



# **Tools and Techniques for Considering Transmission Corridor Options to Accommodate Large Scale Renewable Energy Resources**

*Final Project Report*

**Power Systems Engineering Research Center**

*Empowering Minds to Engineer  
the Future Electric Energy System*



# **Tools and Techniques for Considering Transmission Corridor Options to Accommodate Large Scale Renewable Energy Resources**

## **Final Project Report**

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## **Executive Summary**

The increase in economic and environmental concerns has resulted in the fast growth of renewable resource penetration in the electric power grid. In order to ensure increased penetration of renewable resources several states have a mandated renewable portfolio standard (RPS) which requires a certain percentage of the load to be served by renewable resources. The RPS also states by which year the standard has to be met. In California, for example, the RPS requirement is 20% by the year 2012 and 33% by the year 2020. This project addresses three important aspects of integrating renewable resources in the bulk power system:

1. Transmission expansion planning including renewable resources
2. Impact of large scale energy storage
3. The impact of flexible AC transmission systems (FACTS)

Each of these three aspects is discussed in separate parts of the report. A summary related to each aspect is presented below.

### **Part 1: Transmission Expansion Planning Including Renewable Resources**

Due to economic and environmental reasons, several states in the United States of America have a mandated renewable portfolio standard which requires that a certain percentage of the load served has to be met by renewable resources of energy such as solar, wind and biomass. Renewable resources provide energy at a low variable cost and produce less greenhouse gases as compared to conventional generators. However, some of the complex issues with renewable resource integration are due to their intermittent and non-dispatchable characteristics. Furthermore, most renewable resources are location constrained and are usually located in regions with insufficient transmission facilities. In order to deal with the challenges presented by renewable resources as compared to conventional resources, the transmission network expansion planning procedures need to be modified. New high voltage lines need to be constructed to connect the remote renewable resources to the existing transmission network to serve the load centers. Moreover, the existing transmission facilities may need to be reinforced to accommodate the large scale penetration of renewable resource.

This part of the report proposes a methodology for transmission expansion planning with large-scale integration of renewable resources, mainly solar and wind generation. An optimization model is used to determine the lines to be constructed or upgraded for several scenarios of varying levels of renewable resource penetration. The various scenarios to be considered are obtained from a production cost model that analyses the effects that renewable resources have on the transmission network over the planning horizon. A realistic test bed was created using the data for solar and wind resource penetration in the state of Arizona. The results of the production cost model and the optimization model were subjected to tests to ensure that the North American Electric Reliability Corporation (NERC) mandated *N-1* contingency criterion is satisfied. Furthermore, a cost versus benefit analysis was performed to ensure that the proposed transmission plan is economically beneficial.

Different planning methods and models are used by the power industry to plan transmission for renewable resources. For example, the Midwest ISO mainly uses a power flow tool for transmission expansion planning with renewable resources. In order to ensure reliability of the proposed

expansion plan dynamic simulations, voltage stability and small signal oscillation analysis tools are also employed. The transmission planning process for renewable resources employed at the Midwest ISO can be summarized as follows:

- a) Renewable resource forecasting and placement in power flow models
- b) Copper sheet analysis (power flow with no limits on transmission capacity) to identify a preliminary transmission plan. This preliminary plan is supplemented with area based contour plots that take into consideration areas lacking in transmission facilities that do not show up in the copper sheet analysis.
- c) Use production cost model to identify an expansion plan.
- d) Perform reliability assessment and a cost versus benefit analysis for the proposed set of transmission paths to come up with a consolidated transmission plan.

The main tool used for transmission planning for large scale renewable resource integration, as proposed in this part of the report, is a linear, mixed-integer optimization model which is based on the DC optimal power flow formulation. The optimization model includes a time variable that is used to account for the hourly and seasonal fluctuations in renewable energy availability. However, given the complexity of the power grid and the time horizon of transmission planning, it is proposed that rather than using the entire planning horizon data, a few scenarios be selected to be input to the optimization model. These scenarios are selected using a production cost model which identifies weeks within the planning horizon with increased renewable energy availability. The optimization model is run for each of the scenarios identified and a corresponding optimal set of transmission paths to be constructed is obtained. The results of the optimization model are then combined to form a comprehensive transmission expansion plan, which in turn needs to be checked for economic benefits as well as reliability over the entire planning horizon.

## **Part 2: Power Flow Control and Probabilistic Load Flow**

The existing electric power grid was built for a situation in which power was injected at a few locations by dispatchable bulk power plants to supply inflexible loads. It was designed for reoccurring power flow patterns and has limited control capabilities. In the meantime, the increased consumption, the liberalization of electricity markets and the increase in variable renewable generation have led to a situation in which the constraints imposed by the transmission grid result in economically suboptimal generation dispatches to avoid overloads in the system. However, it often is not just a matter of insufficient transmission line capacities but because power flows are governed by Kirchhoff's Laws a single line reaching its limit restricts how the entire system is operated. To enable the transition to an efficient sustainable electric energy supply system, the transmission grid needs to be able to adjust to the increasingly varying power flows. Power flow control enabled for example by FACTS devices provides the opportunity to influence where power is flowing and therefore to improve the usage of the existing transmission system for varying generation in-feeds.

In this part of the project, the focus is on using power flow control to enable a flexible transmission grid. The work includes (1) the derivation of a decentralized control scheme to determine the optimal steady-state settings of power flow control devices in order to fully utilize the existing transmission infrastructures and (2) an analysis of the flexibility achieved in the transmission

grid. The control approach is a two-stage algorithm based on regression analysis: in the offline stage, a regression function is determined which gives the optimal device setting as a function of a few key measurements in the system. In the online stage, the regression function and the values of the key measurements can be used to locally determine the optimal setting of the device without carrying out the optimal power flow calculation. The method is tested for thyristor-controlled series compensators in the IEEE 14 bus system. The analysis of the flexibility achieved by these devices includes the consideration of the improvement of the transmission capacity usage and the increased range of possible generation dispatches.

Another aspect considered in this part of the project corresponds to probabilistic load flow calculations. The probabilistic nature of variable renewable generation has led to the introduction of probabilistic load flow calculations which are based on probability distributions of the renewable generation output. While probability distributions for wind generation are available, they do not provide information about the correlation between the in-feeds of power generation from two not co-located wind plants. Hence, a further contribution of this work is an initial study on how to mathematically model two  $\beta$ -distributions with a specific correlation.

### **Part 3: Large Scale Energy Storage**

This part of the project technical report concerns the impact of large scale energy storage on interconnected electric power systems, especially systems with high penetration of renewable energy generation. The rapidly increasing integration of renewable energy source into the grid is driving greater attention towards electrical energy storage systems which can serve many applications like economically meeting peak loads, providing spinning reserve. Economic dispatch is performed with bulk energy storage with wind energy penetration in power systems allocating the generation levels to the units in the mix, so that the system load is served and most economically. The results obtained in previous research to solve for economic dispatch uses a linear cost function for a Direct Current Optimal Power Flow (DCOPF). This report uses quadratic cost function for a DCOPF implementing quadratic programming (QP) to minimize the function. A Matlab program was created to simulate different test systems including an equivalent section of the WECC system, namely for Arizona, summer peak 2009.

A mathematical formulation of a strategy of when to charge or discharge the storage is incorporated in the algorithm. In this report various test cases are shown in a small three bus test bed and also for the state of Arizona test bed. The main conclusions drawn from the two test beds is that the use of energy storage minimizes the generation dispatch cost of the system and benefits the power system by serving the peak partially from stored energy. It is also found that use of energy storage systems may alleviate the loading on transmission lines which can defer the upgrade and expansion of the transmission system.

### **Project Publications:**

G. Heydt. “The Next Generation of Power Distribution Systems.” IEEE Transactions on Smart Grid, v. 1, No. 3, December, 2010, pp. 225 – 235.

G. Heydt. *Smart Grids: Infrastructure, Technology and Solutions*. Stuart Borlaise editor, Chapter 4, “Smart Grid Barriers and Critical Success Factors,” CRC Press, Taylor and Francis Book Co. New York, 2012

Rui Yang, Gabriela Hug. “Optimal Usage of Transmission Capacity with FACTS Devices in the Presence of Wind Generation: A Two-Stage Approach.” PES General Meeting, San Diego, USA, 2012.

Rui Yang, Gabriela Hug. “Regression-based FACTS Control for Optimal Usage of Transmission Capacity.” TechCon Conference, Austin, 2012.

### **Student Theses:**

Harald Franchetti. *Probabilistic Load Flow for Correlated Wind Power Outputs*. Masters thesis in the process of being completed. Anticipated completion and graduation from TU Vienna, December 2012.

Samir Gupta. *Dispatch of Bulk Energy Storage in Power Systems with Wind Generation*. MSEE Thesis, Arizona State University, Tempe, AZ, April, 2012.

Sruthi Hariharan. *Transmission Expansion Planning with Large Scale Renewable Resource Integration*. Arizona State University, MS Thesis, May 2012.

## **Part 1**

# **Transmission Expansion Planning with Large Scale Renewable Resource Integration**

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## Nomenclature

$branch_{xij}$	Reactance of branch between bus $i$ and bus $j$
$C_G$	Generator cost function
$C_T$	Scaled transmission line construction cost
$c_{ij}$	Coefficient corresponding to the $i^{th}$ decision variable (general representation of the objective function),
$CLMP$	Congestion component of the location marginal price
$DC$	Direct current
$ELMP$	Energy component of the location marginal price
$F$	Fixed cost of generator
$f_{ij}$	Line flow from bus $i$ to bus $j$ in the DC formulation
$f_x$	Latitude of bus $f$
$f_y$	Longitude of bus $f$
$G_{ij}$	Conductance of the line between bus $i$ and bus $j$
$ISO$	Independent systems operator
$L_j$	Load at bus $j$
$L_{ft}$	Length of line from bus $f$ to bus $t$
$LLMP$	Loss component of the location marginal price
$LMP$	Location marginal price
$M$	A very large number
$n$	Number of sub-periods to consider within a year for planning
$NERC$	North American Electric Reliability Corporation
$NPV$	Net present value (cost of constructing the transmission line)
$P_{ij}$	Real power flow from bus $i$ to bus $j$
$P_{min}, P_{max}$	Minimum and maximum capacity of generator
$PV$	Photovoltaic
$Q_{ij}$	Reactive power flow from bus $i$ to bus $j$
$r$	Annual rate of interest
$RPS$	Renewable portfolio standard
$t_x$	Latitude of bus $t$
$t_y$	Longitude of bus $t$
$TEP$	Transmission expansion planning
$V_{OM}$	Variable cost coefficient
$V_i$	Voltage magnitude at bus $i$
$WECC$	Western Electricity Coordinating Council
$WREZ$	Western Renewable Energy Zones
$x$	Binary variable to decide if a line should be added to a right of way

## Nomenclature (continued)

$X_i$	Decision variable (general representation of constraints and objective function)
$x_{ij}$	Reactance of line between bus $i$ and bus $j$
$y$	Typical life time of transmission line, usually 25-30 years
$\theta_i$	Voltage angle at bus $i$
$\theta_{ij}$	$\theta_i - \theta_j$

# **1 Introduction**

---

## **1.1 Motivation**

Over the last decade, the increase in economic and environmental concerns has resulted in the fast growth of renewable resource penetration in the electric power grid. In order to ensure increased penetration of renewable resources several states have a mandated renewable portfolio standard (RPS) which requires a certain percentage of the load to be served by renewable resources. The RPS also states by which year the standard has to be met. In California, for example, the RPS requirement is 20% by the year 2012 and 33% by the year 2020 [1]. As a result of accelerated increase of renewable resource development, there is a need for sufficient transmission facilities to deliver this renewable energy to the load centers.

Transmission expansion planning (TEP) addresses the problem of expanding an existing transmission network to serve load centers subject to a set of economic and technical constraints. The problem of insufficient export capability of the transmission system could occur for any type of generation interconnected to the grid. However, the variable and intermittent nature of renewable resources would affect the transmission expansion planning procedure. Hence, the inclusion of renewable resources needs to be treated differently as compared to conventional sources of energy while upgrading the transmission system over the planning horizon. Furthermore, there is a significant variation in the available renewable energy, especially solar and wind energy over a year's time period.

Taking into consideration this intermittent nature of renewable resources, a procedure for transmission expansion planning has been developed in this report. The procedure was tested using a realistic test bed created with the renewable resource information for the state of Arizona, USA.

## **1.2 Research objectives**

The main objectives of this report on TEP for large scale renewable resource penetration are:

- To identify locations that have already been projected for likely development of large scale renewable resources in the Western Electricity Coordinating Council (WECC) region of the USA.
- To develop a system theoretic basis for the identification of new transmission corridors to accommodate these large scale renewable energy resources.
- To develop a realistic test bed to test the proposed planning procedure.

## **1.3 Organization of the report**

The principal contents of the report are developed in 7 chapters and one supplemental section. Chapter 1 presents an overview of the motivation for the study and the study objectives. Chapter 2 presents a literature review of pertinent topics that include previously proposed TEP methods,

renewable resource integration, and a brief introduction of the various software tools used in this report. Chapter 3 deals with the identification of locations in the WECC region that have been projected for likely development of large scale renewable energy resources, with a focus on wind and solar resources. Chapter 4 outlines the specialized TEP procedure proposed and discusses the various steps involved in the same. Chapter 5 deals with the optimization model proposed to determine the most economical and feasible set of lines to be included in the grid to best accommodate the renewable resources. The realistic test bed created for the purpose of testing the TEP procedure is discussed in Chapter 6 along with the results of the simulations and studies for the test bed. The required reliability test and a cost versus benefit analysis are also discussed in Chapter 6. Finally, suitable conclusions of the research work are drawn in Chapter 7 along with the scope for future work in this field.

## 2 Literature review

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### 2.1 Transmission planning methods proposed in literature

Transmission planning models can be broadly classified into optimization models, heuristic models, or a combination of these two types of models [2]. The formulation of the optimization model includes an objective function, which needs to be either minimized or maximized while ensuring that the constraint equations of the model are not violated. In the case of TEP, the objective is usually to minimize the sum of the cost of construction of new lines, the cost of reinforcing existing transmission lines and the operational costs of generators over the planning horizon. The constraint equations of the optimization model ensure that the system is modeled in compliance with the power flow equations and operates reliably.

The main mathematical optimization formulations used for transmission planning are the transportation model, the DC model, the AC model, or a hybrid of these three models [3]. The AC model is the most accurate representation of the system as it models reactive power calculations and system losses, which the other two formulations do not model these aspects. However, since the AC formulation for transmission planning is non-linear and has non-convex constraints, it is the most computationally complex formulation. Furthermore, the non-linear characteristics of the AC model cannot ensure a solution which is the global optimum. The DC model and the transportation model are simplified versions of the AC model that can be represented with linearized system constraints, and hence they are computationally less complex to solve and guarantee a global optimum solution.

Heuristic models usually use a sensitivity index or perform local searches with some logical guidelines specified. Furthermore, heuristic models are usually experience based techniques used to speed up the process of finding a satisfactory solution where an exhaustive search is impractical or the problem is computationally complex. However, heuristic models, unlike linear optimization models cannot guarantee an optimal solution. Many heuristic algorithms have been suggested in the literature to reduce the complexity of the AC model and obtain a solution. These heuristics include a constructive heuristic algorithm implemented for the interior point method [4], a genetic algorithm approach [5], a greedy randomized adaptive search technique [6], and a tabu search approach [7]. Some other methods that have been suggested to solve the optimization problem include a Benders decomposition technique [8] [9] and a Monte Carlo simulation method that considers the uncertainties in long term transmission planning [10]. Additionally, since the optimization model is usually formulated as a mixed integer problem, several heuristics that use the branch and bound algorithm for transmission planning have been proposed in literature [11] [12] [13] [14].

