criterion because the MMC criterion is too conservative and does not consider non-extreme scenarios. On the other hand, in the second example, there exists a very risky decision that performs well under one scenario and extremely poorly under the other. In a planning problem where risks should be controlled, such decisions are usually not desirable, but they may affect the maximum regret of other decisions and distort the final decision.

The two-stage structure of our robust optimization models can capture both the planning and operation stages of the transmission planning problem very well. They can be formulated as special cases of trilevel optimization problems. However, due to their non-linear, non-convex structure, they are very difficult to solve. In previous researches [4, 17], the authors use Bender's decomposition to reformulate the problem into a master problem and a bilinear subproblem, which is then solved with outer approximation. However, the outer approximation approach cannot handle the binary variables in the subproblem when the MMR criterion is used. In [18], statistical upper bounds are used to complement the outer approximation approach. We propose a two-layer algorithm where we decompose our problem into a master problem and a bilevel subproblem. The master problem is updated with a branch and cut type procedure, where new constraints and variables are iteratively generated and then solved as a mixed-integer program. This algorithm is a special case of the bilevel optimization algorithm [36]. Similar algorithms are proposed in [37, 38]. It works faster than the traditional Bender's decomposition approach with the use of primal information instead of dual variables. The subproblem is a mixed-integer bilevel optimization problem, which is more difficult to solve. In [27], the difficulty of solving a bilevel linear optimization program is discussed and several heuristics are proposed. We use the Karush-Kuhn-Tucker (KKT) conditions [6] to reformulate the bilevel problem into a single level problem with complementarity constraints, which is then reformulated into a mixed-integer programming problem [15].

2.1.3 Deterministic model

In the deterministic model, consideration of uncertainty is avoided by assuming perfect information for all parameters. For example, in the following deterministic model, the load is fixed as \bar{d} and the new generation capacity is fixed as \bar{P}^N .

$$\min \sum_{i,j,t} \hat{A}_{ij,t}^T x_{ij} + \sum_{i,k,t,m} \hat{A}_{i,k,t,m} (1 + \rho)^t (c_{i,k,t}^P p_{i,k,t,m} + c_{i,t}^L r_{i,t,m}) \quad (1)$$

$$\text{s. t. } \sum_k \hat{A}_{i,k,t,m} p_{i,k,t,m} + \sum_j \hat{A}_{ji,t,m} f_{ji,t,m} - \sum_j \hat{A}_{ij,t,m} f_{ij,t,m} = \bar{d}_{i,t,m} - r_{i,t,m}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, m \in \mathcal{M} \quad (2)$$

$$f_{ij,t,m} - B_{ij}(q_{i,t,m} - q_{j,t,m}) - (1 - x_{ij})M \cdot 0, \quad \forall i, j \in \mathcal{N} \quad (3)$$

$$B_{ij}(q_{i,t,m} - q_{j,t,m}) - f_{ij,t,m} - (1 - x_{ij})M \cdot 0, \quad \forall i, j \in \mathcal{N} \quad (4)$$

$$f_{ij,t,m} = B_{ij}(q_{i,t,m} - q_{j,t,m}), \quad \forall i, j \in \mathcal{L} \quad (5)$$

$$f_{ij,t,m} \cdot F_{ij}^{\max} - x_{ij} \cdot F_{ij}^{\max} \leq 0, \quad \forall i, j \in \mathcal{N} \quad (6)$$

$$f_{ij,t,m} \cdot F_{ij}^{\max} - x_{ij} \cdot F_{ij}^{\max} \geq 0, \quad \forall i, j \in \mathcal{N} \quad (7)$$

$$f_{ij,t,m} \cdot F_{ij}^{\max} - x_{ij} \cdot F_{ij}^{\max} \leq 0, \quad \forall i, j \in \mathcal{L} \quad (8)$$

$$-f_{ij,t,m} \cdot F_{ij}^{\max} - x_{ij} \cdot F_{ij}^{\max} \leq 0, \quad \forall i, j \in \mathcal{L} \quad (9)$$

$$p_{k,t,m} (P_{k,t}^{\max} + \bar{P}_{k,t}^N), \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, m \in \mathcal{M}, k \in \mathcal{K} \quad (10)$$

$$q_{\min} \leq q_{i,t,m} \leq q_{\max}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, m \in \mathcal{M} \quad (11)$$

$$x \text{ binary} \quad (12)$$

The objective function (1) is the transmission capital investment cost and total operational cost (including cost of power production and load shedding) over the planning horizon. This model is a static model, in which the total operational cost over the planning horizon is estimated by extrapolating from $|T|$ years. A similar approach has been used by several other related studies [13, 21, 22]. Constraint (2) requires that the net influx at a node should be equal to the net outflow. Constraints (3) and (4) are equivalent to the equation $f_{ij,t,m} = x_{ij}B_{ij}(q_{i,t,m} - q_{j,t,m})$, which is nonlinear and complicates the model. We introduce the constant M to linearize this equation [21]. When $x_{ij} = 1$, then the two constraints are reduced to $f_{ij,t,m} = B_{ij}(q_{i,t,m} - q_{j,t,m})$, where the value of M does not matter. When $x_{ij} = 0$, then we need to make sure that M is large enough so that no additional constraints are imposed. On the other hand, if M is too large, it may cause computational difficulties. In our experiments, we set it to be ten times the largest value of F_{ij}^{\max} . Equation (5)

calculates the power flow on existing transmission lines. Constraints (6)-(9) dictate that the power flow on transmission lines does not exceed their limits. Constraint (10) specifies the generation capacity on each node. Constraint (11) limits the range of phase angles at a node.

To facilitate algorithmic development and simplify the notations, we abstract the deterministic model as follows:

$$\min_{x,z} c^T x + b^T z \quad (13)$$

$$\text{s. t. } Ax + C_1 z \cdot g_1 \quad (14)$$

$$By + C_2 z \cdot g_2 \quad (15)$$

$$Jz = d \quad (16)$$

In this more concise abstract formulation, we use x to represent the binary variable indicating whether or not a transmission line should be built, y to represent the amount of new generation and z to represent operational variables including power production, phase angles, power flow and load curtailment. Vectors c and b represent coefficients of variables in the objective function. Matrices A, B, C_1, C_2, J are the coefficients of variables in the constraints. Vectors g_1, g_2 are the right-hand-side parameters in the constraints. Constraint (14) corresponds to equations (3)-(11). Constraint (15) corresponds to (10). Constraint (16) corresponds to (2).

2.1.4 The MMC model

In the two-stage MMC model, given the first-stage decisions, the second-stage problem is commonly known as the recourse problem [23], where the optimal operation decisions are identified. The feasible set of the recourse problem is defined as follows:

$$Z(x, d, y) = \{z : C_1 z \cdot g_1 - Ax, C_2 z \cdot g_2 - By, Jz = d\}$$

The uncertainty set is defined as

$$U = \{(d, y) : Q_1 d \cdot q_1, Q_2 y \cdot q_2\}$$

The matrices Q_1, Q_2 are the coefficients of d and y in the uncertainty set. Vectors q_1, q_2 are the right-hand-side parameters. They can contain information including the lower and upper bounds of the uncertain parameters, the lower and upper bounds of the linear combination of the uncertain parameters, etc. Such information can be obtained from historical data or statistical tests on historical data. In this project we consider both the uncertainty caused by load forecast and the uncertainty caused by future generation expansion. In addition, other types of sources of uncertainty can be easily plugged into the model without affecting the algorithm.

The MMC model can be formulated as:

$$\min_{x \text{ binary}} \quad c^{\top} x + \max_{(d,y) \in U} \min_{z \in Z(x,d,y)} \quad b^{\top} z \quad . \quad (17)$$

2.1.5 The MMR model

Unlike the MMC model, the MMR model aims to minimize the worst-case regret under all possible scenarios. Before presenting the MMR model, we first define the feasible set of the perfect information solution $G(d, y)$ and the perfect information cost $G(d, y)$ as follows:

$$G(d, y) = \{(\hat{x}, \hat{z}) : A\hat{x} + C_1\hat{z} \cdot g_1, C_2\hat{z} \cdot g_2 - By, J\hat{z} = d\}.$$

$$G(d, y) = \min_{\hat{x}, \hat{z} \in G(d, y)} \quad c^{\top} \hat{x} + b^{\top} \hat{z} \quad .$$

We can see that $G(d, y)$ is only dependent on the uncertain parameters (d, y) and can only be known after the uncertainty realizations are observed. We call it the perfect information solution because $G(d, y)$ can only be achieved if perfect information about the sources of uncertainty is available.

Then we can define the MMR model as follows:

$$\min_{x \text{ binary}} c^> x + \max_{(d,y) \in U} \min_{z \in Z(x,d,y)} b^> z - G(d,y) \quad . \quad (18)$$

Comparing the MMC model and MMR model side by side, their similarities are very noticeable. The difference between them lies in their definition of the “worst-case scenario”. With the MMC criterion, the worst-case scenario is defined as the scenario with the highest cost, while the MMR criterion defines the worst-case scenario where regret is the highest.