The optimization models suggested in literature for transmission planning tend to use test systems that are small and not representative of a realistic large scale system. A realistic test system usually comprises of an area or multiple areas and could contain thousands of elements (buses, branches, loads etc.). Furthermore, the planning procedure requires several power system software packages to perform reliability studies and economic analyses of transmission plans before they can be approved for construction. An optimization model may be developed to take into consideration all of these factors. However, optimization solvers are not sophisticated enough to efficiently solve for an optimal expansion plan, incorporating all planning

requirements, for a realistic system. The use of an optimization model along with other software to ensure reliability requires system data to be available in all input formats. In order to avoid all of these complications, despite the vast array of transmission planning methods suggested in literature, most utilities prefer to use a case based transmission planning procedure. A limited number of cases are considered over the planning horizon and simulations (mainly power flows) are run for these scenarios along with transient stability studies and short circuit studies. The planner then determines the most economical transmission additions to the grid that will not affect the reliability of the system [15].

## **2.2 Specialized planning algorithms for renewable resource integration**

The idea of using a modified procedure for transmission expansion planning with renewable resource interconnection has been previously proposed in literature. Different planning methods and models are used by the power industry to plan transmission for renewable resources. For example, the Midwest ISO mainly uses a power flow tool for transmission expansion planning with renewable resources [16]. In order to ensure reliability of the proposed expansion plan dynamic simulations, voltage stability and small signal oscillation analysis tools are also employed. The transmission planning process for renewable resources employed at the Midwest ISO can be summarized as follows:

1. Renewable resource forecasting and placement in power flow models
2. Copper sheet analysis (power flow with no limits on transmission capacity) to identify a preliminary transmission plan. This preliminary plan is supplemented with area based contour plots that take into consideration areas lacking in transmission facilities that do not show up in the copper sheet analysis.
3. Use production cost model to identify an expansion plan.
4. Perform reliability assessment and a cost versus benefit analysis for the proposed set of transmission paths to come up with a consolidated transmission plan.

In order to model the intermittent nature of renewable resources, a stochastic model to economically plan transmission expansion was proposed in [16]. This paper highlights the importance of developing a comprehensive transmission planning framework which considers RPS requirements, the available renewable generation in the form of the interconnection queues, and the location of load pockets in the system.

## **2.3 Software tools**

This section briefly describes the key features of the various software tools used in this report.

### **2.3.1 AMPL**

AMPL is a modeling language for linear and nonlinear optimization problems, in discrete or continuous variables [17]. AMPL has the capability to interface with several solvers that include CPLEX, CONOPT, KNITRO, and GUROBI. The optimization model developed in this report is modeled in AMPL as a linear, mixed integer problem and is solved using the GUROBI solver.

GUROBI is a commercial software package that is capable of solving optimization problems with linear constraints and linear or quadratic objective functions [18].

Some non-linear models evaluated in this report are solved using the KNITRO solver since GUROBI cannot handle non-linear constraints in the optimization model. KNITRO is an effective solver for non-linear optimization problems and is capable of handling mixed integer problems as well [19].

### **2.3.2 MATLAB**

MATLAB is a high level technical computing language used for algorithm development, data visualization, data analysis, and numerical computations. Some of the features of MATLAB used for this research are listed below:

- Shape files

A shape file is a digital vector storage format for storing geometric locations and associated attributed information. MATLAB is capable of reading and performing operations on the information in the shape files. The shape files were used to read in the bus, branch, generator, and load information of the system to be studied. This information includes the latitude and longitude of all the buses in the system which was used to calculate the lengths of the transmission lines in the system.

- Read/write Excel and \*.dat files

MATLAB has inbuilt function that can read in data from Microsoft files, perform calculations on them and then output them in any specified format to either an Excel file or a data file. This function comes in extremely handy while handling large amounts of data that cannot be processed manually. Furthermore, since the transmission planning process requires the data to be available to several power system software packages, MATLAB is an excellent medium to read in data from one software package and output to a file format compatible with other software packages. In this report, using shape files and Microsoft Excel files as input to the MATLAB code, the input files to the optimization model (bus and branch data) were created in MATLAB in the data file

### **2.3.3 PowerWorld**

The PowerWorld simulator is a power system simulation package designed to simulate high voltage power systems operation. PowerWorld supports map projections on the one line diagram, i.e., elements on the one line diagram can be represented on a map according to the element's latitude and longitude. This map view helps visualize the power system effectively. Furthermore, PowerWorld is capable of performing the optimal power flow (OPF), transient stability studies and static  $N-1$  reliability tests, and visualizing contour plots which are useful to observe trends across the grid.

### **2.3.4 PROMOD**

PROMOD is a package used for production cost modeling. PROMOD IV is a generator and portfolio modeling system used for nodal LMP forecasting and transmission analysis. PROMOD

takes into consideration the detailed generating unit operations characteristics, renewable generation profiles over the time period under consideration, load variations in the system, transmission grid topology and constraints, and market system operations.

### **2.3.5 PSLF**

The GE PSLF software is designed to perform power flow studies, dynamic simulations and short circuit analyses [20]. Large systems, up to 60,000 buses, can be modeled in PSLF. In this report, The SSTOOLS in PSLF may be used to perform  $N-1$  contingency studies to ensure that the outage of a line or generator does not result in overloading in the rest of the system. The ProvisoHD software tool was used to analyze post-contingency data produced by SSTOOLS. ProvisoHD reads the output produced by the SSTOOLS and presents them in an excel file format, clearly indicating those lines that are overloaded in the contingency study.

### 3 Locating renewable generation in WECC

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The Western Electricity Coordinating Council [22] is a regional reliability entity in the United States responsible for coordinating the bulk electric system in the Western Interconnection. The WECC has the largest geographic area and most diverse system of the eight regional entities under the purview of the North American Electric Reliability Corporation (NERC). Figure 1 shows the WECC region [22]. This chapter of the report presents the potential for likely development of large scale solar PV, solar thermal and wind energy generation in the WECC region.

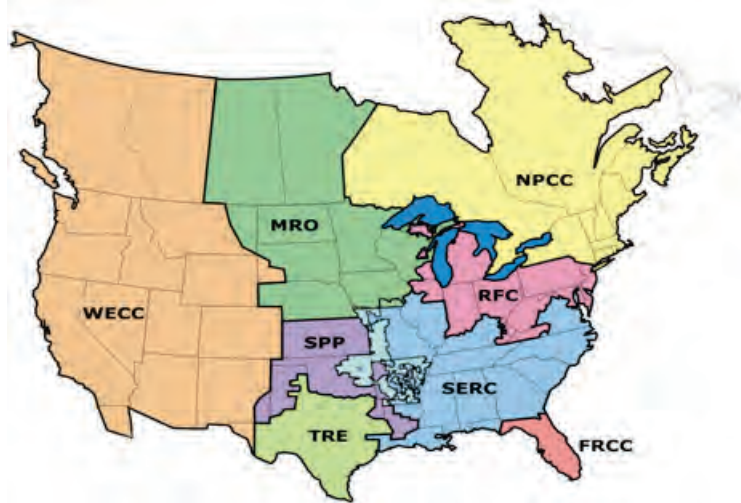


Figure 1: Western Electricity Coordinating Council region [22]

In order to ensure integration of large scale renewable resources in the power grid, several states mandate a Renewable Portfolio Standard (RPS). The RPS is a regulation which states that a specific percentage of the demand in an area has to be met by renewable energy resources. As of March 2009, RPS requirements or goals have been established in 33 states in the US [23]. There is tremendous diversity among these states with respect to the minimum requirements of renewable energy, implementation timing, and eligible technologies and resources. The feasibility of complying with these renewable standards depends on several factors which include the availability of renewable sources of energy, the ability to develop these sources and interconnect them to the grid, and the availability of sufficient transmission capacity to deliver this renewable energy to the load centers. Figure 2 shows the RPS requirements, implementation timings, and the potential solar and wind power generation for different states in the WECC region.

Although there is abundant scope for renewable resources across the WECC, it is important to ensure that the inclusion of these resources is an economical decision and does not result in an increase in costs to the system. Several initiatives like the Western Renewable Energy Zones project (WREZ) are in place to identify the impacts of renewable resource penetration in WECC.

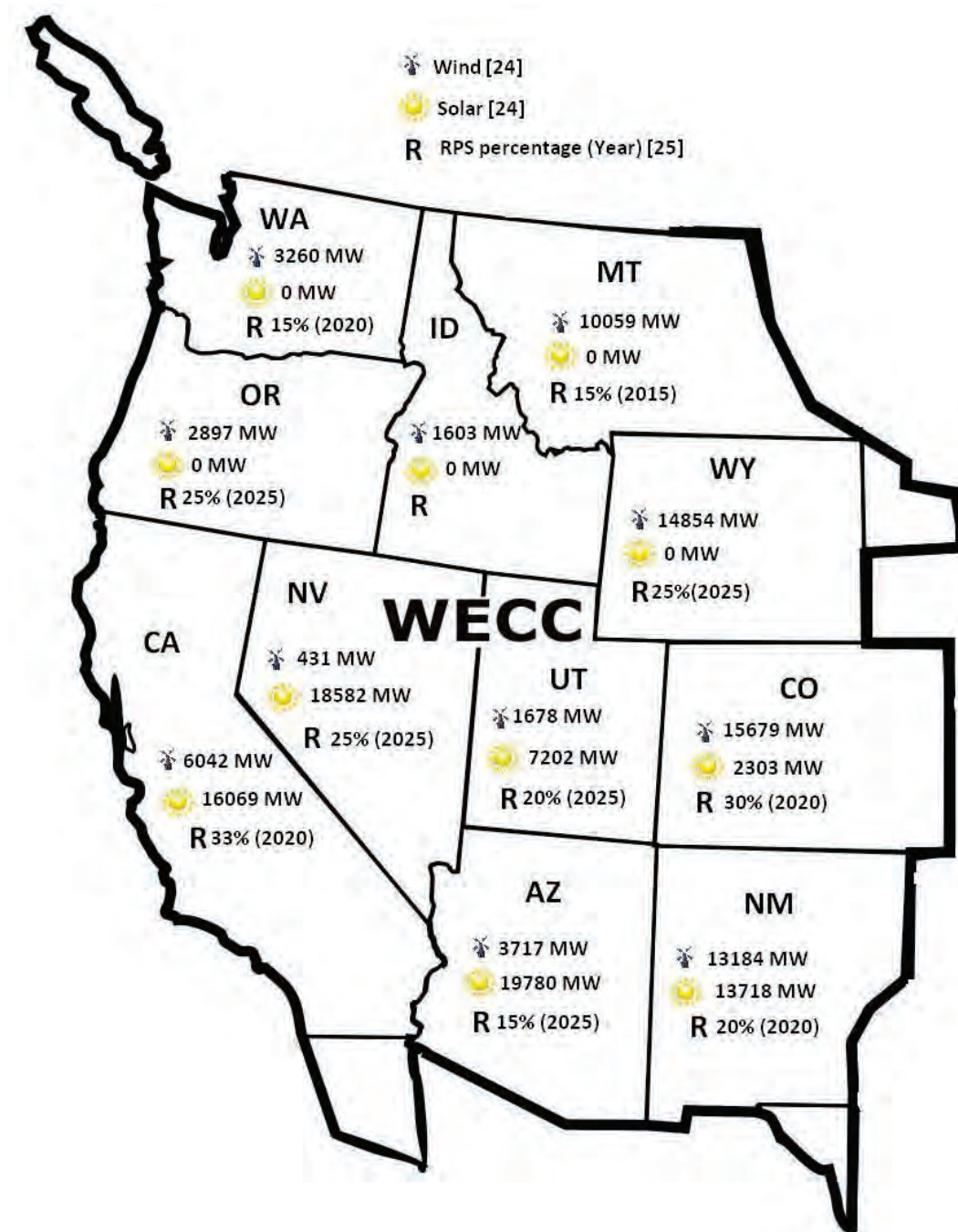


Figure 2: Map of solar and wind generation capacity and RPS requirements in WECC region [23] [24]

## 4 Proposed transmission planning procedure

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This chapter of the report discusses the proposed transmission planning procedure for the inclusion of large scale renewable resources. The various steps of the planning procedure and the software tools used are discussed below.

### 4.1 Step 1: Locating renewable resources

Using the test bed information, a corresponding case is created in PowerWorld. Figure 3 shows a screen shot of the PowerWorld simulator one line diagram.

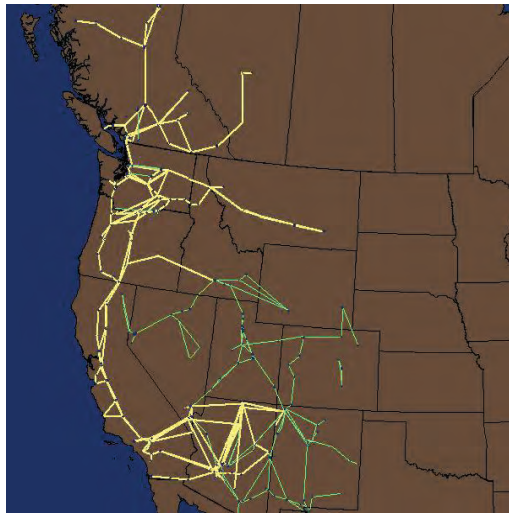


Figure 3: PowerWorld simulator screen shot of the WECC system one-line diagram

The system bus, branch, generator and load parameters along with the corresponding geographic coordinates are output to a shape file. The shape files were processed in MATLAB to calculate the line lengths of the available paths for transmission expansion planning. Furthermore, MATLAB code was written to create the input files to the optimization model.

### 4.2 Step 2: Production cost modeling

Production cost modeling software solves the optimal dispatch of all the power plants in a region over a time period while taking into consideration not only the variable cost of operating each plant, but also the large number of generator and system constraints. Although the production cost simulation may not represent the actual operations of the power system, it may be used to study the impacts of large scale renewable resource penetration in the system. In this report, a production cost model is used to determine scenarios to be considered in the transmission planning process with the large scale integration of renewable resources. It is also used to determine a set of transmission paths to be considered for expansion planning. PROMOD IV is

used to perform production cost simulations over the planning horizon and the results are used to identify transmission congestion in the test system.

The location marginal price (LMP) at a location is the cost of serving an incremental amount of load at the location. LMPs result from the application of a linear programming process, which minimizes the total energy costs for the entire region under consideration, subject to a set of constraints reflecting physical limitations of the power system. The process yields three components of the LMP at every bus as:  $LMP (\$/MWh) = \text{Energy component (ELMP)} + \text{Loss component (LLMP)} + \text{Congestion component (CLMP)}$ . The ELMP is the same for all buses in the system. The LLMP reflects the marginal cost of system losses specific to each location, while the CLMP represents the individual locations marginal transmission congestion cost. In a lossless model, the LMP at any bus is the sum of the energy cost of the system and the congestion component at that bus. In PROMOD, LMPs may be reported for selected zones, or user defined hubs; this may be further broken down into a reference price, a congestion price (showing individual flow gate contributions to congestion), and a marginal loss price. The CLMP is noteworthy in the case of transmission planning as it can be used to decide paths to be considered for transmission expansion. The CLMP represents the cost of congestion for the binding constraints in the market model of the system. If none of the lines in the system are operating at their limits, then the CLMP will be zero for all the buses.

The CLMP obtained from the production cost model is plotted as a contour map in PowerWorld to identify a set of paths that require additional transmission capacity to accommodate large scale renewable resource penetration over the planning horizon. Furthermore, contour maps that exhibit a large difference in the congestion component were observed to represent those scenarios in the planning horizon with a high availability of renewable resources (mainly solar and/or wind).

### **4.3 Step 3: Optimization model**

An optimization model is used to determine an optimum set of lines to be constructed to accommodate large scale penetration of renewable resources. A binary, linear optimization formulation of the DC model was developed. The optimization model was developed in AMPL and solved using the linear solver GUROBI. The input to the optimization model includes the bus and branch data of the system along with the available right of ways for TEP determined in the production cost modeling stage.

The objective of the optimization model is to minimize the cost of construction of new lines and the operational cost of the system with the availability of large scale renewable resources. The optimization model is run for several scenarios identified in the production cost modeling step. The results obtained from all these scenarios are combined to form a comprehensive expansion plan for the planning horizon. Chapter 5 presents a more detailed description of the optimization model developed.

### **4.4 Step 4: Test to ensure N-1 reliability**

Once a comprehensive expansion plan is found using the scenarios from the production cost model and the optimization model results, it is necessary to ensure that the system is robust

against contingencies. According to the NERC standards, power systems are required to be planned and operated such that they can withstand one contingency, i.e., the  $N-1$  contingency criterion. A contingency is defined as the unexpected failure or outage of a system element such as a generator, transmission line, circuit breaker, or switch. To ensure that the inclusion of the proposed plans in the system is  $N-1$  secure, it is required to ensure that a contingency in the system does not cause any system limits to be violated. For example, the outage of any one transmission line in the system should not cause the loading on the other transmission lines to exceed their emergency ratings. The  $N-1$  contingency studies in this report were performed using PSLF.

#### **4.5 Step 5: Cost versus benefit analysis**

The comprehensive transmission expansion plan was devised considering only the scenarios identified in the production cost modeling stage. Hence it is important to justify the construction of new lines for the whole planning horizon, which includes those scenarios that don't have high levels of penetration of renewable resources. This justification is provided through means of a cost versus benefit analysis, which compares the cost of expanding the existing transmission infrastructure and the operational cost savings with the inclusion of renewable resources.

The expected benefits with the integration of large scale renewable resources are:

1. Decrease in operational costs of the system due to the zero fuel costs of the renewable resources, and
2. Greater possibility of meeting the state mandated renewable portfolio standard.

#### 4.6 Summary of transmission planning procedure

A flowchart summarizing the planning procedure is shown in Figure 4.

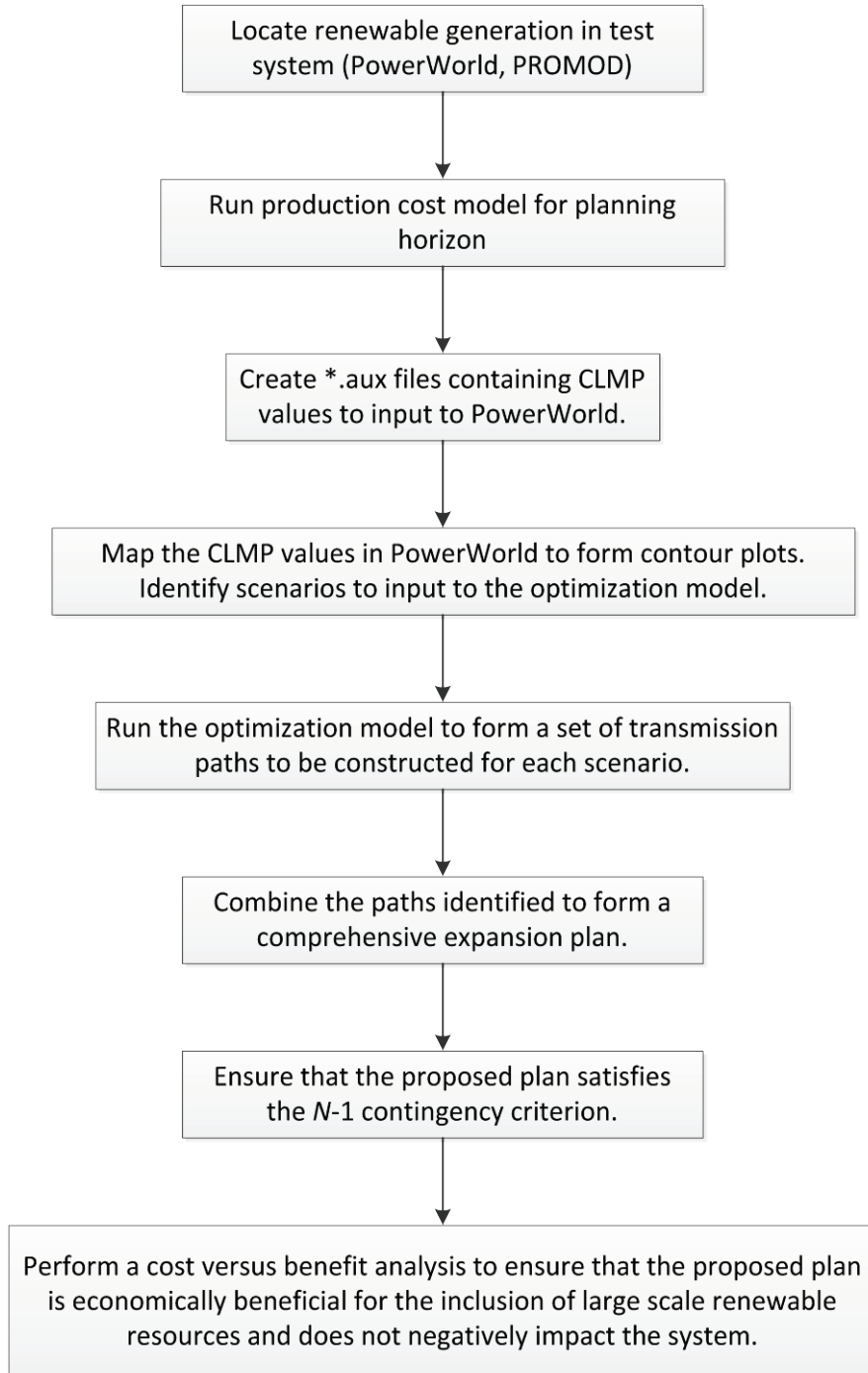


Figure 4: Summary of proposed transmission planning procedure

## 5 Optimization model

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### 5.1 Optimization formulations for TEP

The main mathematical optimization formulations used for transmission planning are the transportation model, the DC model, the AC model, or a hybrid of these three models [3]. The objective function in these three models aims to minimize the cost of construction of new lines in the system. Some of the constraints specified include a line flow constraint, a power balance constraint, and a constraint to limit the generator dispatch values. These formulations are described below along with a comparative study using three test cases in order to determine the most appropriate model to be developed for transmission expansion planning with renewable resource penetration.

- AC model

The AC model for TEP is a non-linear, mixed integer formulation. The AC model is the most accurate representation of the power system. It takes into consideration both the real and reactive power equations that govern the operation of the power system. However, due to its computational complexity, full blown AC models are usually considered only in the later stages of the planning procedure. Furthermore, the non-linear nature of the AC optimization model could result in a solution that is not the global optimum.

The non-linear line flow equations of the AC model are shown below in equation (1) and (2).

$$P_{ij} = V_i^2 G_{ij} - V_i V_j (G_{ij} \cos(\theta_{ij}) - B_{ij} \sin(\theta_{ij})) \quad (1)$$

$$Q_{ij} = -V_i^2 B_{ij} - V_i V_j (G_{ij} \sin(\theta_{ij}) + B_{ij} \cos(\theta_{ij})) \quad (2)$$

where

$P_{ij}$  = real power flow from bus  $i$  to bus  $j$

$Q_{ij}$  = reactive power flow from bus  $i$  to bus  $j$

$V_i$  = voltage magnitude at bus  $i$

$\theta_i$  = voltage phase angle at bus  $i$

$\theta_{ij} = \theta_i - \theta_j$

$G_{ij}$  = conductance of the line between bus  $i$  and bus  $j$

$B_{ij}$  = susceptance of the line between bus  $i$  and bus  $j$

- DC model

The DC power flow model for transmission expansion planning can be represented as a linear, mixed integer optimization model. The DC formulation for transmission expansion planning is an approximation of the AC model that considers only the real power components of the power

system. Furthermore, the DC model assumes a voltage magnitude of 1 per unit at all buses in the system. The line flow equation is approximated as follows

$$f_{ij} = \frac{1}{branch\_x_{ij}} (\theta_{ij}) \quad (3)$$

where

$f_{ij}$  = real power line flow between bus  $i$  and bus  $j$

$branch\_x_{ij}$  = reactance of line between bus  $i$  and bus  $j$

$\theta_i$  = voltage angle at bus  $i$

$\theta_{ij} = \theta_i - \theta_j$

Although the DC model is not as accurate a representation of the system as the AC model, it is computationally less complex. Furthermore, since the DC formulation can be represented as a set of linear constraints, with a linear objective function for a feasible set of data this formulation guarantees a global optimum solution as compared to the AC formulation which can only provide a local optimal solution.

- Transportation model

The transportation model for transmission expansion planning is obtained by relaxing the branch real power flow equation of the DC model. Thus, the line flow calculation equations considered in the AC and DC model are ignored in the transportation model. Only the line limit constraints are used to limit the power flow in the transmission lines. The transportation model could result in an optimal expansion plan which may not be feasible for the DC or AC model of the system.

The three mathematical formulations for transmission expansion planning were tested using three test systems to determine the most suitable model for the transmission planning process with a realistic system. The three test beds are the Garver's 6 bus model [3], the IEEE 14 bus system [25] and IEEE 118 bus system [26].

- Garver's 6 bus test system

The 6 bus test system is one of the most popular test systems in transmission expansion planning research endeavors. The system has 6 buses and 15 right-of-ways for the addition of new circuits. The network topology of the 6 bus system is shown below in Figure 5. The data for this system is given in Appendix B.

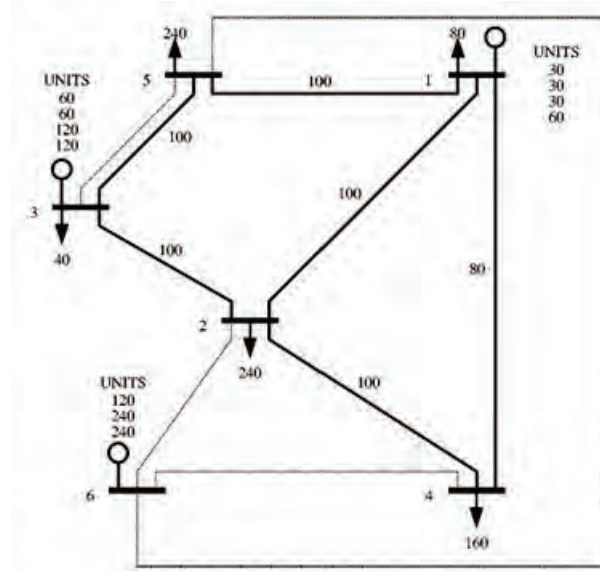


Figure 5: Garver's 6 bus test system

Table 1: TEP Optimization model results for the 6 bus test system

Test model	Model type	Objective function value	Computational time (s)	Results
DC	Non-linear	100	0.281	6,11,14,14
Transportation	Linear	80	0.156	11,14,14
AC	Non-linear	Infeasible	N/A	N/A

- IEEE 14 bus test system

The IEEE 14 bus test case represents a small system in the Midwest region of the American Electric Power Co. system. The system has 14 buses and 19 branches. The bus, branch, generator and load data is shown in Appendix A. The one line diagram of the 14 bus system is shown in Figure 6.



Table 3: TEP Optimization model results for the 118 bus test system

Test Model	Model Type	Objective Function Value	Computational time (s)
DC	Non-linear	47.51	132.203
Transportation	Linear	40.12	0.796
AC	Non-linear	Infeasible	N/A

The conclusions to be drawn from the above comparative study are as follows:

- The transportation model solves the fastest among the three models. However, when the decision variables obtained from the transportation model were tried on a DC and AC power flow formulation, it was found that the transportation model is not necessarily feasible and results in an infeasible AC and DC power flow solution.
- The DC formulation solves faster than the AC model and is more accurate than the transportation model. The solution obtained in the DC model is closer to the actual optimal power flow solution than the transportation model solution.
- Although the test systems represent feasible systems, the AC solution indicates the test systems are infeasible. The AC model results are greatly dependent on the initial conditions provided. Based on these initial conditions a solution that is locally optimal is obtained. The non-linear characteristics of the AC formulation cannot guarantee a global optimum solution.

The need for an approximate DC formulation arises mainly as a result of the limitations of existing optimization solvers and solution techniques that are used for non-linear formulations. Thus, based on the above observations a linear, mixed-integer, DC formulation based optimization model was developed for this report. The details of the developed model are further elaborated upon in Section 5.2 of this report.

## 5.2 Optimization model details

A linear, binary optimization model based on the DC model is formulated in AMPL to solve for an optimum set of transmission lines to be constructed to accommodate renewable resources. The optimization model is solved using the GUROBI solver, which is capable of solving linear, mixed-integer problems. The model needs to consider all the planning scenarios identified in the production cost modeling stage. Hence, it is run for each scenario. The input to the optimization model, the objective function, the system constraints, the output of the optimization model, and other aspects of the optimization model developed are further elaborated upon in the following sub-sections. The full AMPL code written is shown in Appendix C.

### 5.2.1 Input to the optimization model

MATLAB is used to generate the input files to the optimization model. The input is split over three data files: static bus data that does not change with time, branch data, and generator

capacity and load requirement values that vary over time. The different fields included in each of these data files are listed below in Table 4

Table 4: Input to the optimization model

Static bus data (For each bus)	Static branch data (For each branch)	Time varying data (at each bus for every hour of scenario time period)
<ul style="list-style-type: none"> <li>• Bus number</li> <li>• Slack bus (If slack, then 1, else 0),</li> <li>• Generator type</li> <li>• Generator cost function coefficients</li> </ul>	<ul style="list-style-type: none"> <li>• From bus</li> <li>• To bus</li> <li>• Initial state (existing (1) or available for expansion planning (0))</li> <li>• Admittance</li> <li>• Real power limit</li> <li>• Cost of construction</li> </ul>	<ul style="list-style-type: none"> <li>• Max MW generation capacity</li> <li>• Load MW</li> </ul>

### 5.2.2 Decision variables

The purpose of an optimization model is to find the values for the decision variables such that all the constraints are satisfied and the objective function is optimized. The objective function is a function of the decision variables and it is up to the solver to determine appropriate values of the decision variables to ensure that an optimal solution set is obtained. These decision variables can be of different types: binary variables, integer variables, or real variables. The type of decision variables in an optimization model will affect the method used to solve the problem.

The decision variables for the optimization model used for transmission expansion planning are:

1. A binary variable to decide if a line should be added to a right of way ( $x$ ),
2. Bus voltage angle ( $\theta$ ), in radians, required to calculate branch flows in the optimization model,
3. Branch real power flows ( $f$ ) in per unit, and
4. Generator real power dispatch ( $bus\_pgen$ ) in per unit.