To shed more light on which criterion is more appropriate under different situations, we can classify scenarios into two categories: regretful vs. regretless. We use x^C to denote the MMC solution and x^R to denote the MMR solution. $R(x, s)$ is the regret of decision x under scenario s . If $R(x^C, s) \geq R(x^R, s)$, then we call scenario s a regretful scenario for decision x^C . Otherwise, we say it is regretless. When MMR is used, regret is redistributed among the scenarios. The regretful ones become less regretful and the costs in the regretless scenarios increase. As an uncertainty set consists of both regretful scenarios and regretless scenarios, it is unpractical to predict accurately which type of scenario will occur in the future. However, the classification of scenarios compares and illustrates the advantages of the MMR and MMC approaches for decision-makers to choose the criterion more appropriately for their specific problem. For example, if they are more confident that regretful scenarios will occur, they can choose the MMR criterion. Otherwise, they may choose the MMC criterion.

2.2 Model B: More realistic generation uncertainty

2.2.1 Nomenclature

Sets and indices

I	Set of nodes
L	Set of existing transmission lines
N	Set of candidate transmission lines
T	Set of time periods in the planning horizon
M	Set of load blocks
K	Set of technology types

Parameters

$P_{i,k}$	Capacity of existing and future generators of technology k at bus i
$c_{i,j,t}^T$	Cost of building the new transmission line (i, j) at time t
$c_{i,t}^L$	Cost of load curtailment at node i at time t
$c_{i,k,t}^P$	Cost of power production of technology k at node i
$F_{k,t,m}^C$	Average capacity factor of generation technology k at year t load block m
$F_{i,j}^{\max}$	The maximum power flow on transmission line (i, j)
$B_{i,j}$	Susceptance of transmission line (i, j)
M, M^0	Big constants used to linearize the power flow constraints and the products between two variables
$d_{i,t,m}$	Demand at node i at time period t load block m
q^{\min}	The lower bound of voltage angles
q^{\max}	The upper bound of voltage angles
I	Cash flow interest rate (with inflation)
$g_{i,k,0}$	Current availability of generators of technology type k at bus i

Variables

$x_{i,j,t}$	Binary variables indicating whether a transmission line is built at time t
$g_{i,k,t}$	Binary variables indicating whether a generator of technology type k exists at bus i in time period t
$f_{i,j,t,m}$	Power flow from node i to node j at year t load block m
$p_{i,k,t,m}$	Power production of technology k at node i at year t load block m
$r_{i,t,m}$	The amount of load shedding at node i at time t load block m
$q_{i,t,m}$	Voltage angle at node i at time t load block m
z	Abstract variable representing all dispatch variables, including $f_{ij,t,m}$, $p_{i,k,t,m}$, $r_{i,t,m}$, and $q_{i,t,m}$
$C^I(x)$	Total investment cost
$C^O(z)$	Total operation cost

2.2.2 Overview

In this section, we present a new multi-period robust optimization framework for transmission planning considering generation expansion and retirement uncertainty. To tackle robust TP problems under generation profile uncertainty, our proposed model has the following features:

1. We minimize the sum of investment cost and estimated operation cost under the worst-case scenario – including generation cost and load curtailment cost, over the planning horizon.
2. To model generation profile uncertainty, we assume a predefined set of new generators in addition to the existing generators. In the uncertainty set, we use binary variables to represent whether a new generator is going to be built, or if an existing generator is going to be retired. Policy and total capacity constraints can be added to the uncertainty set as needed.
3. The model enables multiple decision time points, at the beginning of which new investment decisions can be made.
4. The two-stage robust optimization framework mimics the real decision making process.

Transmission investment plans is proposed in the first stage, before uncertainty realization. Operation decisions are made in the second stage, after uncertainty has been observed. In addition, the minimax structure adds a layer of protection against uncertainty by identifying the worst-case scenario.

The model is decomposed into a master problem and a subproblem using the column and constraint procedure [38]. The subproblem is a bilevel optimization problem. As is demonstrated in [9], using KKT reformulation to solve the subproblem is not very efficient computationally. In this project, we first dualize the subproblem, resulting in a nonlinear mixed-integer optimization problem. It is then linearized by exploiting the binary variable in the nonlinear term. The model and algorithm is then tested on the WECC 240-bus system. The effects of policy natural gas price change is also explored in the case study.

2.2.3 Model formulation

To account for the long planning horizon and multiple sources of uncertainty, transmission planning problems are usually formulated as two-stage problems. In this section, we present a two-stage robust TP model in which uncertainty in future generation profile are considered. A summary of the modeling framework is shown in Figure 1.

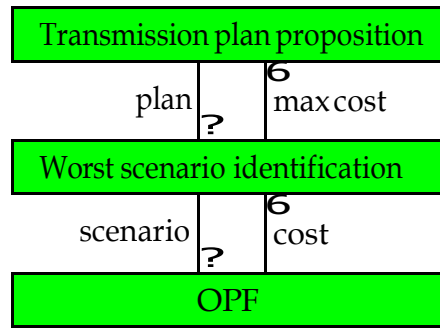


Figure 1: Summary of the robust optimization modeling framework

In our robust optimization model, candidate lines are selected for construction with limited information on uncertainty during the first decision making stage. In the second stage, operation decisions are made after uncertainty realization. The goal of robust optimization is to identify a

combination of new transmission lines that can achieve the lowest total cost, including construction cost and estimated operation cost.

$$\min_{x \in X} C^I(x) + \max_{g \in G} \min_{z \in Z(x, g)} C^O(z) \quad (19)$$

where

$$X = \{x | \hat{A}_{i,j,t}^{x_{i,j,t}} \cdot 1, \forall (i, j) \in N, x \text{ Binary}\} \quad (20)$$

$$C^I(x) = \sum_{t,i,j} \hat{A}_{i,j,t} (1+l)^{-t} c_{i,j,t}^T x_{i,j,t} \quad (21)$$

$$C^O(z) = \sum_{i,t,m} \hat{A}_{i,t,m} (1+l)^{-t} [\sum_k \hat{A}_{i,k,t}^P p_{i,k,t,m} + c_{i,t,m}^L r_{i,t,m}] \quad (22)$$

and

$$Z(x, g) = \{z : f_{i,j,t,m} = B_{i,j}(q_{i,t,m} - q_{j,t,m}), \forall (i, j) \in L, [\mu^{(1)}] \quad (23)$$

$$|f_{i,j,t,m} - B_{i,j}(q_{i,t,m} - q_{j,t,m})| \cdot (1 - \hat{A}_{i,j,k}^{x_{i,j,k}}) M, \forall (i, j) \in N, [\mu^{(1)}] \quad (24)$$

$$-F_{i,j}^{\max}(\sum_{k=1}^t \hat{A}_{i,j,k}^{x_{i,j,k}}) \cdot f_{i,j,t,m} \cdot F_{i,j}^{\max}(\sum_{k=1}^t \hat{A}_{i,j,k}^{x_{i,j,k}}), \forall (i, j) \in N, [\mu^{(2)}, \mu^{(3)}] \quad (25)$$

$$-F_{i,j}^{\max} \cdot f_{i,j,t,m} \cdot F_{i,j}^{\max}, \forall (i, j) \in L, [\mu^{(2)}, \mu^{(3)}] \quad (26)$$

$$0 \cdot p_{i,k,t,m} \cdot F_{i,k,t,m}^C \cdot p_{i,k,t,m}^P \cdot g_{i,k,t} \quad (27)$$

$$q_{i,t,m}^{\min} \cdot q_{i,t,m} \cdot q_{i,t,m}^{\max}, \forall (i, t, m) \quad [\mu^{(5)}, \mu^{(6)}] \quad (28)$$

$$\sum_k \hat{A}_{i,k,t} p_{i,k,t,m} + \sum_j \hat{A}_{j,i,t} f_{j,i,t,m} - \sum_j \hat{A}_{i,j,t} f_{i,j,t,m} = d_{i,t,m} - r_{i,t,m}, \forall (i, t, m) \quad [\mu^{(7)}] \quad (29)$$

From the objective function (19), we can see the two-stage structure of the robust TP model. It minimizes the investment cost and the projected operation cost under the most costly scenario. Investment and operation costs are represented by equations (21) and (22) respectively. The operation cost includes generation cost and load curtailment cost. The set G is the uncertainty set,

which can be defined by planners' belief on uncertain parameters. Constraints (20) means that a transmission line only needs to be built once in the planning horizon.

Constraints (23)-(29) defines $Z(x, g)$, the set of feasible power flow solutions once new transmission and generation is fixed. Equation (23) defines the power flow on existing transmission lines, while constraints (24) defines the power flow on candidate lines. It is equivalent to the constraint $f_{i,j,t,m} = (\hat{\mathbf{A}}_{k=1}^t x_{i,j,k}) B_{i,j}(q_{i,t,m} - q_{j,t,m})$, which is nonlinear and can complicate the computation. By using the auxiliary parameter M , it is linearized. Constraints (25)-(26) impose power flow limits on candidate and existing transmission lines. Constraints (27) requires power generation to be within the capacity limit of generators after capacity factor is considered. Constraint (28) limits the range of phase angles at each node. Constraint (29) requires the net influx at a node to be equal to the net outflow.