### 5.2.3 Objective function

The objective function of an optimization model is the value that needs to be either minimized or maximized without violating the system constraints specified. The objective function needs to be a function of at least one decision variable. The general form of the optimization model is

$$\text{minimize } \sum_{i=1}^n c_i X_i \quad (4)$$

where

$c_i$  = coefficient corresponding to the  $i^{\text{th}}$  variable

$X_i$  = decision variable

For the purpose of transmission expansion planning, it is desired to determine an expansion plan that minimizes the sum of the operation costs of the generators and the cost to construct new lines required for large scale renewable resource penetration. The operational cost of generators is represented as a linear function of the real power output of the generator. The generator cost model is defined by equation (5).

$$C(P_i) = F + V_{OM} P_i \quad (5)$$

where

$P_i$  = Real power output of generator

$F$  = Fixed cost of generator

$V_{OM}$  = Variable cost coefficient

The cost of constructing transmission lines per unit length varies according to voltage levels. In order to make the operational cost of the system over the time frame of the scenarios considered comparable to the cost of constructing new lines, the cost of transmission line construction is scaled as described by equation (6) [27].

$$C_T = \frac{r^* NPV}{n \left[ 1 - \left( \frac{1}{1 + \frac{r}{n}} \right)^{nY} \right]} \quad (6)$$

where

$C_T$  = scale transmission line construction cost

$NPV$  = net present value of transmission line

$Y$  = typical life time of transmission line, usually 25-30 years

$n$  = number of sub-periods to consider within a year

$r$  = annual rate of interest

The  $NPV$  is the cost of construction of the transmission line. It is represented as the sum of a time series of present values ( $C_T$ ) calculated for a scenario's time period. The present values calculated are paid as a series of installments over the lifetime of the transmission line, which is usually assumed to be around 25-30 years. This scaling method is often used to determine the value of an investment over a period of time, especially for long term projects. A discount rate ( $r$ ) is applied to this calculation to adjust for risk and variations of  $C_T$  over time [28]. One of the major drawbacks of using the  $NPV$  method to scale transmission costs to each scenario's duration is that the value of  $C_T$  is very sensitive to the discount rate. Minor variation in  $r$  will result in significant variations in  $C_T$ .

### 5.2.4 Constraints

The constraints of the optimization model place a bound on the values of the decision variables or ensure that their values are found in keeping with certain system conditions. System constraints usually take on the following general form:

$$\text{Subject to } a_{ij}X_i \leq b_j \quad j=1,2,3..n \quad (7)$$

where

$X_i$  = decision variable

$a_{ij}$  = the coefficient of  $X_i$  in the constraint, and

$b_j$  = the right hand side coefficient

$n$  = the number of constraints

The set of constraints for the transmission planning model are to ensure that the solution obtained does not violate node and branch equations of the power system. Furthermore, they impose bounds on generator output and line flows. Each of the constraints included in the optimization model for expansion planning with renewable resource integration are elaborated upon below.

1. Real power conservation at each node

$$\sum_{(k,i)} f_{ki} - \sum_{(:,k)} f_{ik} - P_i + L_i = 0 \quad \forall i \in \text{Bus}, (k,i) \in \text{Branch} \quad (8)$$

2. Line Flow constraints

$$\left| f_{ij} - \frac{1}{\text{branch}_{-}x_{ij}} (\theta_{ij}) \right| \leq M(1 - x_{ij}) \quad (9)$$

$$|f_{ij}| \leq f \max_{ij} x_{ij} \quad (10)$$

3. Generator dispatch limits

$$P_{\min} \leq P \leq P_{\max} \quad (11)$$

4. Angle constraint

$$|\theta_i - \theta_j| \leq 0.6 \quad \forall (i,j) \in \text{Branch} \quad (12)$$

5. RPS constraint, if applicable

$$\sum_i P_i \geq RPS * \sum_j L_j \quad \forall j \in \text{Bus}, i \in \text{Renewable generator} \quad (13)$$

where

$f_{ij}$  = real power flow from bus  $i$  to bus  $j$

$P_i$  = real power generation dispatched at bus  $i$

$L_i$  = real power load at bus  $i$

$RPS$  = renewable portfolio standard, represented as a fraction

$branch\_x_{ij}$  = reactance of line between bus  $i$  and bus  $j$

$M$  = a very large number

$\theta$  = bus voltage phase angle

### 5.2.5 Output of optimization model

The optimization model determines the optimum transmission expansion plan for each of the input scenarios. These output sets are all combined suitably to formulate a comprehensive transmission expansion plan for the planning horizon considered.

## 6 Realistic test bed

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One of the main objectives of this report was to test the proposed transmission expansion planning procedure with a realistic test system. Based on the planning procedure outlined in Chapter 4, the realistic test system was tested and an optimum transmission expansion plan was obtained. The results obtained at each stage of the planning process are discussed below.

### 6.1 Step 1: Creation of a realistic test bed

A test system was created using the renewable resource information for the state of Arizona in the US. The bus, branch, generator, and load data for the WECC region were available. An equivalent system was created in PowerWorld considering all elements within Arizona as the study system and the elements in the other areas as the external system. The external system was modeled as equivalent loads at the inter area tie line buses. A figure of the equivalent system is shown in Figure 7.

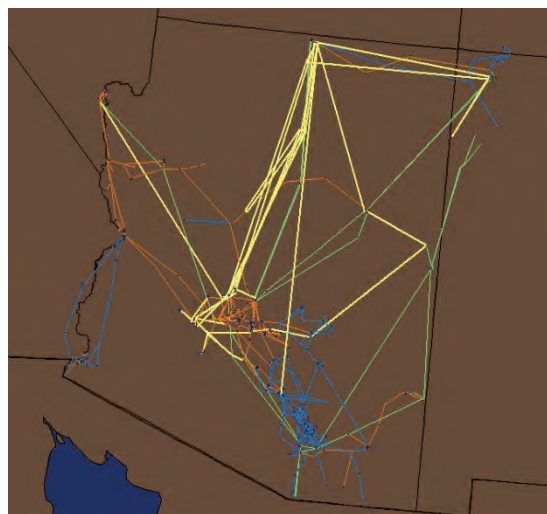


Figure 7: Equivalent test system (AZ) in PowerWorld

Since the slack bus of the WECC system is located outside the state of Arizona, the bus to which the largest generator is connected was defined as the slack bus for the equivalent system. Table 5 summarizes the key parameters of the equivalent system obtained.

Table 5: System parameters of the AZ test bed

<b>No. of Buses</b>	822
<b>No. of Branches</b>	1079
<b>Number of generators</b>	227
<b>Slack bus</b>	15981 – Navajo 1

The available renewable resource information was obtained from the generation interconnection queues of the Arizona Public Service (Appendix D) and the Salt River Project (Appendix D). The renewable resources from these interconnection queues were modeled in the PowerWorld equivalent model. PowerWorld has a GIS interface that can depict the system on a map as was seen in Figure 8 above. A summary of the interconnected renewable resources is presented below in Table 6.

Table 6: Renewable resource integration in test system

Renewable generation type	Connected capacity (MW)
Wind	2763
Solar thermal	3555
Solar PV	3690

## 6.2 Step 2: Production cost modeling

In order to limit the scenarios to be considered for transmission planning by the optimization model a production cost model was used. A case was created in PROMOD that contains information regarding the renewable resources interconnected. The planning horizon considered in this case was the year 2020 since all the renewable resources are expected to be interconnected by 2020. The production cost model was simulated and weekly reports were generated containing the generation output, generation costs and the congestion component of the LMP's at all the buses of the test system. The CLMP was plotted in PowerWorld as a contour plot to identify scenarios that result in congestion in the transmission system. Furthermore, buses that exhibit very high or very low (negative) CLMP were combined to form a set of transmission paths that can be used for transmission expansion planning. A preliminary study of these contour plots for different time periods over the planning horizon revealed four scenarios that could be considered by the optimization model. The contour plots for these four scenarios are shown below in Figures 8-11. Appendix D shows the contour plots of some of the other weeks of the planning period not considered for the optimization model.

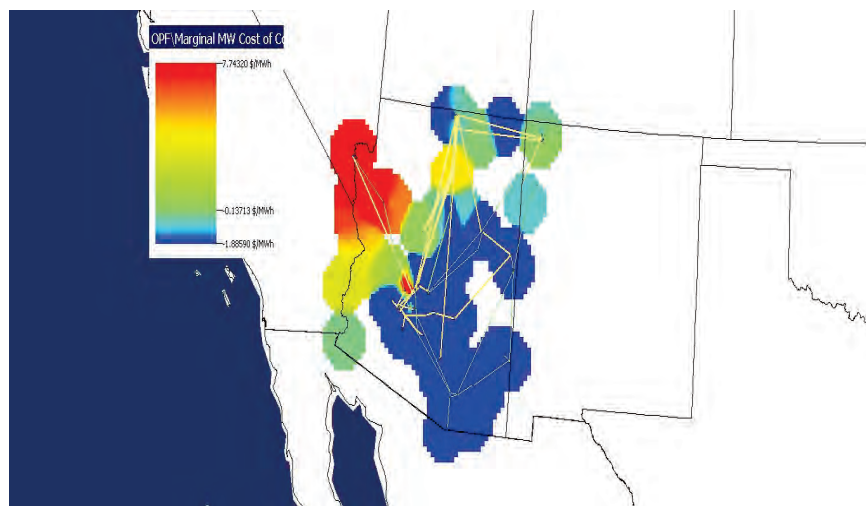


Figure 8: CLMP contour plot for Scenario 1

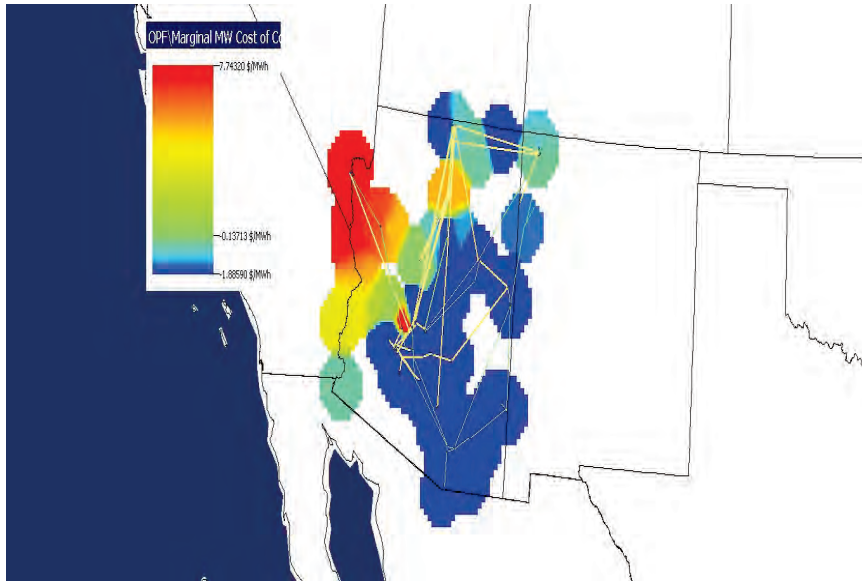


Figure 9: CLMP contour plot for Scenario 2

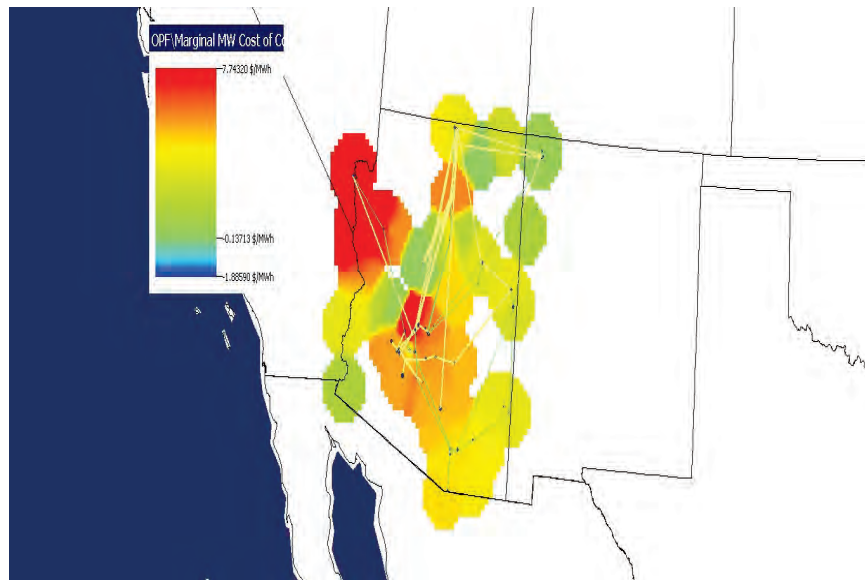


Figure 10: CLMP contour plot for Scenario 3

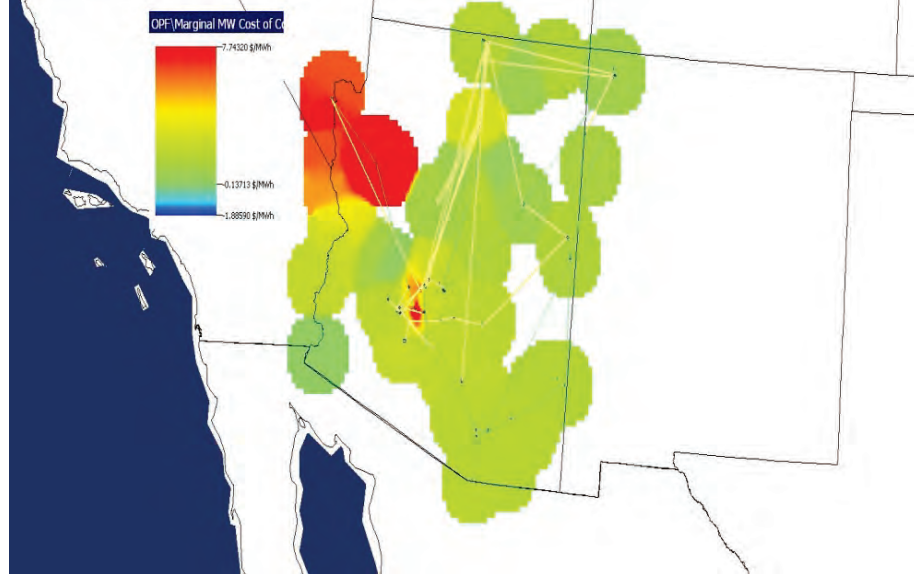


Figure 11: CLMP contour plot for Scenario 4

### 6.3 Step 3 Optimization model

The input files for the optimization model were created using MATLAB. The bus, branch, generator and load shape files from PowerWorld were read in MATLAB. Using the latitude and longitude information of each bus the length of each transmission line was calculated using equation (9)

$$L_{ft} = 3963.1 \left[ a \cos(\sin f_x \sin t_x) + \cos f_x \cos t_x \cos(t_y - t_x) \right] \text{miles} \quad (14)$$

where

$L_{ft}$  = length of line from bus  $f$  to bus  $t$

$f_x$  = latitude of bus  $f$ , in radians

$t_x$  = latitude of bus  $t$ , in radians

$f_y$  = longitude of bus  $f$ , in radians

$t_y$  = longitude of bus  $t$ , in radians

The fuel cost values of various types of generation and the transmission line construction cost values used in the optimization model for this test system are attached in Appendix F. The results of the four scenarios considered in the optimization model are listed below in Table 7.

Table 7: Optimization model results for all scenarios considered

Scenario	Week	Objective function value for week(M\$)	Lines to be constructed	
1	4	11.5382	14235-14238 (2)	
			14007-14238	
2	6	11.9603	14235-14238 (2)	14000-14008
			14007-14238	
3	17	11.3313	14235-14238 (2)	
			14007-14238	
4	34	12.588	No lines to be added	

It was observed from the optimization model results that the suggested set of lines proposed for each scenario was very similar for all of the scenarios. Hence, a union set of the individual lines proposed for each of the scenarios was chosen for the comprehensive transmission expansion plan for the entire planning horizon. The comprehensive expansion plan, along with the key parameters of the lines to be constructed is listed in Table 8.

Table 8: Comprehensive transmission expansion plan for the realistic test bed

From bus	To bus	Voltage (kV)	No. of lines to be constructed	Cost of constructing one line (M\$)
14235	14238	230	2	1.3848
14000	14008	500	1	1.7630
14007	14238	500	1	5.8394

#### 6.4 Step 4: N-1 contingency criterion compliance

A study in PSLF to ensure that the proposed plan satisfies the NERC recommended *N-1* Contingency criterion on the WECC heavy summer case revealed no overloading beyond the emergency limit rating on any lines of the system due to large scale renewable resource penetration. It was also seen that the voltage magnitudes on some of the buses exceeded the permissible limit of 1.05 p.u. and further study is required in this field to ensure that there are no voltage violations for the proposed transmission plan. This static contingency study was performed using the SSTOOLS in PSLF and the data was presented in an excel file format using the ProvisoHD tool.

## 6.5 Step 5: Cost versus benefit analysis

A cost versus benefit analysis was performed on the proposed transmission expansion plan to ensure that it is economically beneficial to construct these lines in order to better facilitate the inclusion of large scale renewable resources in the system.

Table 9 shown below presents the increase in the amount of renewable resource penetration for each scenario considered in the test system with the construction of the lines proposed in the expansion plan. Table 9 shows that the inclusion of the lines proposed in the expansion plan significantly increases the wind resource penetration and thereby decreases the operational cost of generation for the scenarios identified. Furthermore, from the results presented it can also be inferred that there is sufficient transmission capacity for concentrated solar power and solar photo-voltaic resource penetration and the additional lines to be constructed are mainly to facilitate wind resource penetration.

Table 9: Comparative study of output of optimization model before and after the construction of lines proposed

Scenario	Operational cost (M\$/week)		Wind (GWh)		Solar photovoltaic (GWh)		Concentrated solar power (GWh)	
	Before	After	Before	After	Before	After	Before	After
1	10.976	10.673	32.408	82.141	38.946	38.946	16.248	16.248
2	13.294	12.957	37.843	37.843	45.025	45.025	10.506	10.506
3	12.140	11.536	64.017	164.66	81.724	81.724	21.703	21.703
4	12.588	12.588	56.837	56.837	70.677	70.677	15.601	15.601

The net cost of construction of the lines proposed = M\$ 8.9872.

Savings obtained in the operational cost for the 4 weeks considered = M\$ 1.244

Thus, since just 4 weeks of renewable resource penetration results amount to about 14% payback in terms of savings in operational cost, it can be clearly seen that over the life expectancy of the transmission line (25-30 years) the inclusion of the proposed lines will ensure that cheaper renewable generation will be dispatched in the system and hence the overall operation cost of the generators will be reduced. The cost versus benefit analysis presented here is just a preliminary evaluation to ensure that the proposed plan is cost effective and further study is required in this field.

Additional cost factors that need to be considered include reactive power capacity of lines, availability of increased ancillary services to offset the intermittency of renewable resources, and cost of setting up renewable resource generators as compared to conventional generators. On the other hand, the additional benefits provided by renewable resource integration that need to be

considered include increased ease in achieving the RPS, possible profits from carbon credits, and the additional environmental benefit of reduced greenhouse gases.

## 7 Conclusion and future work

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The WECC region has great potential for large scale development of renewable resources. There is an urgent need for transmission grid expansion to accommodate these resources. Renewable resources like wind and solar differ from conventional sources of energy in that they are usually location constrained, intermittent and non-dispatchable. These factors indicate a need for a specialized transmission planning framework that differs from traditional transmission planning for conventional resources.

The expansion planning procedure proposed in this report uses a production cost model to determine scenarios with large scale renewable resources that cause congestion in the existing transmission grid. These scenarios are identified using the CLMP values which are generated for all the buses in the study system over the planning horizon. One of the major drawbacks with the CLMP, as discussed in [29], is that the value of the CLMP may change when a different slack bus is chosen for the study system. Furthermore, different power markets across the world use different methods to calculate the LMP and the CLMP. Therefore, although the CLMP values observed over a long period of time may be used to identify areas prone to transmission line congestion in the system, further work is required to study the impact of the choice of slack bus and the method of calculation of the CLMP on the scenarios identified as input to the optimization model.

The optimization model developed to identify a set of lines to be built for each scenario is based on the DC formulation of the transmission planning procedure. This model is a binary, linear optimization problem that aims to minimize the sum of the operation cost of all the generation dispatched in the system and the cost of transmission line construction. A linear optimization model ensures that the output for a feasible system will be globally optimal. Furthermore, since the optimization model developed takes into consideration the hourly fluctuations in the renewable energy capacity available, it ensures that the savings in operational cost obtained from renewable resource penetration is greater than the cost of constructing lines to accommodate these resources. The optimization model developed in AMPL assumes a lossless system. Several linear loss models have been developed in the literature. However, modeling losses could negatively impact the computational complexity of the optimization model and further work is required to study the impact of losses on the transmission plan obtained.

A major area of concern with renewable resource penetration is the reactive power imbalance created in the system with operating renewable resources. The expansion method proposed in this report takes into consideration only the real power component. In order to have a linear optimization model, the DC formulation assumes a voltage magnitude of 1 per unit at all the buses and considers just the real power equations as constraints. However, it is important to ensure that the expansion plan proposed is AC feasible and does not cause voltage or reactive power imbalance in the system.

In order to make the construction cost of new lines comparable to the operational cost of generators in each scenario, a formula (equation (6)) was used to scale the transmission line costs. A more accurate representation of this formula would be as shown in equation (15), where rather than calculating the cost of construction as annual payments equally divided for all weeks across each year, the payments are calculated as equal weekly payments over the entire life period of the line. In other words, in the formula used in the report, the interest is calculated

annually and then divided by 52 to represent weekly payments. In the formula represented by equation (15), the scaled cost is calculated assuming weekly payments.

$$C = \frac{r^* NPV}{n \left[ 1 - \left( \frac{1}{1 + \frac{r}{n}} \right)^{ny} \right]} \quad (15)$$

where

$C$  = cost of transmission line to be considered for each scenario

$NPV$  = net present value of transmission line

$y$  = typical life time of transmission line, usually 25-30 years

$n$  = number of sub-periods to consider within a year

$r$  = annual rate of interest

Further study is required to determine the most accurate formulation of the objective function since the transmission plan obtained is directly dependent on this formulation.

One of the challenges faced by transmission planners today is the limitations of the software packages needed to plan transmission. No commercially available software is currently capable of handling all the different phases required while planning transmission. Thus, considering the magnitude of most power grids, large amounts of data need to be maintained in order to accurately represent the system in all the different software and any changes made in one software package need to be reflected in all the other software packages. This drawback of maintaining and manually editing large amounts of data is overcome in this report with the use of MATLAB code that reads in the output of one stage of the planning process and creates the necessary input files with the data modifications for the next stage of the planning process.

Presently, the accelerated increase in renewable resource penetration in the US is mainly policy driven. In order to encourage renewable resource integration, several incentives like carbon credits are being offered to renewable generator owners. Carbon credits are tradable certificates that permit the emission of greenhouse gases. Efforts like renewable resource integration that produce lesser greenhouse gases are granted carbon credits and these credits may be traded in the energy market. Since these incentives issued to renewable resources are fairly recent, they need to be studied further to ensure that their short term benefits are taken into consideration. Future work is also required to determine how changes in public policy concerned with renewable resources, lack of incentives, and achieving the RPS may impact the need for transmission expansion for renewable resource penetration.

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## Appendix A: Test systems data

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### 7.1 A.1: Bus test system

Table 10: 6 Bus test system bus data

Bus	Slack	Max Gen (p.u.)	Max Load (p.u.)
1	0	1.5	0.8
2	0	0	2.4
3	0	3.6	0.4
4	0	0	1.6
5	0	0	2.4
6	1	6	0

Table 11: 6 Bus test system branch data

From bus	To bus	$n_0$	$x$	$n_{\max}$	$P_{\max}$	Cost
1	2	1	0.4	2	1	40
1	4	1	0.6	2	0.8	60
1	5	1	0.2	2	1	20
2	3	1	0.2	2	1	20
2	4	1	0.4	2	1	40
2	6	0	0.3	2	1	30
3	5	1	0.2	2	1	20
4	6	0	0.3	2	1	30
2	1	1	0.4	2	1	40
4	1	1	0.6	2	0.8	60
5	1	1	0.2	2	1	20
3	2	1	0.2	2	1	20
4	2	1	0.4	2	1	40
6	2	0	0.3	2	1	30
5	3	1	0.2	2	1	20
6	4	0	0.3	2	1	30
1	3	0	0.38	0	1	38
1	6	0	0.68	0	0.7	68
2	5	0	0.31	0	1	31
3	4	0	0.59	0	0.82	59
3	6	0	0.48	0	1	48
4	5	0	0.63	0	0.75	63
5	6	0	0.61	0	0.78	61
3	1	0	0.38	0	1	38
6	1	0	0.68	0	0.7	68
5	2	0	0.31	0	1	31
4	3	0	0.59	0	0.82	59
6	3	0	0.48	0	1	48
5	4	0	0.63	0	0.75	63
6	5	0	0.61	0	0.78	61

## 7.2 A.2: 14 Bus test system

Table 12: 14 Bus test system bus data

Bus	Slack	Bus_pmax (p.u.)	Bus_pload (p.u.)
1	1	6	0
2	0	1.5	0
3	0	1.5	0
4	0	0	0
5	0	0	5
6	0	0	5
7	0	0	0
8	0	7.5	0
9	0	0	0
10	0	0	0
11	0	0	0
12	0	0	0
13	0	0	0
14	0	0	5

Table 13: 14 Bus test system branch data

From bus	To bus	$n_0$	$r$	$X$	$P_{\max}$	Cost
1	6	0	0.019999	0.145999	1.526808	1.4562
1	7	0	0.029999	0.18	0.585774	1.58864
2	4	0	1	0.022859	1.500062	1.191302
3	4	0	1	0.022859	1.000058	1.191195
4	10	0	0.0065	0.07	2.500118	1.588457
4	10	0	0.0065	0.07	2.500118	1.588457
5	11	0	1	0.316599	0.146305	1.5881
6	5	0	0.014999	0.1	1.083266	1.455851
7	8	0	0.019999	0.15	0.592821	1.588702
9	8	0	0.019999	0.15	0.932316	1.588321
10	6	0	0.5	0.01143	2.456932	1.720897
10	6	0	0.5	0.01143	2.456932	1.720897
11	12	0	0.095459	0.253399	0.148736	1.058768
12	13	0	0.158999	0.422369	0.152785	1.058795
13	14	0	0.0636	0.16895	0.154405	0.926468
1	6	1	0.019999	0.145999	1.526808	1.4562
1	7	1	0.029999	0.18	0.585774	1.58864
1	5	1	0.0013	0.07268	10	8.022044
1	5	1	0.0013	0.07268	10	8.022044
1	5	1	0.0013	0.07268	10	8.022044
1	5	1	0.0013	0.07268	10	8.022044
1	5	1	0.0013	0.07268	10	8.022044
2	4	1	1	0.022859	1.500062	1.191302
3	4	1	1	0.022859	1.000058	1.191195
5	9	1	0.029999	0.18	0.914862	1.588259
5	11	1	1	0.316599	0.146305	1.5881
6	5	1	0.014999	0.1	1.083266	1.455851
7	8	1	0.019999	0.15	0.592821	1.588702
8	14	1	0.0013	0.07268	10	4.102237
8	14	1	0.0013	0.07268	10	4.102237
8	14	1	0.0013	0.07268	10	4.102237
8	14	1	0.0013	0.07268	10	4.102237
8	14	1	0.0013	0.07268	10	4.102237
9	8	1	0.019999	0.15	0.932316	1.588321
11	12	1	0.095459	0.253399	0.148736	1.058768
12	13	1	0.158999	0.422369	0.152785	1.058795
13	14	1	0.0636	0.16895	0.154405	0.926468

### 7.3 A.3: 118 Bus test system

Table 14: 118 Bus test system bus data

Bus	Slack	Pmax	Pload
1	0	0	0.51
2	0	0	0.2
3	0	0	0.39
4	0	1	0.3
5	0	0	0
6	0	1	0.52
7	0	0	0.19
8	0	1	0
9	0	0	0
10	0	5	0
11	0	0	0.7
12	0	3	0.47
13	0	0	0.34
14	0	0	0.14
15	0	1	0.9
16	0	0	0.25
17	0	0	0.11
18	0	1	0.6
19	0	1	0.45
20	0	0	0.18
21	0	0	0.14
22	0	0	0.1
23	0	0	0.07
24	0	1	0
25	0	5	0
26	0	5	0
27	0	1	0.62
28	0	0	0.17
29	0	0	0.24
30	0	0	0
31	0	1	0.43
32	0	1	0.59
33	0	0	0.23
34	0	1	0.59
35	0	0	0.33
36	0	1	0.31
37	0	0	0
38	0	0	0
39	0	0	0.27

Table 14: 118 Bus test system bus data (continued)

40	0	1	0.2
41	0	0	0.37
42	0	1	0.37
43	0	0	0.18
44	0	0	0.16
45	0	0	0.53
46	0	1	0.28
47	0	0	0.34
48	0	0	0.2
49	0	3	0.87
50	0	0	0.17
51	0	0	0.17
52	0	0	0.18
53	0	0	0.23
54	0	1	1.13
55	0	1	0.63
56	0	1	0.84
57	0	0	0.12
58	0	0	0.12
59	0	3	2.77
60	0	0	0.78
61	0	3	0
62	0	1	0.77
63	0	0	0
64	0	0	0
65	0	5	0
66	0	5	0.39
67	0	0	0.28
68	0	0	0
69	1	5	0
70	0	1	0.66
71	0	0	0
72	0	1	0
73	0	1	0
74	0	1	0.68
75	0	0	0.47
76	0	1	0.68
77	0	1	0.61
78	0	0	0.71
79	0	0	0.39
80	0	5	1.3
81	0	0	0

Table 14: 118 Bus test system bus data (continued)

82	0	1	0.54
83	0	0	0.2
84	0	0	0.11
85	0	1	0.24
86	0	0	0.21
87	0	1	0
88	0	0	0.48
89	0	5	0
90	0	1	0.78
91	0	1	0
92	0	1	0.65
93	0	0	0.12
94	0	0	0.3
95	0	0	0.42
96	0	0	0.38
97	0	0	0.15
98	0	0	0.34
99	0	1	0
100	0	5	0.37
101	0	0	0.22
102	0	0	0.05
103	0	1	0.23
104	0	1	0.38
105	0	1	0.31
106	0	0	0.43
107	0	1	0.28
108	0	0	0.02
109	0	0	0.08
110	0	1	0.39
111	0	1	0
112	0	1	0.25
113	0	1	0
114	0	0	0.08
115	0	0	0.22
116	0	1	0
117	0	0	0.2
118	0	0	0.33

Table 15: 118 Bus test system branch data

1	4	11	0	0.0209	0.0688	0.641	68.8
2	5	6	0	0.0119	0.054	0.884	54
3	8	5	0	0	0.0267	3.382	26.7
4	5	11	0	0.0203	0.0682	0.771	68.2
5	6	7	0	0.0046	0.0208	0.354	20.8
6	7	12	0	0.0086	0.034	0.164	34
7	8	9	0	0.0024	0.0305	4.452	30.5
8	8	30	0	0.0043	0.0504	0.745	50.4
9	9	10	0	0.0026	0.0322	4.5	32.2
10	11	12	0	0.0059	0.0196	0.342	19.6
11	11	13	0	0.0225	0.0731	0.349	73.1
12	12	14	0	0.0215	0.0707	0.181	70.7
13	12	16	0	0.0212	0.0834	0.076	83.4
14	12	117	0	0.0329	0.014	0.201	14
15	13	15	0	0.0744	0.2444	0.006	244.4
16	14	15	0	0.0595	0.195	0.04	195
17	15	17	0	0.0132	0.0437	1.034	43.7
18	15	19	0	0.012	0.0394	0.11	39.4
19	15	33	0	0.038	0.1244	0.054	124.4
20	16	17	0	0.0454	0.1801	0.175	180.1
21	17	18	0	0.0123	0.0505	0.792	50.5
22	30	17	0	0	0.0388	2.312	38.8
23	17	31	0	0.0474	0.1563	0.113	156.3
24	17	113	0	0.0091	0.0301	0.088	30.1
25	18	19	0	0.0112	0.0493	0.184	49.3
26	19	20	0	0.0252	0.117	0.103	117
27	19	34	0	0.0752	0.247	0.055	247
28	20	21	0	0.0183	0.0849	0.285	84.9
29	21	22	0	0.0209	0.097	0.429	97
30	22	23	0	0.0342	0.159	0.539	159
31	23	24	0	0.0135	0.0492	0.121	49.2
32	23	25	0	0.0156	0.08	1.68	80
33	23	32	0	0.0317	0.1153	0.907	115.3
34	24	70	0	0.1022	0.4115	0.039	411.5
35	24	72	0	0.0488	0.196	0.029	196
36	26	25	0	0	0.0382	0.902	38.2
37	25	27	0	0.0318	0.163	1.422	163
38	26	30	0	0.008	0.086	2.238	86
39	27	28	0	0.0191	0.0855	0.312	85.5
40	27	32	0	0.0229	0.0755	0.128	75.5
41	27	115	0	0.0164	0.0741	0.209	74.1