3 Solution techniques

In this section, we develop three customized trilevel optimization algorithms to solve the robust optimization problems, which we decompose into two levels: the master problem and the subproblem. We first present the algorithm for the MMC model. This algorithm is then modified for the MMR model. Since we use a cutting plane procedure that does not require duality information, we can reformulate the sub-problem as a mixed-integer linear programming problem. In [18], the worst-case scenarios are identified via statistical upper bounds with Monte Carlo simulation. In contrast, our algorithm provides a theoretical global optimality guarantee to find the worst-case scenarios as the entire problem is solved as a mixed-integer linear programming problem after reformulating the sub-problem. Then we present the solution methods for our Model B. Bilinear terms exists in the subproblem. However with the discrete structure of the uncertain parameter, the bilinear terms can be linearized without incurring too much computational burden.

3.1 Algorithm for Model A

The master problem is designed to provide a relaxation of the MMC model (17), in which the search for the worst-case scenario is restricted to be within a given finite set of scenarios, $W^C = \{(d^i, y^i), \forall i = 1, \dots, |W^C|\}$, rather than the complete set of scenarios, U . As such, the master problem yields a lower bound of the MMC model (17). We denote the master problem as $M^C(W^C)$, and it is formulated as the following single level mixed integer linear program.

$$\min_{x, z^i} c^T x + x \quad (30)$$

$$\text{s.t.} \quad x \leq b^T z^i \quad \forall i = 1, \dots, |W^C| \quad (31)$$

$$Ax + C_1 z^i \leq g_1 \quad \forall i = 1, \dots, |W^C| \quad (32)$$

$$By^i + C_2 z^i \leq g_2 \quad \forall i = 1, \dots, |W^C| \quad (33)$$

$$J z^i = d^i \quad \forall i = 1, \dots, |W^C| \quad (34)$$

$$x \text{ binary.} \quad (35)$$

The subproblem is defined as the MMC model (17) with a given first-stage decision, x . As such, the subproblem yields an upper bound of the MMC model (17). We denote the subproblem as $S^C(x)$, and it is formulated as the following bilevel linear program.

$$\max_{(d, y) \in U} \min_{z \in Z(x, d, y)} b^T z. \quad (36)$$

This model can be further reformulated as the following linear program with complementarity

constraints (LPCC).

$$\max_{d,y,z,a,b,g} \quad b^>z \quad (37)$$

$$\text{s. t.} \quad Q_1 d \cdot q_1 \quad (38)$$

$$Q_2 y \cdot q_2 \quad (39)$$

$$0 \cdot g_1 - Ax - C_1 z \cdot a \cdot 2 : 0 \quad (40)$$

$$0 \cdot g_2 - By - C_2 z \cdot b \cdot 2 : 0 \quad (41)$$

$$Jz = d \quad (42)$$

$$C_1^> a + C_2^> b + J^> g + b = 0 \quad (43)$$

LPCC problems can be solved by several algorithms ([15] Branch-and-Bound, Bender's, Big-M). The big-M approach [15] was found to be one of the most computationally efficient in our computational experiments. This approach reformulates (37)-(43) as the following mixed-integer linear program (MILP).

$$\max_{d,y,z,a,b,g,w} \quad b^>z \quad (44)$$

$$\text{s. t.} \quad \text{Constraints (38), (39), (42), (43)} \quad (45)$$

$$0 \cdot g_1 - Ax - C_1 z \cdot Mw_1 \quad (46)$$

$$0 \cdot a \cdot M(1 - w_1) \quad (47)$$

$$0 \cdot g_2 - By - C_2 z \cdot Mw_2 \quad (48)$$

$$0 \cdot b \cdot M(1 - w_2) \quad (49)$$

Here, M is a sufficiently large constant (big-M) and w_1 and w_2 are auxiliary binary variables that are introduced to enforce the complementarity conditions in (40) and (41).

The proposed algorithm for the MMC model, which we call Alg^{MMC} , is an iterative one, in which the master problem is solved to provide an increasing series of lower bound solutions, and

then the subproblem is solved to provide a series of decreasing upper bound solutions using the solution from the master problem, x , as an input. The input for the master problem, W^C , is iteratively enriched by the solutions from the subproblem until the gap between the lower and upper bounds falls below a tolerance, e . Detailed steps of this algorithm are described as follows:

Alg^{MMC}($c, b, A, B, C_1, C_2, g_1, g_2, J, Q_1, q_1, Q_2, q_2$)

Step 0 : Initialization. Create W^C that contains at least one selected scenario. Set $LB = -\infty$, $UB = \infty$, and $k = 1$. Go to Step 1.

Step 1 : Update $k \leftarrow k + 1$. Solve the master problem $M^C(W^C)$ and let (x^k, x^k) denote its optimal solution. Update the lower bound as $LB \leftarrow c^> x^k + x^k$ and go to Step 2.

Step 2 : Solve the sub-problem $S^C(x^k)$ and let (d^k, y^k, z^k) denote its optimal solution. Update $W^C \leftarrow W^C \cup \{(d^k, y^k)\}$, and $UB \leftarrow c^> x^k + b^> z^k$.

Step 3 : If $UB - LB > e$, go to Step 1; otherwise return (x^k, d^k, y^k, z^k) as the optimal solution to (17) and LB as the optimal value.

The MMR model can be solved using a similar algorithmic framework to Alg^{MMC} after the following simplifying yet equivalent reformulation.

$$\begin{aligned}
 & \min_{x \text{ binary}} c^> x + \max_{(d,y) \in U} \min_{z \in Z(x,d,y)} b^> z - G(d,y) \\
 & = \min_{x \text{ binary}} c^> x + \max_{(d,y) \in U} \min_{z \in Z(x,d,y)} b^> z - \min_{(\hat{x}, \hat{z}) \in G(d,y)} c^> \hat{x} + b^> \hat{z} \\
 & = \min_{x \text{ binary}} c^> x + \max_{(\hat{x}, \hat{z}) \in G(d,y)} \min_{z \in Z(x,d,y)} b^> z - (c^> \hat{x} + b^> \hat{z})
 \end{aligned}$$

To solve this reformulation of the MMR model, which is structurally similar to the MMC model (17), only slight modifications to the master and sub-problems are required. The master problem is defined for a different set of input scenarios, $W^R = \{(d^i, y^i, \hat{x}^i, \hat{z}^i), \forall i = 1, \dots, |W^R|\}$, in which the

two additional variables, \hat{x}^i and \hat{z}^i , represent the optimal investment and recourse decisions with hindsight of the uncertainty realization (d^i, y^i) . We denote the master problem as $\mathbf{M}^R(W^R)$, and it is formulated as the following single level mixed integer linear program.

$$\min_{x, \hat{x}, \hat{z}^i} \quad c^> x + x \quad (50)$$

$$\text{s. t. } x \geq b^> \hat{z}^i - (c^> \hat{x}^i + b^> \hat{z}^i) \quad \forall i = 1, \dots, |W^R| \quad (51)$$

$$Ax + C_1 \hat{z}^i \cdot g_1 \quad \forall i = 1, \dots, |W^R| \quad (52)$$

$$By^i + C_2 \hat{z}^i \cdot g_2 \quad \forall i = 1, \dots, |W^R| \quad (53)$$

$$J \hat{z}^i = d^i \quad \forall i = 1, \dots, |W^R| \quad (54)$$

$$x \text{ binary.} \quad (55)$$

We denote the subproblem as $\mathbf{S}^R(x)$, and it is formulated as the following bilevel linear program.

$$\max_{(d, y) \in U} \min_{(\hat{x}, \hat{z}) \in G(d, y)} b^> \hat{z} - (c^> \hat{x} + b^> \hat{z}) \quad , \quad z \in Z(x, d, y)$$

which can be solved using the same big-M approach with the following MILP.

$$\max_{d, y, \hat{z}, \hat{x}, \hat{z}, a, b, g, w} \quad b^> \hat{z} - (c^> \hat{x} + b^> \hat{z}) \quad (56)$$

$$\text{s. t.} \quad \text{Constraints (45)-(49)} \quad (57)$$

$$A \hat{x} + C_1 \hat{z} \cdot g_1 \quad (58)$$

$$B y + C_2 \hat{z} \cdot g_2 \quad (59)$$

$$J \hat{z} = d \quad (60)$$

$$\hat{x} \text{ binary.} \quad (61)$$

With the new definitions of master and sub-problems, the same algorithm Alg^{MMC} can be used to solve the MMR model with the following minor modification to Step 2, besides the apparent need to change the superscript “C” to “R”:

Step 2 : Solve the sub-problem $\mathbf{S}^R(x^k)$ and let $(d^k, y^k, z^k, \hat{x}^k, \hat{z}^k)$ denote its optimal solution. Update $W^R = W^R \cup \{(d^k, y^k, \hat{x}^k, \hat{z}^k)\}$, and $UB = c^> x^k + b^> z^k - (c^> \hat{x}^k + b^> \hat{z}^k)$.