Table 15: 118 Bus test system branch data (continued)

42	28	29	0	0.0237	0.0943	0.14	94.3
43	29	31	0	0.0108	0.0331	0.1	33.1
44	30	38	0	0.0046	0.054	0.628	54
45	31	32	0	0.0298	0.0985	0.266	98.5
46	113	31	0	0	0.1	0.086	100
47	32	113	0	0.0615	0.203	0.06	203
48	32	114	0	0.0135	0.0612	0.092	61.2
49	33	37	0	0.0415	0.142	0.177	142
50	34	36	0	0.0087	0.0268	0.302	26.8
51	34	37	0	0.0026	0.0094	0.976	9.4
52	34	43	0	0.0413	0.1681	0.027	168.1
53	35	36	0	0.0022	0.0102	0.009	10.2
54	35	37	0	0.011	0.0497	0.341	49.7
55	38	37	0	0	0.0375	2.264	37.5
56	37	39	0	0.0321	0.106	0.437	106
57	37	40	0	0.0593	0.168	0.332	168
58	38	65	0	0.009	0.0986	1.664	98.6
59	39	40	0	0.0184	0.0605	0.161	60.5
60	40	41	0	0.0145	0.0487	0.05	48.7
61	40	42	0	0.0555	0.183	0.227	183
62	41	42	0	0.041	0.135	0.325	135
63	42	49	0	0.0715	0.323	0.523	323
64	42	49	0	0.0715	0.323	0.523	323
65	42	49	0	0.0715	0.323	0.523	323
66	43	44	0	0.0608	0.2454	0.155	245.4
67	44	45	0	0.0224	0.0901	0.317	90.1
68	45	46	0	0.04	0.1356	0.36	135.6
69	45	49	0	0.0684	0.186	0.51	186
70	46	47	0	0.038	0.127	0.305	127
71	46	48	0	0.0601	0.189	0.15	189
72	47	49	0	0.0191	0.0625	0.121	62.5
73	47	69	0	0.0844	0.2778	0.548	277.8
74	48	49	0	0.0179	0.0505	0.352	50.5
75	49	50	0	0.0267	0.0752	0.497	75.2
76	49	51	0	0.0486	0.137	0.616	137
77	49	54	0	0.073	0.289	0.33	289
78	49	54	0	0.0869	0.291	0.331	291
79	49	54	0	0.073	0.289	0.33	289
80	49	66	0	0.018	0.0919	1.036	91.9
81	49	66	0	0.018	0.0919	1.036	91.9
82	49	66	0	0.018	0.0919	1.036	91.9

Table 15: 118 Bus test system branch data (continued)

83	49	69	0	0.0985	0.324	0.449	324
84	50	57	0	0.0474	0.134	0.32	134
85	51	52	0	0.0203	0.0588	0.268	58.8
86	51	58	0	0.0255	0.0719	0.158	71.9
87	52	53	0	0.0405	0.1635	0.086	163.5
88	53	54	0	0.0263	0.122	0.145	122
89	54	55	0	0.0169	0.0707	0.098	70.7
90	54	56	0	0.0027	0.0096	0.276	9.6
91	54	59	0	0.0503	0.2293	0.211	229.3
92	55	56	0	0.0049	0.0151	0.28	15.1
93	55	59	0	0.0474	0.2158	0.257	215.8
94	56	57	0	0.0343	0.0966	0.194	96.6
95	56	58	0	0.0343	0.0966	0.038	96.6
96	56	59	0	0.0825	0.251	0.205	251
97	56	59	0	0.0803	0.239	0.215	239
98	56	59	0	0.0825	0.251	0.205	251
99	59	60	0	0.0317	0.145	0.403	145
100	59	61	0	0.0328	0.15	0.491	150
101	63	59	0	0	0.0386	1.432	38.6
102	60	61	0	0.0026	0.0135	1.123	13.5
103	60	62	0	0.0123	0.0561	0.063	56.1
104	61	62	0	0.0082	0.0376	0.308	37.6
105	64	61	0	0	0.0268	0.322	26.8
106	62	66	0	0.0482	0.218	0.334	218
107	62	67	0	0.0258	0.117	0.2	117
108	63	64	0	0.0017	0.02	1.437	20
109	64	65	0	0.0027	0.0302	1.768	30.2
110	65	66	0	0	0.037	0.399	37
111	65	68	0	0.0014	0.016	0.078	16
112	66	67	0	0.0224	0.1015	0.486	101.5
113	68	69	0	0	0.037	1.371	37
114	68	81	0	0.0018	0.0202	0.392	20.2
115	68	116	0	0.0003	0.0041	1.841	4.1
116	69	70	0	0.03	0.127	1.047	127
117	69	75	0	0.0405	0.122	1.067	122
118	69	77	0	0.0309	0.101	0.552	101
119	70	71	0	0.0088	0.0355	0.152	35.5
120	70	74	0	0.0401	0.1323	0.163	132.3
121	70	75	0	0.0428	0.141	0	141
122	71	72	0	0.0446	0.18	0.091	180
123	71	73	0	0.0087	0.0454	0.06	45.4

Table 15: 118 Bus test system branch data (continued)

124	74	75	0	0.0123	0.0406	0.522	40.6
125	75	77	0	0.0601	0.1999	0.371	199.9
126	75	118	0	0.0145	0.0481	0.39	48.1
127	76	77	0	0.0444	0.148	0.646	148
128	76	118	0	0.0164	0.0544	0.057	54.4
129	77	78	0	0.0038	0.0124	0.577	12.4
130	77	80	0	0.017	0.0485	0.71	48.5
131	77	80	0	0.0294	0.105	0.323	105
132	77	80	0	0.017	0.0485	0.71	48.5
133	77	82	0	0.0298	0.0853	0.059	85.3
134	78	79	0	0.0055	0.0244	0.135	24.4
135	79	80	0	0.0156	0.0704	0.53	70.4
136	81	80	0	0	0.037	0.392	37
137	80	96	0	0.0356	0.182	0.158	182
138	80	97	0	0.0183	0.0934	0.233	93.4
139	80	98	0	0.0238	0.108	0.254	108
140	80	99	0	0.0454	0.206	0.16	206
141	82	83	0	0.0112	0.0366	0.359	36.6
142	82	96	0	0.0162	0.053	0.124	53
143	83	84	0	0.0625	0.132	0.203	132
144	83	85	0	0.043	0.148	0.367	148
145	84	85	0	0.0302	0.0641	0.316	64.1
146	85	86	0	0.035	0.123	0.172	123
147	85	88	0	0.02	0.102	0.448	102
148	85	89	0	0.0239	0.173	0.662	173
149	86	87	0	0.02828	0.2074	0.04	207.4
150	88	89	0	0.0139	0.0712	0.94	71.2
151	89	90	0	0.0518	0.188	0.419	188
152	89	90	0	0.0238	0.0997	0.797	99.7
153	89	90	0	0.0518	0.188	0.419	188
154	89	92	0	0.0099	0.0505	1.224	50.5
155	89	92	0	0.0393	0.1581	0.385	158.1
156	89	92	0	0.0099	0.0505	1.224	50.5
157	91	90	0	0.0254	0.0836	0.028	83.6
158	91	92	0	0.0387	0.1272	0.129	127.2
159	92	93	0	0.0258	0.0848	0.628	84.8
160	92	94	0	0.0481	0.158	0.573	158
161	92	100	0	0.0648	0.295	0.343	295
162	92	102	0	0.0123	0.0559	0.475	55.9
163	93	94	0	0.0223	0.0732	0.497	73.2
164	94	95	0	0.0132	0.0434	0.45	43.4

Table 15: 118 Bus test system branch data (continued)

165	94	96	0	0.0269	0.0869	0.245	86.9
166	94	100	0	0.0178	0.058	0.053	58
167	95	96	0	0.0171	0.0547	0.027	54.7
168	96	97	0	0.0173	0.0885	0.081	88.5
169	98	100	0	0.0397	0.179	0.088	179
170	99	100	0	0.018	0.0813	0.263	81.3
171	100	101	0	0.0277	0.1262	0.197	126.2
172	100	103	0	0.016	0.0525	1.203	52.5
173	100	104	0	0.0451	0.204	0.571	204
174	100	106	0	0.0605	0.229	0.605	229
175	101	102	0	0.0246	0.112	0.422	112
176	103	104	0	0.0466	0.1584	0.324	158.4
177	103	105	0	0.0535	0.1625	0.429	162.5
178	103	110	0	0.0391	0.1813	0.597	181.3
179	104	105	0	0.0099	0.0378	0.496	37.8
180	105	106	0	0.014	0.0547	0.087	54.7
181	105	107	0	0.053	0.183	0.269	183
182	105	108	0	0.0261	0.0703	0.247	70.3
183	106	107	0	0.053	0.183	0.239	183
184	108	109	0	0.0105	0.0288	0.226	28.8
185	109	110	0	0.0278	0.0762	0.145	76.2
186	110	111	0	0.022	0.0755	0.36	75.5
187	110	112	0	0.0247	0.064	0.695	64
188	114	115	0	0.0023	0.0104	0.012	10.4
189	4	11	1	0.0209	0.0688	0.641	68.8
190	5	6	1	0.0119	0.054	0.884	54
191	8	5	1	0	0.0267	3.382	26.7
192	5	11	1	0.0203	0.0682	0.771	68.2
193	6	7	1	0.0046	0.0208	0.354	20.8
194	7	12	1	0.0086	0.034	0.164	34
195	8	9	1	0.0024	0.0305	4.452	30.5
196	8	30	1	0.0043	0.0504	0.745	50.4
197	9	10	1	0.0026	0.0322	4.5	32.2
198	11	12	1	0.0059	0.0196	0.342	19.6
199	11	13	1	0.0225	0.0731	0.349	73.1
200	12	14	1	0.0215	0.0707	0.181	70.7
201	12	16	1	0.0212	0.0834	0.076	83.4
202	12	117	1	0.0329	0.014	0.201	14
203	13	15	1	0.0744	0.2444	0.006	244.4
204	14	15	1	0.0595	0.195	0.04	195
205	15	17	1	0.0132	0.0437	1.034	43.7

Table 15: 118 Bus test system branch data (continued)

206	15	19	1	0.012	0.0394	0.11	39.4
207	15	33	1	0.038	0.1244	0.054	124.4
208	16	17	1	0.0454	0.1801	0.175	180.1
209	17	18	1	0.0123	0.0505	0.792	50.5
210	30	17	1	0	0.0388	2.312	38.8
211	17	31	1	0.0474	0.1563	0.113	156.3
212	17	113	1	0.0091	0.0301	0.088	30.1
213	18	19	1	0.0112	0.0493	0.184	49.3
214	19	20	1	0.0252	0.117	0.103	117
215	19	34	1	0.0752	0.247	0.055	247
216	20	21	1	0.0183	0.0849	0.285	84.9
217	21	22	1	0.0209	0.097	0.429	97
218	22	23	1	0.0342	0.159	0.539	159
219	23	24	1	0.0135	0.0492	0.121	49.2
220	23	25	1	0.0156	0.08	1.68	80
221	23	32	1	0.0317	0.1153	0.907	115.3
222	24	70	1	0.1022	0.4115	0.039	411.5
223	24	72	1	0.0488	0.196	0.029	196
224	26	25	1	0	0.0382	0.902	38.2
225	25	27	1	0.0318	0.163	1.422	163
226	26	30	1	0.008	0.086	2.238	86
227	27	28	1	0.0191	0.0855	0.312	85.5
228	27	32	1	0.0229	0.0755	0.128	75.5
229	27	115	1	0.0164	0.0741	0.209	74.1
230	28	29	1	0.0237	0.0943	0.14	94.3
231	29	31	1	0.0108	0.0331	0.1	33.1
232	30	38	1	0.0046	0.054	0.628	54
233	31	32	1	0.0298	0.0985	0.266	98.5
234	113	31	1	0	0.1	0.086	100
235	32	113	1	0.0615	0.203	0.06	203
236	32	114	1	0.0135	0.0612	0.092	61.2
237	33	37	1	0.0415	0.142	0.177	142
238	34	36	1	0.0087	0.0268	0.302	26.8
239	34	37	1	0.0026	0.0094	0.976	9.4
240	34	43	1	0.0413	0.1681	0.027	168.1
241	35	36	1	0.0022	0.0102	0.009	10.2
242	35	37	1	0.011	0.0497	0.341	49.7
243	38	37	1	0	0.0375	2.264	37.5
244	37	39	1	0.0321	0.106	0.437	106
245	37	40	1	0.0593	0.168	0.332	168
246	38	65	1	0.009	0.0986	1.664	98.6

Table 15: 118 Bus test system branch data (continued)

247	39	40	1	0.0184	0.0605	0.161	60.5
248	40	41	1	0.0145	0.0487	0.05	48.7
249	40	42	1	0.0555	0.183	0.227	183
250	41	42	1	0.041	0.135	0.325	135
251	42	49	1	0.0715	0.323	0.523	323
252	42	49	1	0.0715	0.323	0.523	323
253	42	49	1	0.0715	0.323	0.523	323
254	43	44	1	0.0608	0.2454	0.155	245.4
255	44	45	1	0.0224	0.0901	0.317	90.1
256	45	46	1	0.04	0.1356	0.36	135.6
257	45	49	1	0.0684	0.186	0.51	186
258	46	47	1	0.038	0.127	0.305	127
259	46	48	1	0.0601	0.189	0.15	189
260	47	49	1	0.0191	0.0625	0.121	62.5
261	47	69	1	0.0844	0.2778	0.548	277.8
262	48	49	1	0.0179	0.0505	0.352	50.5
263	49	50	1	0.0267	0.0752	0.497	75.2
264	49	51	1	0.0486	0.137	0.616	137
265	49	54	1	0.073	0.289	0.33	289
266	49	54	1	0.0869	0.291	0.331	291
267	49	54	1	0.073	0.289	0.33	289
268	49	66	1	0.018	0.0919	1.036	91.9
269	49	66	1	0.018	0.0919	1.036	91.9
270	49	66	1	0.018	0.0919	1.036	91.9
271	49	69	1	0.0985	0.324	0.449	324
272	50	57	1	0.0474	0.134	0.32	134
273	51	52	1	0.0203	0.0588	0.268	58.8
274	51	58	1	0.0255	0.0719	0.158	71.9
275	52	53	1	0.0405	0.1635	0.086	163.5
276	53	54	1	0.0263	0.122	0.145	122
277	54	55	1	0.0169	0.0707	0.098	70.7
278	54	56	1	0.0027	0.0096	0.276	9.6
279	54	59	1	0.0503	0.2293	0.211	229.3
280	55	56	1	0.0049	0.0151	0.28	15.1
281	55	59	1	0.0474	0.2158	0.257	215.8
282	56	57	1	0.0343	0.0966	0.194	96.6
283	56	58	1	0.0343	0.0966	0.038	96.6
284	56	59	1	0.0825	0.251	0.205	251
285	56	59	1	0.0803	0.239	0.215	239
286	56	59	1	0.0825	0.251	0.205	251
287	59	60	1	0.0317	0.145	0.403	145