3.2 Algorithm for Model B

The master problem is a relaxation of the robust TP problem and provide a first-stage solution. The original trilevel problem requires the most costly scenario in the entire uncertainty set G . In the master problem, the search for master problem is restricted in a smaller set of scenarios $S \subset G$. We denote the master problem as $\mathbf{M}(S)$, and it is formulated as the following mixed-integer linear program:

$$\min_{x, z^s} C^I(x) + z \quad (62)$$

$$\text{s. t. } x \in X \quad (63)$$

$$z \leq C^O(z^s), \forall s \in S \quad (64)$$

$$z^s \in Z(x, g^s), \forall s \in S \quad (65)$$

When $S = G$, the master problem $\mathbf{M}(S)$ is equivalent to the trilevel formulation (19). However, due to the enormous number of possible scenarios in G , it is unrealistic to search the entire uncertainty set. Instead, we search the subset S . As such, the master problem provides a lower bound to the actual optimal objective value. New scenarios are iteratively added to the set S and the lower bound increases at each iteration.

The subproblem is defined as the robust TP problem with a given first-stage solution x . Since the performance of such solution can only be worse than the optimal first-stage solution, the subproblem provides an upper bound to the optimal objective value for the original trilevel problem. We denote the subproblem as $\mathbf{R}(x)$ and it can be formulated as the following bilevel linear program:

$$\max_{g \in G} \min_{z \in Z(x, g)} C^O(z) \quad (66)$$

The subproblem defined in (66) is a bilevel optimization problem. In order to solve it, we write out the dual of the operation cost minimization problem $\min_{z \in Z(x,g)} C^O(z)$, denoted as $\mathbf{D}(x, g)$, where $\mathbf{D}(x, g)$ can be expressed as the following optimization problem:

$$\max_{\mu} \hat{\mathbf{A}}_{t,m} - \hat{\mathbf{A}}_{(i,j)2L[N]} F_{i,j}^{\max}(\mu_{i,j,t,m}^{(2)} + \mu_{i,j,t,m}^{(3)}) - \hat{\mathbf{A}}_{i,k} F_{k,t,m}^C P_{i,k} g_{i,k,t} \mu_{i,k,t,m}^{(4)} - \hat{\mathbf{A}}_i (q_{i,t,m}^{\max} \mu_{i,t,m}^{(5)} - q_{i,t,m}^{\min} \mu_{i,t,m}^{(6)}) - \hat{\mathbf{A}}_i d_{i,t,m} \mu_{i,t,m}^{(7)} \quad (67)$$

$$\text{s. t. } \mu_{i,k,t,m}^{(4)} + \mu_{i,t,m}^{(7)} \leq (1 + \epsilon)^{-t} c_{i,k,t}^P, \quad \forall i, k, t, m \quad [p] \quad (68)$$

$$\mu_{i,t,m}^{(7)} \leq (1 + \epsilon)^{-t} c_{i,t}^L, \quad \forall i, t, m \quad [r] \quad (69)$$

$$\mu_{i,j,t,m}^{(1)} + \mu_{i,j,t,m}^{(2)} - \mu_{i,j,t,m}^{(3)} - \mu_{i,t,m}^{(7)} + \mu_{j,t,m}^{(7)} = 0, \quad \forall (i, j) \in L[N, t, m] \quad [f] \quad (70)$$

$$\mu_{i,t,m}^{(5)} - \mu_{i,t,m}^{(6)} - \hat{\mathbf{A}}_{(i,j)2L} B_{i,j} \mu_{i,j,t,m}^{(1)} + \hat{\mathbf{A}}_{(j,i)2L} B_{j,i} \mu_{j,i,t,m}^{(1)} - \hat{\mathbf{A}}_{(i,j)2N} B_{i,j} (\hat{\mathbf{A}}_{k=1}^t x_{i,j,k}) \mu_{i,j,t,m}^{(1)} + \hat{\mathbf{A}}_{(j,i)2N} B_{j,i} (\hat{\mathbf{A}}_{k=1}^t x_{j,i,k}) \mu_{j,i,t,m}^{(1)} = 0, \quad \forall i, t, m \quad [q] \quad (71)$$

$$\mu^{(k)} \geq 0, \quad k = 2, 3, 4, 5, 6. \quad (72)$$

In this formulation, $\mu^{(k)}$ are the dual variables of constraints (23)-(29). Note that because $x_{i,j,t}$ is a parameter, we can replace (24) with their equivalent form $f_{i,j,t,m} = x_{i,j,t} B_{i,j} (q_{i,t,m} - q_{j,t,m})$ to simplify the formulation. In addition, with the previous replacement, we can combine constraints (25) and (26) by removing the term $\hat{\mathbf{A}}_{k=1}^t x_{i,j,k}$.

Then the bilevel subproblem is simplified as the following single level optimization problem

$$\max_{g \in G} \mathbf{D}(x, g)$$

In the objective function (67), the terms $F_{k,t,m}^C P_{i,k} g_{i,k,t} \mu_{i,k,t,m}^{(4)}$ are bilinear. Since they are multiplications between a continuous variable and a binary variable, they can be replaced by the follow-

ing equations.

$$g_{i,k,t} \mu_{i,k,t,m}^{(4)} = h_{i,k,t,m} \quad (73)$$

$$\text{s. t. } h_{i,k,t,m} \leq \mu_{i,k,t,m} - M^0(1 - g_{i,k,t}) \quad (74)$$

$$h_{i,k,t,m} \geq 0 \quad (75)$$

Then the subproblem can be easily solved as a mixed-integer linear program.

The algorithm uses the column and constraint generation procedure to iteratively generate new scenarios to be included in the subset S . The master problem is solved in each iteration to provide an increasing series of lower bounds and a first-stage solution. Given the aforementioned first-stage solution, the subproblem is solved to provide a decreasing series of upper bounds and a worst-case scenario to be included in set S . The algorithm terminates after the gap between the upper bound and the lower bound falls below a user defined threshold ϵ . A flow chart of the algorithm is summarized in Figure 2. Detailed steps of this algorithm are described as follows:

Step 0 : Initialization. Create S that contains at least one selected scenario. Set lower bound

$LB = -\infty$, $UB = \infty$, and $s = 1$. Go to Step 1.

Step 1 : Update $s = s + 1$. Solve the master problem $\mathbf{M}(S)$ and let (x^s, z^s) denote its optimal solution. Update the lower bound as $LB = z^s$. Go to Step 2.

Step 2 : Solve the subproblem $\mathbf{R}(x)$ and let (g^s, z^s) denote the optimal solution. Update $S = S \cup \{g^s\}$, and $UB = C^0(z^s)$.

Step 3 : If $UB - LB > \epsilon$, go to Step 1; otherwise return x^s as the optimal investment plan and $C^I(x^s) + z^s$ as the optimal value.

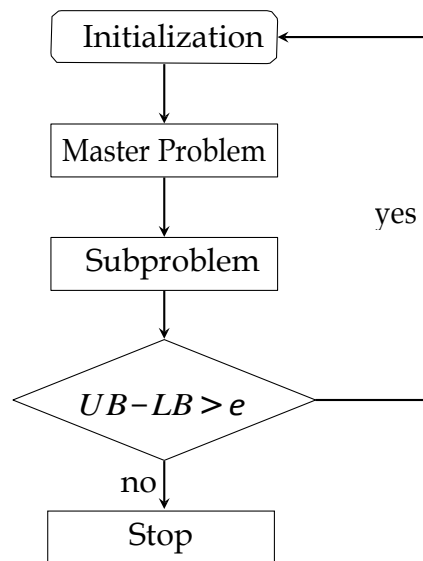


Figure 2: Flow chart of the decomposition algorithm

4 Case study

In this section, we apply our decision-support system to an IEEE 118-bus test system and the WECC 240-bus system. The model with simplified representation of load and generation investment uncertainty is applied to the IEEE 118-bus system. The performances of the model using different optimization criteria is compared. The model with the more detailed representation of generation investment and retirement uncertainty is applied to the WECC 240-bus system. The transmission plan obtained by this model is tested under various scenarios with different renewable energy policy mandates and fuel prices.

4.1 IEEE 118-bus test system

The IEEE 118-bus test system consists of 186 transmission lines, 5 wind farms, 5 coal plants, 5 gas plants and 33 loads. The network data is available in [1]. We consider 10 candidate lines. The operation costs are calculated based on the data of 4 load blocks. We consider a planning horizon of 20 years, with the operation cost extrapolated from the cost of year 1. Then the operation cost is assumed to increase at the same rate each year. The characteristics of generation and candidate lines are summarized in Table 4 and Table 5 respectively. In our case study, generation capacity data in the system is set to be able to satisfy all demand levels if there is no network congestion.

Uncertainty in future generation capacity consists of two parts, expansion and retirement. For wind and natural gas fired plants, we set the lower and upper bounds for new capacity. For coal plants, the range of reduced capacity is also provided. We use negative capacity to depict coal retirement. In addition to bounds on individual plants, we also set the lower bound and upper bound on total new generation capacity to control the randomness of the uncertainty set. The mean of our demand (\bar{d}) and the capacity factor are modified based on the real data from WECC [32].