Table 15: 118 Bus test system branch data (continued)

288	59	61	1	0.0328	0.15	0.491	150
289	63	59	1	0	0.0386	1.432	38.6
290	60	61	1	0.0026	0.0135	1.123	13.5
291	60	62	1	0.0123	0.0561	0.063	56.1
292	61	62	1	0.0082	0.0376	0.308	37.6
293	64	61	1	0	0.0268	0.322	26.8
294	62	66	1	0.0482	0.218	0.334	218
295	62	67	1	0.0258	0.117	0.2	117
296	63	64	1	0.0017	0.02	1.437	20
297	64	65	1	0.0027	0.0302	1.768	30.2
298	65	66	1	0	0.037	0.399	37
299	65	68	1	0.0014	0.016	0.078	16
300	66	67	1	0.0224	0.1015	0.486	101.5
301	68	69	1	0	0.037	1.371	37
302	68	81	1	0.0018	0.0202	0.392	20.2
303	68	116	1	0.0003	0.0041	1.841	4.1
304	69	70	1	0.03	0.127	1.047	127
305	69	75	1	0.0405	0.122	1.067	122
306	69	77	1	0.0309	0.101	0.552	101
307	70	71	1	0.0088	0.0355	0.152	35.5
308	70	74	1	0.0401	0.1323	0.163	132.3
309	70	75	1	0.0428	0.141	0	141
310	71	72	1	0.0446	0.18	0.091	180
311	71	73	1	0.0087	0.0454	0.06	45.4
312	74	75	1	0.0123	0.0406	0.522	40.6
313	75	77	1	0.0601	0.1999	0.371	199.9
314	75	118	1	0.0145	0.0481	0.39	48.1
315	76	77	1	0.0444	0.148	0.646	148
316	76	118	1	0.0164	0.0544	0.057	54.4
317	77	78	1	0.0038	0.0124	0.577	12.4
318	77	80	1	0.017	0.0485	0.71	48.5
319	77	80	1	0.0294	0.105	0.323	105
320	77	80	1	0.017	0.0485	0.71	48.5
321	77	82	1	0.0298	0.0853	0.059	85.3
322	78	79	1	0.0055	0.0244	0.135	24.4
323	79	80	1	0.0156	0.0704	0.53	70.4
324	81	80	1	0	0.037	0.392	37
325	80	96	1	0.0356	0.182	0.158	182
326	80	97	1	0.0183	0.0934	0.233	93.4
327	80	98	1	0.0238	0.108	0.254	108
328	80	99	1	0.0454	0.206	0.16	206

Table 15: 118 Bus test system branch data (continued)

329	82	83	1	0.0112	0.0366	0.359	36.6
330	82	96	1	0.0162	0.053	0.124	53
331	83	84	1	0.0625	0.132	0.203	132
332	83	85	1	0.043	0.148	0.367	148
333	84	85	1	0.0302	0.0641	0.316	64.1
334	85	86	1	0.035	0.123	0.172	123
335	85	88	1	0.02	0.102	0.448	102
336	85	89	1	0.0239	0.173	0.662	173
337	86	87	1	0.02828	0.2074	0.04	207.4
338	88	89	1	0.0139	0.0712	0.94	71.2
339	89	90	1	0.0518	0.188	0.419	188
340	89	90	1	0.0238	0.0997	0.797	99.7
341	89	90	1	0.0518	0.188	0.419	188
342	89	92	1	0.0099	0.0505	1.224	50.5
343	89	92	1	0.0393	0.1581	0.385	158.1
344	89	92	1	0.0099	0.0505	1.224	50.5
345	91	90	1	0.0254	0.0836	0.028	83.6
346	91	92	1	0.0387	0.1272	0.129	127.2
347	92	93	1	0.0258	0.0848	0.628	84.8
348	92	94	1	0.0481	0.158	0.573	158
349	92	100	1	0.0648	0.295	0.343	295
350	92	102	1	0.0123	0.0559	0.475	55.9
351	93	94	1	0.0223	0.0732	0.497	73.2
352	94	95	1	0.0132	0.0434	0.45	43.4
353	94	96	1	0.0269	0.0869	0.245	86.9
354	94	100	1	0.0178	0.058	0.053	58
355	95	96	1	0.0171	0.0547	0.027	54.7
356	96	97	1	0.0173	0.0885	0.081	88.5
357	98	100	1	0.0397	0.179	0.088	179
358	99	100	1	0.018	0.0813	0.263	81.3
359	100	101	1	0.0277	0.1262	0.197	126.2
360	100	103	1	0.016	0.0525	1.203	52.5
361	100	104	1	0.0451	0.204	0.571	204
362	100	106	1	0.0605	0.229	0.605	229
363	101	102	1	0.0246	0.112	0.422	112
364	103	104	1	0.0466	0.1584	0.324	158.4
365	103	105	1	0.0535	0.1625	0.429	162.5
366	103	110	1	0.0391	0.1813	0.597	181.3
367	104	105	1	0.0099	0.0378	0.496	37.8
368	105	106	1	0.014	0.0547	0.087	54.7
369	105	107	1	0.053	0.183	0.269	183

Table 15: 118 Bus test system branch data (continued)

370	105	108	1	0.0261	0.0703	0.247	70.3
371	106	107	1	0.053	0.183	0.239	183
372	108	109	1	0.0105	0.0288	0.226	28.8
373	109	110	1	0.0278	0.0762	0.145	76.2
374	110	111	1	0.022	0.0755	0.36	75.5
375	110	112	1	0.0247	0.064	0.695	64
376	114	115	1	0.0023	0.0104	0.012	10.4

## Appendix B: Generation interconnection queues

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Table 16: APS and SRP generation interconnection queue

Bus number	Bus name	Nameplate rating (MW)	Gen. type
14000	Cholla 500kv	300	Wind
14002	Moenkopi 500kv	1601	Wind
14100	Cholla 345kv	740	Wind
14201	Buckeye 230kv	378	Solar pv
14204	Cholla 230kv	90	Solar PV
14204	Cholla 230kv	442.9	Wind
14209	EagleEye 230kv	40	Solar PV
14228	Surprise 230kv	40	Solar PV
14234	Yavapai 230kv	20	Solar PV
14235	GilaBend 230kv	955	Solar - PV
14235	GilaBend 230kv	1310	Solar CST
14235	GilaBend 230kv	850	Solar PV
14244	Seligman	12	Solar PV
14244	Seligman	260	Wind
14250	WillowLake	20	Solar PV
14250	WillowLake	120	Wind
15090	Hassayampa 500kv	300	Solar CST
15093	Harquahala Valley	400	Solar CLFR
15093	Harquahala Valley	300	Solar CST
15093	Harquahala Valley	60	Solar PV
15094	Harquahala Valley	800	Solar CLFR
15099	SolanaTap 500kv	198	Solar CST
15099	SolanaTap 500kv	40	Solar PV
15102	Asarco	20	Solar
19603	Blythe 161kv	40	solar PV
84832	LagunaTp 69kv	80	Solar PV
84836	NGila 69kv	400	Solar CLFR
84836	NGila 69kv	450	Solar CST

## Appendix C: Contour plots of CLMP

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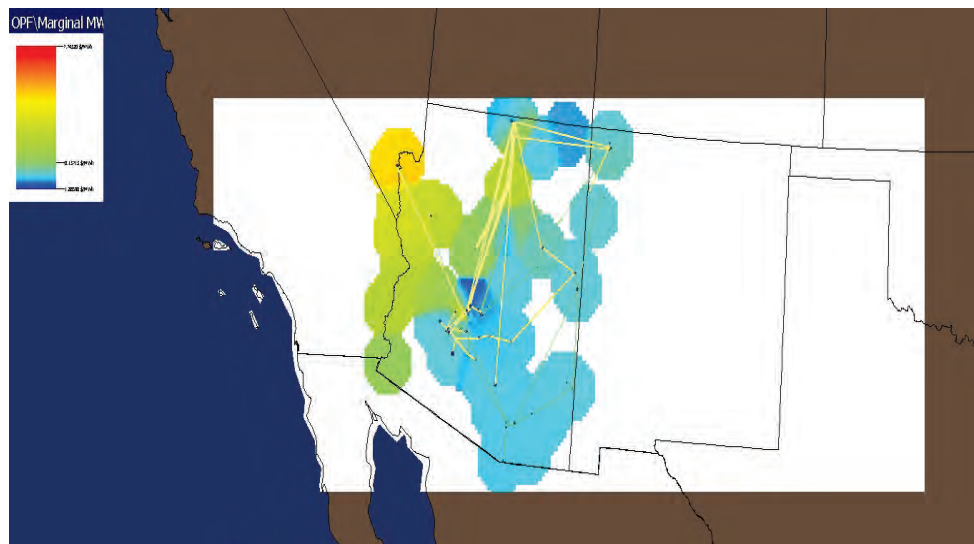


Figure 12: Week 1 of 2020

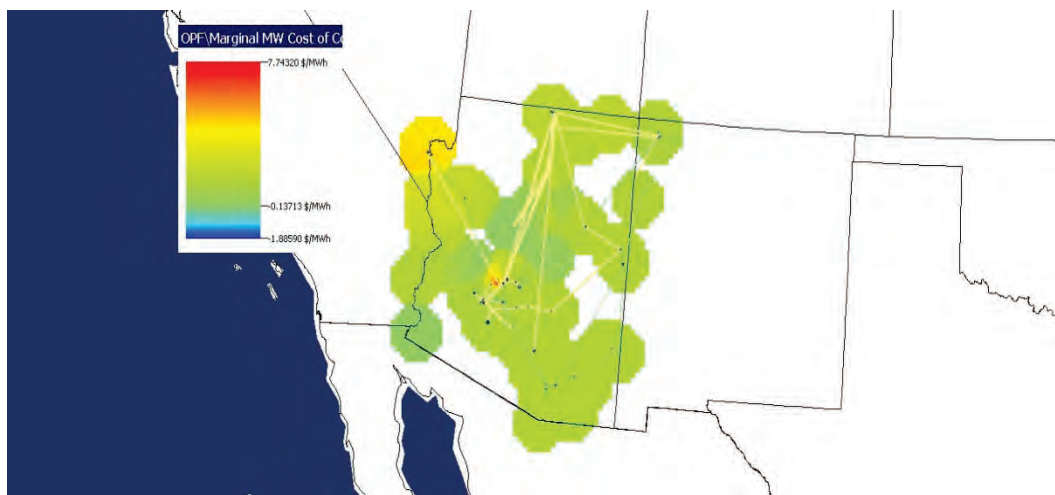


Figure 13: Week 8 of 2020

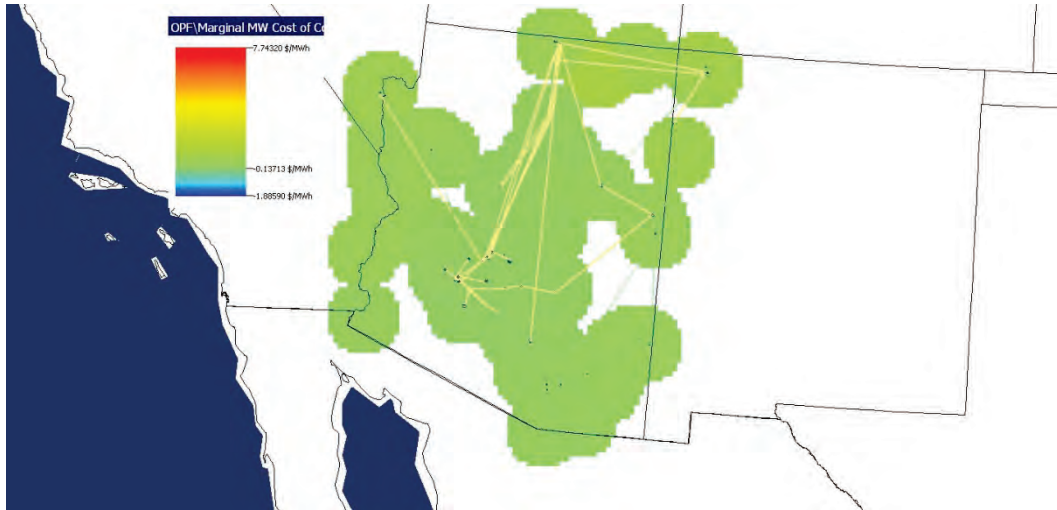


Figure 14: Week 23 of 2020

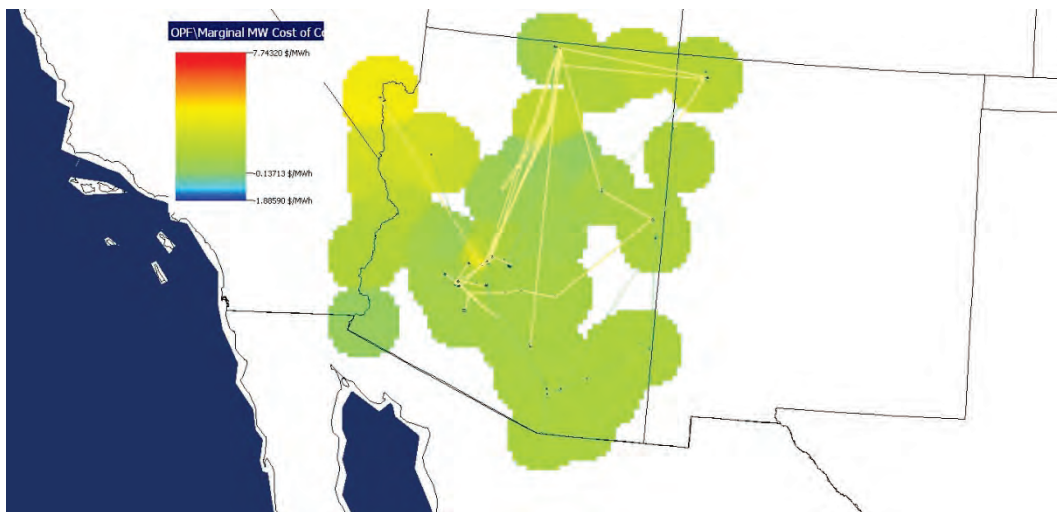


Figure 15: Week 46 of 2020

## Appendix D: Optimization model input data

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Table 17: Operational cost of generators based on fuel type

Type of generation	F	V <sub>OM</sub>
Coal fired	0	1.642
Nuclear	0	2.485
NG (GT)	0	2.4787
NG (ST)	0	1.3077
NG (CT/CA)	0	0.94893
Hydro	0	1.287
Wind	0	0
Solar PV	0	0
Solar thermal	0	0

The transmission expansion costs per mile are shown below. The expansion costs were scaled assuming a 30 year life expectancy for the transmission lines and a 3% annual rate of interest. The scaled costs considered per scenario are also shown below.

Table 18: Transmission line construction costs

Voltage level (kV)	Net present value (M\$)	Scaled costs per scenario (\$)
		$r = 3\%$
500	2.5	1717.3
345	2	1288.0
230	1.5	858.7
69	1	343.5

## **Part 2**

# **Power Flow Control and Probabilistic Load Flow**

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# 1. INTRODUCTION

## 1.1. Motivation

The paradigm shift in power systems in the last decade has been tremendous. Not too long ago, it was acceptable that a large part of our electric power consumption is produced by environmental unfriendly coal-fired power plants. But with the increased public awareness of environmental issues and the demand for sustainable power generation, a rethinking took place.