Table 4: Generation parameters

Bus No.	Type	Current Capacity (MW)	Fuel Cost (\$/MWh)	Min New Capacity (MW)	Max New Capacity (MW)
25	Wind	1,000	0.0	2,000	3,000
31	Wind	500	0.0	150	400
32	Coal	300	43.0	-160	-80
36	Wind	1,100	0.0	800	1,500
49	Wind	1,000	0.0	1,200	2,000
61	Coal	400	28.2	-200	-70
65	Coal	1,500	52.7	-800	-600
66	Coal	300	28.3	-150	-100
76	Gas	200	31.0	80	100
85	Gas	300	63.9	150	200
87	Wind	900	0.0	1,300	2,500
89	Gas	150	49.1	100	150
92	Gas	200	32.0	200	250
99	Coal	300	27.5	-150	-80
113	Gas	200	65.0	200	300
Total		8,350		6,000	8,500

We generate four instances by changing the uncertainty sets. Their definitions are listed as follows:

$$U_1 = \{0.95\bar{d}_{i,t,m} \cdot d_{i,t,m} \cdot 1.05\bar{d}_{i,t,m} \quad (76)$$

$$P_{i,k,t}^{N,\min} \cdot P_{i,k,t}^N \cdot P_{i,k,t}^{N,\max} \quad (77)$$

$$P_{i,k,t}^{N,\min} \cdot \hat{A}_{i,k,t} P_{i,k,t}^{N,\max} \quad (78)$$

$$U_2 = \{0.85\bar{d}_{i,t,m} \cdot d_{i,t,m} \cdot 1.15\bar{d}_{i,t,m} \quad (79)$$

$$\text{Equations (77) - (78)} \quad (80)$$

$$U_3 = \{0.95\bar{d}_{i,t,m} \cdot d_{i,t,m} \cdot 1.05\bar{d}_{i,t,m} \quad (81)$$

$$[1.25 - 0.5 \text{sgn}(P_{i,k,t}^{N,\min})] P_{i,k,t}^{N,\min} \cdot P_{i,k,t}^N \cdot [0.75 + 0.5 \text{sgn}(P_{i,k,t}^{N,\max})] P_{i,k,t}^{N,\max} \quad (82)$$

$$0.75 P_{i,k,t}^{N,\min} \cdot \hat{A}_{i,k,t} P_{i,k,t}^N \cdot 1.15 P_{i,k,t}^{N,\max} \quad (83)$$

$$U_4 = \{0.85\bar{d}_{i,t,m} \cdot d_{i,t,m} \cdot 1.15\bar{d}_{i,t,m} \quad (84)$$

$$\text{Equations (82) - (83)} \quad (85)$$

Table 5: Candidate line parameters

From	To	Susceptance (W ⁻¹)	Transmission Capacity (MW)	Construction Cost (\$M)
25	4	30	390	40.60
25	18	30	390	32.48
25	115	30	390	40.60
32	6	30	390	50.75
36	34	30	390	28.42
36	77	30	390	44.66
70	25	30	390	97.44
86	82	30	390	36.54
87	106	30	390	62.93
87	108	30	390	52.78

where $(p_{i,k,t}^{N,\min}, p_{i,k,t}^{N,\max})$ and $(p^{N,\min}, p^{N,\max})$ are listed in the last two columns in Table 4, with $(p^{N,\min}, p^{N,\max})$ in the last row. The load curtailment cost is set as \$2000/MWh. The interest rate is set to be 0.1.

The experiment is implemented on a computer with Intel Core i5 3.30GHz with 4GB memory and CPLEX 12.5. The computation time of each instance is around 9 hours. The numbers of iterations for solving each instance are summarized in Table 6.

Table 6: Number of iterations for each instance

	U_1	U_2	U_3	U_4
MMC	3	3	2	2
MMR	3	6	2	3

The transmission plans, investment costs and objective values of each criterion under the four uncertainty sets are summarized in Tables 7 and 8. We then compare the performances of the MMC solution and the MMR solution under various scenarios in Tables 9 and 10, where we use D^c , D^r and D^d to denote the optimal MMC solution, the optimal MMR solution and the optimal deterministic solution. The lower cost and regret between D^c and D^r are highlighted. The deterministic solution is derived by setting the mean demand as the load levels and the median of new capacity as the future expansion plans. The scenarios are generated by our algorithms when solving the MMR problems and MMC problems. Each scenario corresponds to an optimal solution

to a sub-problem at an iteration of our algorithm and is the worst-case scenario for the first-stage solution obtained at the same iteration. Scenarios $S_1 - S_3$ and $S_6 - S_{10}$ are generated by solving the MMR problems. Scenarios S_4, S_5, S_{11} and S_{12} are generated when solving the MMC problems. Those scenarios typically have very high costs or regrets, thus are representative of bad scenarios that robust optimization tries to hedge against. The investment and operational costs of both the MMC and MMR decisions under the above scenarios are summarized in Table 11.

Table 7: Transmission plans of the MMC approach

Lines (from bus, to bus)	Uncertainty Set			
	U_1	U_2	U_3	U_4
Candidate Line (25, 4)	1	0	1	1
Candidate Line (25, 18)	1	0	1	1
Candidate Line (25, 115)	0	1	0	0
Candidate Line (32, 6)	0	1	0	0
Candidate Line (36, 34)	1	1	1	1
Candidate Line (36, 77)	1	1	1	1
Candidate Line (70, 25)	0	1	0	0
Candidate Line (86, 82)	1	1	0	0
Candidate Line (87, 106)	1	1	1	1
Candidate Line (87, 108)	1	1	1	1
Investment Cost (\$M)	298	414	262	262
Maximum Cost (\$M)	1,233	1,960	1,927	2,802

Table 8: Transmission plans of the MMR approach

Lines (from bus, to bus)	Uncertainty Set			
	U_1	U_2	U_3	U_4
Candidate Line (25, 4)	1	1	1	1
Candidate Line (25, 18)	1	1	1	1
Candidate Line (25, 115)	1	0	1	0
Candidate Line (32, 6)	0	0	0	0
Candidate Line (36, 34)	1	1	1	1
Candidate Line (36, 77)	1	1	1	1
Candidate Line (70, 25)	0	1	0	1
Candidate Line (86, 82)	1	1	1	1
Candidate Line (87, 106)	1	1	1	1
Candidate Line (87, 108)	1	1	1	1
Investment Cost (\$M)	339	395	339	395
Maximum Regret (\$M)	89	234	110	235

Table9: Comparison of the MMC, MMR and deterministic solutions for uncertainty set U_1

Cost/Regret (\$M)	D^c	D^r	D^d
Scenario S_1	1,137 / 117	1,109 / 89	1,085 / 65
Scenario S_2	1,039 / 0	1,080 / 41	1,263 / 224
Scenario S_3	803 / 0	844 / 41	1,125 / 422
Scenario S_4	1,126 / 0	1,167 / 41	1,325 / 199
Scenario S_5	1,233 / 27	1,272 / 66	1,367 / 161

Table10: Comparison of the MMC, MMR and deterministic solutions for uncertainty set U_2

Cost/Regret (\$M)	D^c	D^r	D^d
Scenario S_6	1,730 / 170	1,597 / 37	1,861 / 301
Scenario S_7	1,375 / 133	1,354 / 112	1,519 / 277
Scenario S_8	1,309 / 497	1,046 / 234	836 / 24
Scenario S_9	1,398 / 288	1,266 / 156	1,473 / 363
Scenario S_{10}	776 / 246	701 / 171	666 / 136
Scenario S_{11}	1,959 / 201	1,862 / 104	2,082 / 324
Scenario S_{12}	1,960 / 31	1,962 / 33	2,173 / 244

From Tables 7 and 8, we can see that as the uncertainty in demand increases, although the numerical value of the maximum regret and worst-case cost increases, the change in the transmission plan is not very substantial. It means many of the candidate lines are necessary regardless of the demand levels with our unchanged depiction of generation capacity uncertainty. The reason is that those candidate lines connect regions with very high locational marginal price differences due to the presence of large amount of wind energy. On the other hand, when uncertainty in generation expansion is increased, although the total cost also increases, fewer lines are actually built with the MMC criterion. That is because in the worst-case scenarios, the system contains less renewable energy capacity and the differences in the locational marginal prices between the otherwise connected regions are not substantial enough to justify new transmission lines. When the MMR criterion is used, however, the final transmission plan does not seem to be sensitive to the change of uncertainty in generation expansion. One possible explanation is that since the MMR criterion does not make decisions only based on the boundary scenarios, it is less sensitive to the changes in uncertainty sets.

From the more detailed comparisons of results from the MMC and MMR approaches in Ta-

Table 11: Investment and operational costs for scenarios (\$M)

	D^c	D^r		D^c	D^r
Invest $S_1 - S_5$	298	339	Invest $S_6 - S_{12}$	414	395
Operation S_1	839	770	Operation S_6	1,316	1,202
Operation S_2	741	741	Operation S_7	961	959
Operation S_3	505	505	Operation S_8	895	651
Operation S_4	828	828	Operation S_9	984	871
Operation S_5	935	933	Operation S_{10}	362	306
			Operation S_{11}	1,545	1,467
			Operation S_{12}	1,546	1,567

bles 9 and 10, we can gain more insights about which criterion is more appropriate under different situations. Both robust optimization solutions outperform the deterministic solution under most of the scenarios. When the uncertainty set is U_1 , the MMC solution outperforms the MMR solution under most of the listed scenarios. When the uncertainty set is U_2 , on the other hand, the MMR solution has a lower total cost under more listed scenarios. The solutions of the cases when the uncertainty sets are U_3 and U_4 yield similar results. According to our definition at the end of section 2.1.2, scenarios $S_2 - S_5$ in Table 9 and scenario S_{12} in Table 10 are regretless scenarios for the MMC decision, while scenarios S_1 and $S_6 - S_{11}$ are regretful ones. Which criterion should be used depends on the decision-maker's perception on the uncertainty sets. In typical regretless scenarios for MMC decisions, there usually exists high demand and low renewable energy penetration. If decision-makers care more about such scenarios or believe they are more likely, then MMC should be used. Otherwise, choosing MMR might be better. Both criteria provide good upper bounds for the total costs under scenarios contained in an uncertainty set. The MMC criterion provides a smaller upper bound with higher average costs while the costs of MMR decisions are lower on average but have higher variability.

From the above results, it is obvious that both the future generation expansion behavior of generation companies and demand uncertainty play important roles in transmission planning. In addition, we can also conclude that both criteria have their merits and can yield relatively reliable expansion plans that guarantee zero curtailment for uncertainty realizations contained in the uncertainty sets. However, depending on the characteristics of uncertainty sets and the preference

of decision-makers, they may outperform each other under different situations. Thus, a comparative analysis of the MMC and MMR criteria can shed more light on better utilization of both approaches.

4.2 WECC 240-bus test system

The topology of the WECC 240-bus test system [32] is shown in Figure 3, which was from [30].

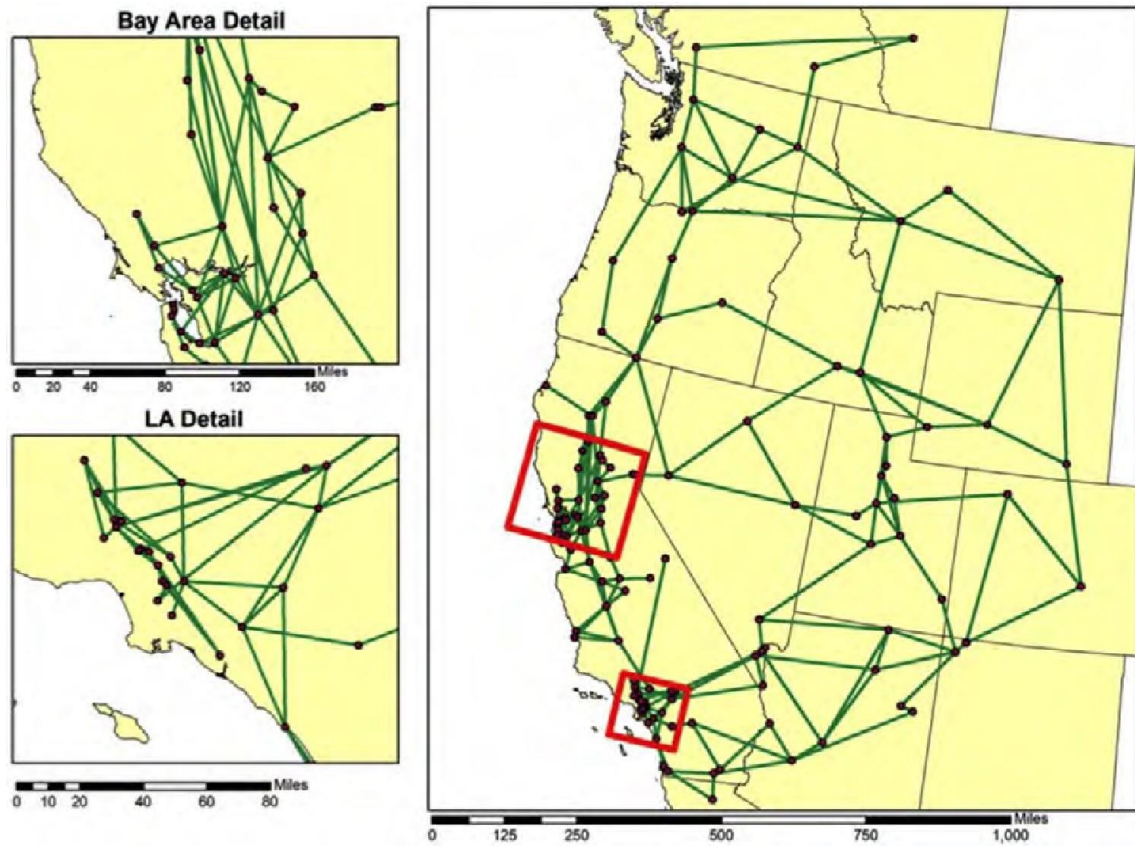


Figure3: WECCfrom[30]

The system has 240 buses, 448 transmission lines, 139 load centers and 124 generation units, including coal, gas-fired, nuclear, hydro, geothermal, biomass, wind and solar generators.

We consider 18 candidate lines. The planning horizon is 20 years, which is divided into four five-year periods. We selected the load of two representative hours from the typical week market data to be the baseline of our demand data. An annual growth rate of 1% is assumed based on the

projections of EIA [11].

Uncertainty in future generation capacity is due to the success in the implementation of new capacity and the timing and amount of capacity retired. We consider 70 uncertain generators, of which 17 are existing coal power plants set to retire over the study period. The rest are possible gas-fired resources and wind and solar farms as well as geothermal units at various locations in the system. In the uncertainty set, we specify several requirements for the uncertain generators. Firstly, we require that all coal plants retire at the end of the planning horizon. Secondly, 50% of the new capacity needs to be included at the end of the planning horizon. Thirdly, the total additional capacity at a decision point cannot be lower than 50% of the total additional capacity at the next decision point. Finally, we require that renewable energy generation account for at least a certain percentage of the total additional capacity. These restrictions on the uncertainty set can be adjusted based on available information as well as planners' belief on future generation profiles. The load curtailment cost is set as \$2000/MWh. The interest rate is set to be 10%.

We visualize the test system as well as the transmission planning problem in Figure 4. The 240 buses are arranged in a circle, with bus #1 starting at the three o'clock position and going counter-clockwise; bus #240 also comes back to the three o'clock position. Although the geographic information is completely distorted by this arrangement, it allows for more informative visualization of the big picture for the transmission planning problem. The 448 transmission lines are represented by black line segments connecting the corresponding bus pairs. The relative lengths of the line segments in the figure do not necessarily represent the actual length of the transmission lines. The 18 candidate lines are also plotted in green. These lines do not exist in the test system yet, and it is up to the planner to decide which (if any) lines should be built and when. The squares surrounding the inner circle represent the 139 load centers, with the areas of the square proportional to the magnitudes of the loads at the corresponding nodes. The colorful dots surrounding squares represent the existing 124 generators, with the areas of the dots proportional to the capacities of the generators at the corresponding nodes. The colorful stars represent the potential new generators that may be added to the corresponding nodes. The color map on the left sub-figure is

for both existing and potential new generators. Multiple generators at the same nodes are plotted at different orbits to avoid overlap.

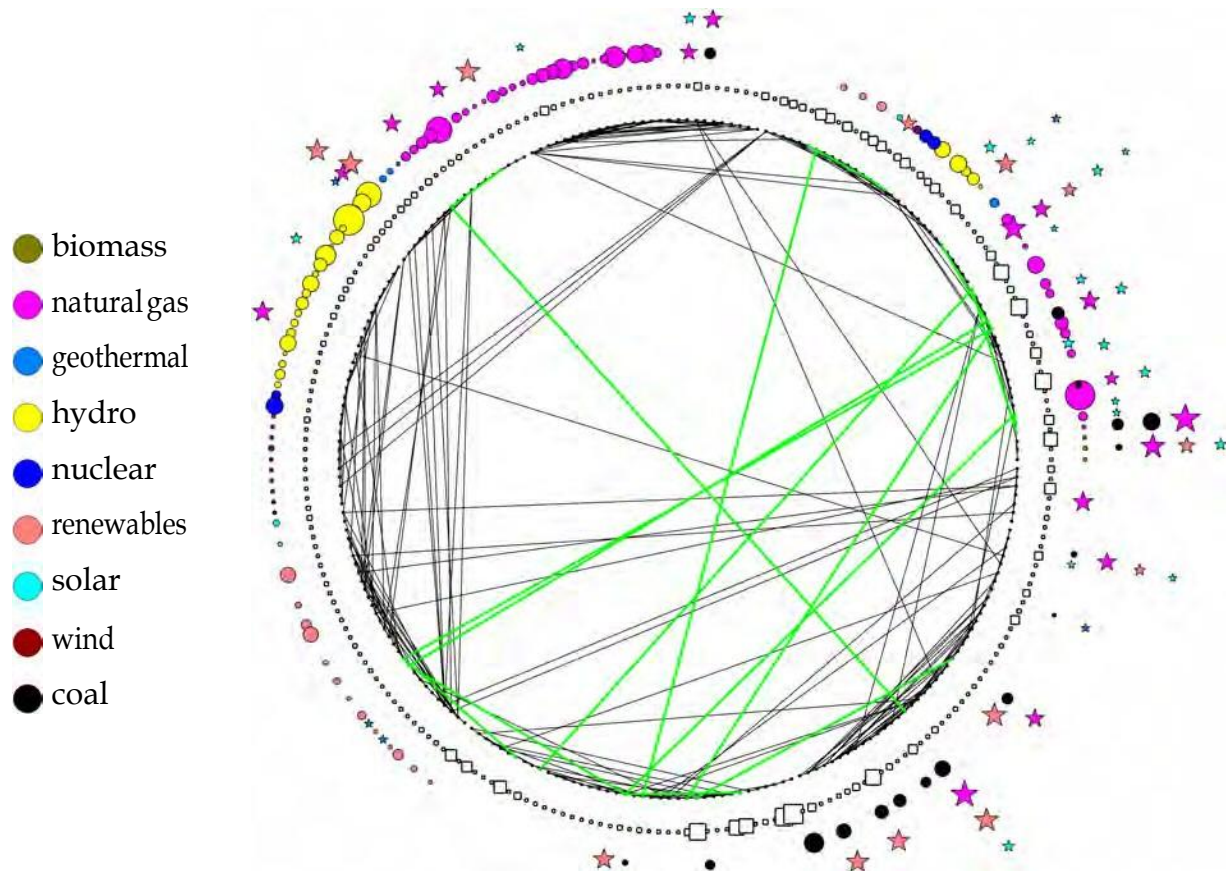


Figure 4: Visualization of the transmission planning problem

Depending on the policy requirement on renewables and the uncertain gas prices over the twenty-year horizon, we define the following four futures, which will be used for sensitivity analysis of our model.

- Future 1: 20% additional renewables and high natural gas prices (double the current price)
- Future 2: 20% additional renewables and low natural gas prices (at the current level throughout the next twenty years)
- Future 3: 40% additional renewables and high natural gas prices

- Future4: 40% additional renewables and low natural gas prices

The experiment was implemented on a desktop computer with Intel Core i7 3.4GHz CPU, 8GB memory and CPLEX 12.5. We adjust the uncertainty sets and natural gas price to create multiple test instances. The computation time of each instance is around 20 hours.

We obtained the following six transmission plans.

- Plan 1: Optimal transmission plan under future 1.
- Plan 2: Optimal transmission plan under future 2.
- Plan 3: Optimal transmission plan under future 3.
- Plan 4: Optimal transmission plan under future 4.
- Plan 5: Too little and too late investment. This is an arbitrarily created transmission plan to represent the case of very little and very late investment in new transmission lines.
- Plan 6: Too much and too early investment. This is an arbitrarily created transmission plan to represent the case of a very high level of investment implemented very early in the planning horizon.

These six transmission plans are visualized in Figure 5. Each column represents a plan, and each row represents a five-year period in the planning horizon. The green lines are to be added to the system in the period according to the transmission plan. Although the added new lines are cumulative, only new additions are shown in green. It can be seen that the first four transmission plans are similar and all build new lines gradually in the first three periods. Plan 5 has only a small number of lines, and plan 6 almost adds all candidate lines at once in the first period.

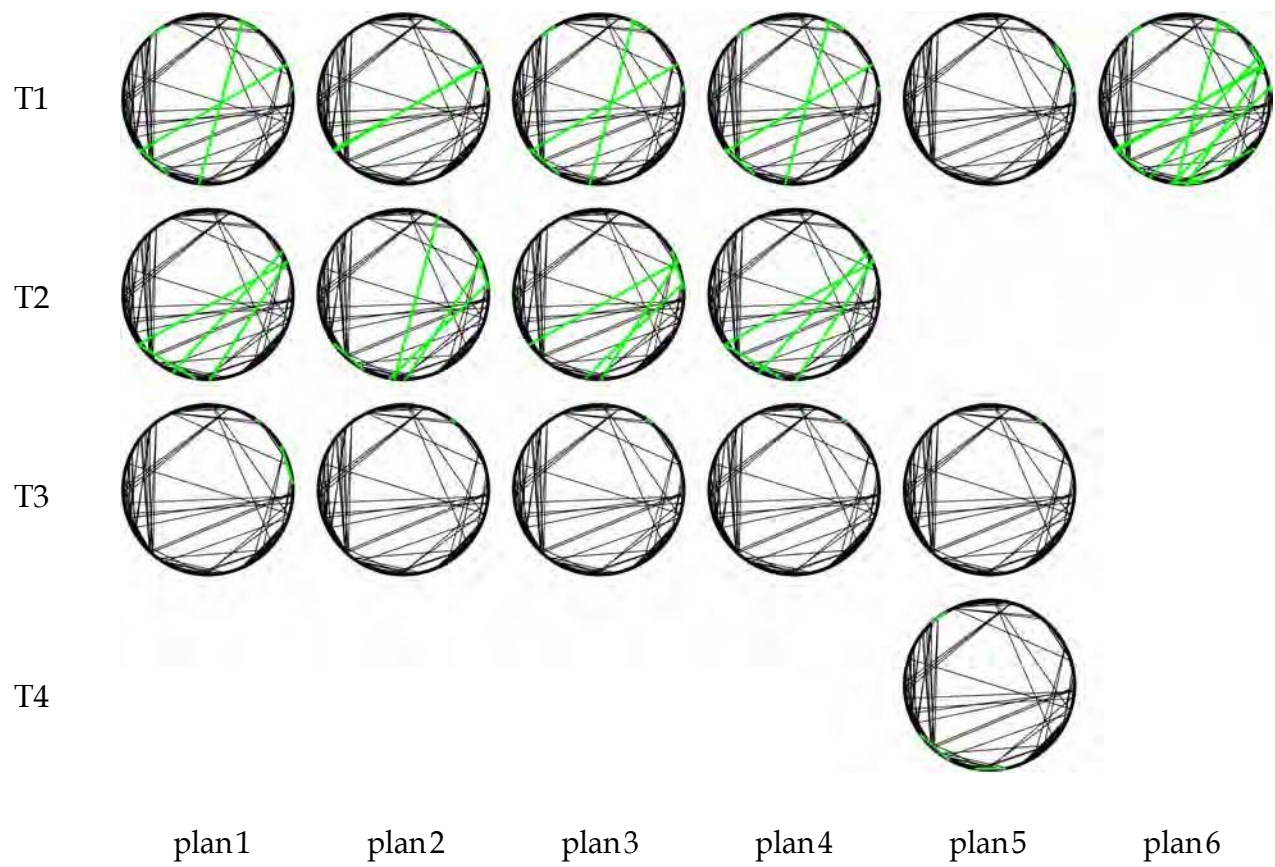


Figure5: Six transmission plan

A by-product of the algorithm from Section 3.2 is a set of scenarios that represent either the best (with lowest cost) or worst (with highest cost) case scenarios for a given transmission plans. We have collected twelve of these scenarios under the four futures and use them to illustrate the robustness performances of the six plans under these scenarios. The scenarios are shown in Figure 6. The four orbits of the stars indicate the periods in which the new generators are added to the generation portfolio. The higher the orbit, the later in time.

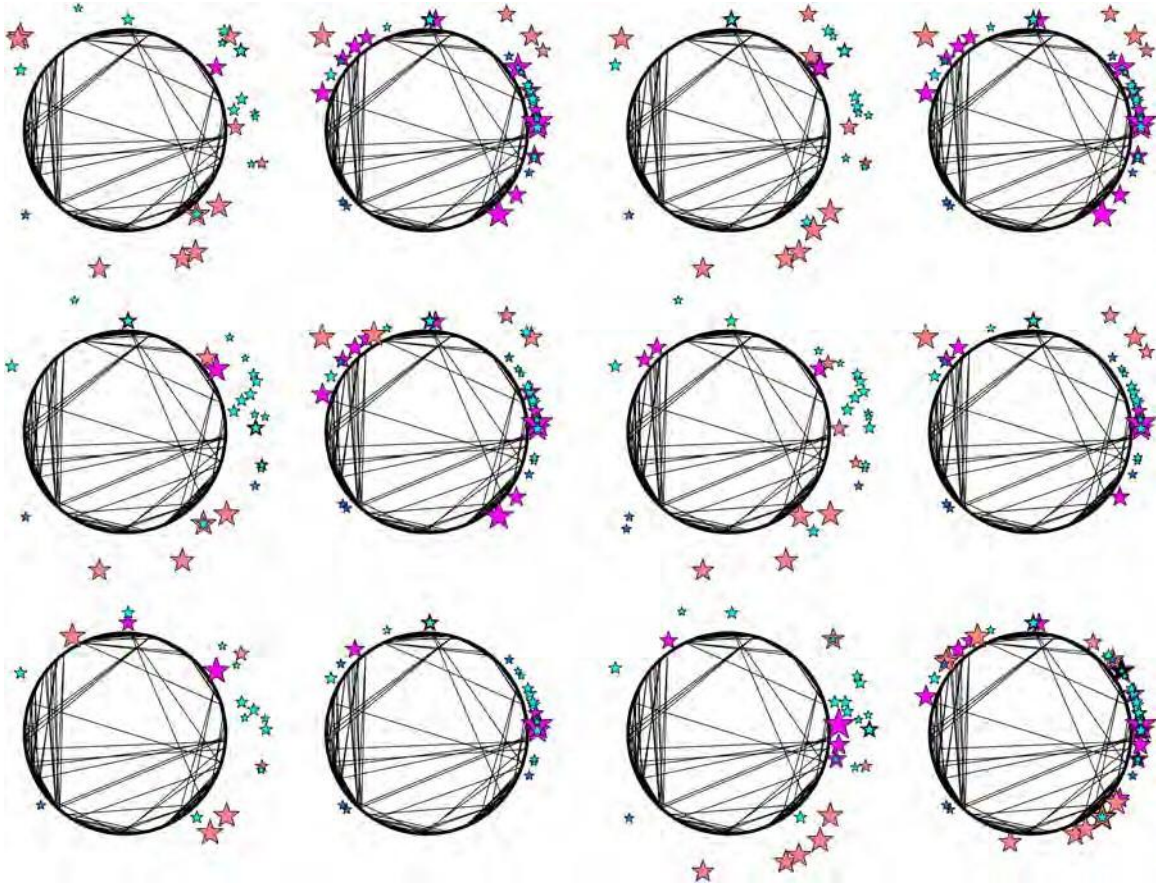


Figure 6: Twelve best and worst case scenarios

Finally, the robustness performances of the six transmission plans under the twelve scenarios were evaluated with respect to the investment cost, the operations cost, and load curtailment (as a percentage of total load) over the planning horizon in Figure 7. The colored regions in the bottom two rows in Figure 7 are outlined by the upper and lower bounds performances under the twelve scenarios. The white curves inside the colored region are the intermediate scenarios. The figure

suggests that the first four plans have similar investment costs, with plan 4 being the least costly. Their performances in operational costs and load curtailments are also comparable. Plan 5 has a negligible amount of investment, which is a \$1.4B savings from plan 4. However, as a consequence of the lack of enough investment, its operational cost is almost \$20B more than plan 4, and its load curtailment in the worst case is also much sever than plan 4. On the other extreme, plan 6 makes about 20% more investment than plan 4, but its performance in operations cost and load curtailment are similar, if not even worse than the cheaper counterpart. These results demonstrate the need for making enough and smart investment in transmission planning in order to reduce the long-term operations cost as well as enhancing the reliability of the grid.

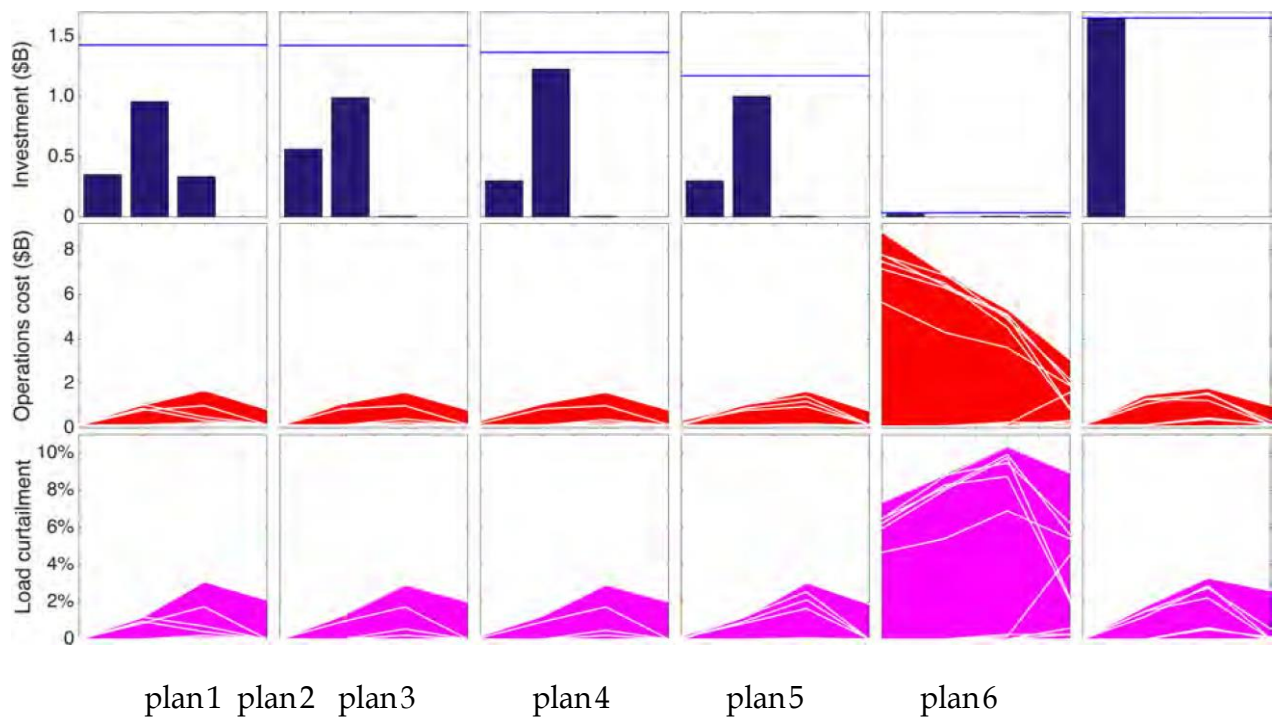


Figure 7: Performance of six plans under twelve scenarios

5 Concluding remarks

In this section we discuss in detail the analytical basis for our decision-support system, analyze the case study results and provide an interpretation of the ramifications of investment decisions made with the our decision-support system.

5.1 Overview

As a facilitator for generation expansion, transmission planning is critical to the reliable and economic operation of power systems. Traditionally, transmission planning is usually studied as deterministic problems. However, in addition to operation level sources of uncertainty including load prediction error and renewable energy intermittency, with the restructuring of the power system, generation expansion plans of generation companies also become uncertain. Increasingly strict environmental legislation imposes additional uncertainty for transmission planners. With the limitations of previous researches, new models and algorithms are needed to address the difficulties occurring with the increased level of uncertainty.

5.2 Findings and conclusions

In this project, we develop a decision-support system for transmission planning, which contains two models, both using robust optimization to address the challenges posed by generation expansion uncertainty. For Model A, we propose two robust optimization framework for the transmission planning problem under uncertainty, where we take into consideration both the high-frequency uncertainty caused by load forecast errors and the low-frequency uncertainty caused by future generation expansion and retirement. We use two criteria: minimax cost and minimax regret, and compare their performances. The uncertain parameters are described by a polyhedral uncertainty set. With this approach, we can derive a transmission plan that is robust under all scenarios. The resulting models can be formulated as trilevel mixed-integer problems. We use a branch and cut type mechanism to decompose the problem into a master problem and a subproblem. The subproblem

generates scenarios and returns them to the master problem to cut off sub-optimal solutions. The bilevel mixed-integer subproblem is reformulated into a single level mixed-integer-programming problem with the KKT conditions to obtain the global optimal solution. Our model and algorithm are then tested on an IEEE 118-bus system, where we compare the results of our MMR and MMC models and analyze their differences. We conclude that the MMR and MMC criteria may outperform each other depending on the uncertainty set and decision-maker's preference.

For Model B, we propose a robust transmission model that can explore the impact of generation expansion uncertainty in a more detailed manner. When describing future generation expansion and retirement, we use binary variables to represent whether a generator exists at a node or not, which enables a more flexible description of the uncertain parameters. Under the new modelling framework, uncertainty concerning both the location and size of generators can be modeled in the uncertainty set. We use a similar column and constraint generation procedure to decompose the problem into a master problem and a subproblem. Due to the special structure of our model, the bilinear subproblem can be equivalently reformulated as a mixed-integer linear problem without utilizing the KKT conditions, which also makes testing a larger test size possible. In the second case study, our model and algorithms are tested on a WECC 240-bus test system. We compared the performance of the robust transmission plans derived by our model to the performance of plans derived from solving deterministic models or other heuristic rules under four different scenarios, which are obtained by adjusting natural gas price and renewable energy policy mandate. The results show that our transmission plans not only out-perform plans obtained from other means but also remain robust under different scenarios.

5.3 Future work

Several directions of possible future work for this project are worth pursuing. In this project, demand and generation profile uncertainty is modeled by uncertainty sets. However, since probability distributions of load may be obtained from historical data, it is possible to model various sources of uncertainty with different methods. We can combine stochastic programming with robust opti-

mization by representing operation level uncertainty such as load and renewable energy generation with probability distribution, and representing other types of uncertainty including generation investments and retirement with uncertainty sets, which reduces the conservativeness of the plan. To solve such a problem, and to improve the computational efficiency of current models, new algorithms and heuristics need to be developed. Another way to extend the work done to date is to incorporate additional details into the model, including more explicit representations of various existing and possible policies, changes in demand caused by demographic development, climate change and recovery from natural disasters.

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