While the government, environmental agencies as well as consumers ask for a significant growth of renewable generation, there are several grand challenges which have to be resolved before the ambitious goal of 20% wind penetration by 2030 defined by the U.S. Department of Energy [1] can be achieved. Among these challenges are:

- *Variability and Intermittency*: The main focus in terms of renewable energy sources lies on wind and solar generation for which the primary energy source is variable and intermittent, i.e., is not continuously available and not necessarily at times most needed. As the energy produced has to match the energy consumed at all instances, this variability has to be balanced.
- *Underrated Infrastructure*: The location of high availability of the primary renewable energy source is often not where the major load is located, e.g. the best wind conditions in the country can be found in central US whereas the load centers are mainly on the east and west coasts [2] (see also Figure 1). Therefore, the power generated by renewable energy sources often will have to be transmitted over long distances from where it is generated to where it is consumed. But the existing infrastructure is not designed to carry this additional load and building the required new lines is difficult for environmental and political reasons.

This part of the project mostly focuses on the second challenge taking into account that the issue is not per se the missing transmission capacity but also the fact that the power currently cannot be rerouted over lines which still have capacity available. Power flow control devices, such as Flexible AC Transmission Systems (FACTS) provide the opportunity to influence voltages and power flows. Hence, these devices allow making better usage of the existing transmission system and to artificially enhance the transfer capacity. The effectiveness of multiple types of FACTS devices for congestion management and transfer capacity enhancements has been investigated and evaluated in several papers. For example, in [3], the benefits of these devices for several power flow control objectives are illustrated, such as unloading a specific transmission line, directing power flows between two regions and increasing the transfer capability. The feasibility and the efficacy of static series synchronous compensator (SSSC) is demonstrated in [4] for alleviating congestion caused by high penetration of wind power in the network. The improvement of total transfer capability (TTC) with thyristor-controlled series compensator (TCSC) and static var compensator (SVC) is shown in [5]. The impacts of various kinds of FACTS devices, such as thyristor-controlled phase shifter (TCPS), SVC and unified power-flow controller (UPFC) are also tested and evaluated for increasing the available transfer capa-

bility (ATC) in [6]. These papers illustrate that power flow control devices play an important role for congestion relief and transmission capability improvement.

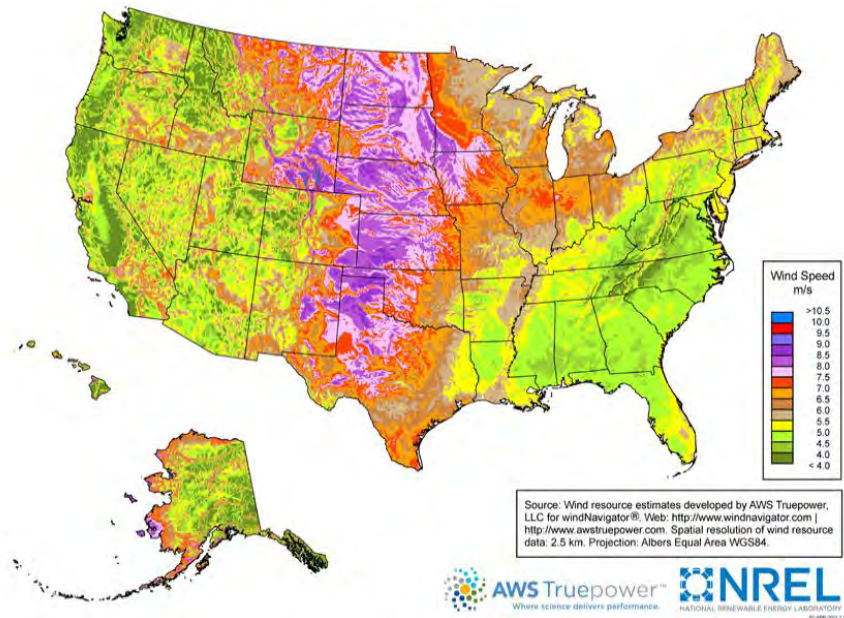


Figure 1: Wind resources in the United States 2

## 1.2. Contributions

This part of the project is composed of three main contributions:

- In order to achieve the optimal utilization of the current transmission network with FACTS devices, a control scheme is needed which determines the optimal steady-state settings of FACTS devices. While a centralized controller would be able to determine these settings using Optimal Power Flow (OPF) calculations, the goal in this project is to provide a novel approach which is in line with the notion of the smart grid consisting of smart local controllers which determine the optimal settings of the FACTS devices with respect to certain objectives by only using a limited amount of local measurements.
- Another important aspect when adding variable renewable generation to the system is that the power flows generally become much more variable and cannot be scheduled very accurately any more. Hence, deterministic power flow calculations do not capture the stochasticity in the resulting power flows. A probabilistic approach is required to capture the expected power flows. Hence, a second part of this volume gives an overview of probabilistic load flow methods and introduces an approach to capture correlation between multiple variable generation infeeds given their probability distribution. Including correlation is an important aspect because generators located spatially close to each other experience similar weather patterns resulting in correlated generation outputs.

- Power flow control devices increase the operational flexibility in the power grid. This is especially important when the uncertainty in the system is increased and/or when power flows become highly variable such as when an increased amount of renewable power generation is integrated into the system. Hence, approaches to compare the increased flexibility enabling a greater range of possible generation dispatches and a reduction in generation cost are provided.

## **2. DECENTRALIZED FACTS CONTROL**

The goal in this part is to provide a scheme which allows determining the optimal setting of a FACTS device without having to carry out an Optimal Power Flow calculation. The basic idea is to map a few key measurements in the system to the optimal setting of the power flow control device [7].

### **2.1. Centralized/Distributed/Decentralized**

In order to achieve optimal usage of the transmission system with FACTS devices, the optimal settings of these devices need to be determined. Traditionally, this can be done by solving an Optimal Power Flow (OPF) problem either in a centralized (e.g. [6]) or distributed way [8]. The centralized approach directly solves the OPF problem for the whole system providing the optimal device settings which benefit the system the most; however, it needs the information from the entire system. Since the whole system is usually operated by several entities, who may not be willing to share all the information, a distributed approach may be used which decomposes the overall OPF into several subproblems and allows each entity to solve a reduced-size OPF but still reaches the optimal values for the device settings with only a limited amount of information exchange with its neighbors.

Unlike the two aforementioned OPF-based approaches, a control approach based on regression analysis for the determination of the optimal device settings is derived [7]. It consists of two stages: 1) offline simulation stage and 2) online decision-making stage. In the offline simulation stage, the controller of the device is trained by solving OPF problems for various generation and loading scenarios of the system. Regression analysis is used to find a function describing the relationship between particular measurements which are identified as key measurements and the optimal device settings. In the online decision making stage, the controller uses this function and the information of local measurements to determine the optimal settings of the power flow control device. In the online operation, the controller only needs to evaluate the locally stored function. It basically results in a decentralized approach.

### **2.2. Overview Two-Stage Approach**

In this section, an overview over the two-stage approach is given before describing the method employed to realize this approach in more detail in the following sections. The process of the approach is shown in Figure 2.

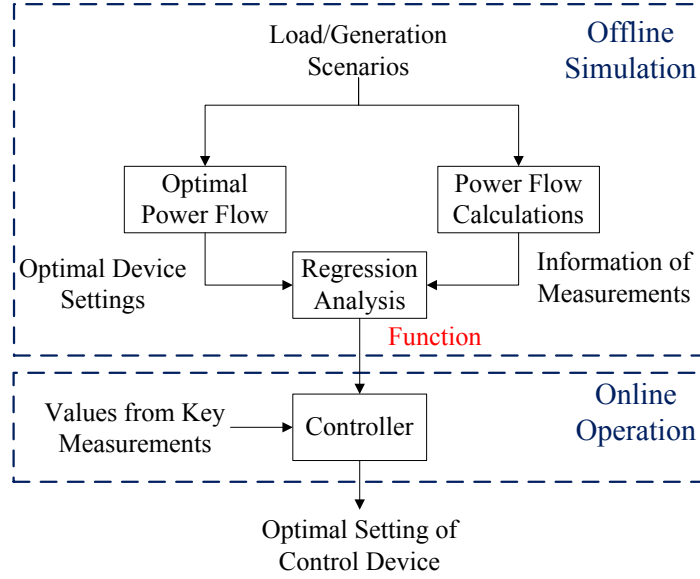


Figure 2: Overview two stage approach 2

*Offline Simulation Stage:* In this stage, a set of load/generation scenarios and initial settings of the FACTS devices is chosen. For each scenario  $s \in \{1, \dots, N_s\}$ , the power flow is solved when the FACTS device  $n$  is set to an initial value  $x_{n,0}^{(s)}$ ,  $n \in \{1, \dots, N_d\}$ , providing the voltage magnitude  $V_k^{(s)}$  and angle  $\theta_k^{(s)}$  at each bus  $k \in \{1, \dots, N_b\}$  as well as the power flow  $P_m^{(s)}$  and current magnitude  $I_m^{(s)}$  through each transmission line  $m \in \{1, \dots, N_l\}$ . Then, the optimal setting  $x_n^{(s)}$  of FACTS device  $n$  is determined by solving the OPF problem. This can be done either by solving the OPF problem for the overall system in a centralized way or if it is not possible, a decomposition approach could be used to achieve the same goal.  $N_s$ ,  $N_d$ ,  $N_b$ , and  $N_l$  represent the number of the load/generation scenarios, the number of the FACTS devices in the system, the number of the buses, and the number of the transmission lines, respectively. The goal in this stage is to find a function  $f_n$  giving the relationship between particular measurements and the optimal settings for each FACTS device.

Since not all of the possible measurements in the system are important for determining the optimal settings of a FACTS device and less measurements required means less communication needed in the online operation, a set of key measurements providing the most information about the optimal device settings needs to be selected. In this work, we integrate the key measurements selection into the regression analysis. Therefore, by employing the regression analysis, which is introduced in the following section, both the key measurements are selected and the corresponding coefficients in the function  $f_n$  are determined.

Assuming the key measurements for FACTS device  $n$  include the voltage magnitude  $V_{p,n}$  at bus  $p$ , voltage angle  $\theta_{q,n}$  at bus  $q$ , the active power flow  $P_{r,n}$  in line  $r$  or current magnitude  $I_{t,n}$  in line  $t$ , the regression function determining the optimal setting of FACTS device  $n$  can be written as

$$x_n = f(V_{p,n}, \theta_{q,n}, P_{r,n}, I_{t,n}, x_{n,0}), \quad n = 1, \dots, N_d \quad (1)$$

As can be seen in (1), the controller of device  $n$  only needs to know the current setting of its own along with several key measurements in the system. Hence, only the communication between the controller and a specific set of measuring units is necessary while no coordination between different FACTS devices is required.

*Online Decision Making Stage:* In this stage, based on the information of the key measurements  $V_{p,n}, \theta_{q,n}, P_{r,n}, I_{t,n}$ , the controller adjusts the device setting  $x_n$  from  $x_{n,0}$  by only evaluating the locally stored function  $f_n$ .

The modeling of the FACTS devices, the mathematical formulation of the optimization problem formulated and regression problem setup in the offline simulation stage are described in the following sections.

### 2.3. OPF Problem Formulation

The Thyristor-Controlled Series Compensator is used as the device to be studied. Hence, the problem formulation is given for this device. It is modeled as a variable reactance  $X_{TCSC}$  in series with the transmission line, i.e., the total reactance of that line results in  $X_{Tot} = X_{Line} + X_{TCSC}$  where  $X_{Line}$  is the reactance of the line itself.

In the offline simulation, the optimal settings of the TCSCs in each load and generation scenario are determined by solving an OPF problem with the following formulation:

$$\begin{aligned} \max_{\eta_{TCSC,n}} \quad & \min(P_{margin}, ij) \\ \text{s. t.} \quad & \eta_{TCSC,n} = \frac{X_{TCSC,n}}{X_{Line,n}} \end{aligned} \quad (2)$$

$$P_{margin,ij} = \frac{F_{ij}^{max} - |P_{ij}|}{F_{ij}^{max}} \quad (3)$$

$$P_{G,i} - P_{L,i} - \sum_j P_{ij} = 0 \quad (4)$$

$$Q_{G,i} - Q_{L,i} - \sum_j Q_{ij} = 0 \quad (5)$$

$$|P_{ij}| \leq F_{ij}^{max} \quad (6)$$

$$V_{k,min} \leq V_k \leq V_{k,max} \quad (7)$$

$$X_{TCSC,n,min} \leq X_{TCSC,n} \leq X_{TCSC,n,max} \quad (8)$$

Table 1 gives an overview over the variables and parameters used in the OPF problem.

Table 1: Variables and Parameters in the OPF Problem

Variable	Description
$X_{TCSC,n}$	reactance of $n$ -th TCSC
$X_{Line,n}$	original reactance of the transmission line where the $n$ -th TCSC is placed
$\eta_{TCSC,n}$	compensation ratio of $n$ -th TCSC
$X_{TCSC,n,min}$ $X_{TCSC,n,max}$	lower and upper limits of the reactance of $n$ -th TCSC
$P_{ij}$	active power flow of the line from bus $i$ to bus $j$
$F_{ij}^{max}$	capacity limit of the line from bus $i$ to bus $j$
$P_{margin,ij}$	capacity margin of the line from bus $i$ to bus $j$
$P_{G,i}, Q_{G,i}$	active and reactive power generation at bus $i$
$P_{L,i}, Q_{L,i}$	active and reactive power consumption at bus $i$
$V_k$	voltage magnitude at bus $k$
$V_{k,min}, V_{k,max}$	lower and upper limits of the voltage magnitude at bus $k$

In the formulation of the OPF problem, the control variables are the reactance of each TCSC, or the compensation ratio which is defined in (2) as the ratio of the reactance of the TCSC to the original reactance of the transmission line in which the TCSC is placed. In this report, the setting of a TCSC refers to its compensation ratio. The objective function is to maximize the minimum value of the capacity margin of the transmission lines. In (3), the capacity margin of a line is defined as the difference between the active power flowing through this line and its capacity limit divided by its capacity limit. The minimum capacity margin indicates how much extra power can flow through the most heavily loaded line without exceeding its capacity. By maximizing the minimum value of the capacity margin, the TCSCs are trying to reduce the loading of the most heavily loaded line as much as possible. Equations (4) and (5) correspond to the active and reactive power balances at bus  $i$  for power injection  $P_{G,i}, Q_{G,i}$  by the generators and consumption  $P_{L,i}, Q_{L,i}$  by the loads.

For the formulation of the optimization problem shown in the previous section, it cannot be solved directly as it is a max/min problem. Hence, a reformulation of the problem is necessary. By introducing another variable  $R$ , the objective function can be formulated as:

$$\begin{aligned} & \max_{\eta_{TCSC,n}} R \\ & s.t. \quad P_{margin,ij} \geq R \quad \text{for all } ij \end{aligned} \quad (9)$$

All the other constraints (2) – (8) remain the same. The solution to this problem is the same as to the initial problem.

With further consideration of the constraints, constraint (6) can be removed and the optimization problem becomes a relaxed problem. The reason is that the value of the objective function is upper bounded by the minimum value of the capacity margin. If a transmission line is overloaded, the capacity margin of this line will be negative according to its definition, resulting in a negative value for the objective function. If congestion occurs in the original system without FACTS devices and the FACTS devices are able to redirect the power flows from the overloaded lines to those non-overloaded ones, a nonnegative value for the objective function of the relaxed problem is reached. Hence, the constraints of the capacity limits of the lines will be satisfied automatically, that is, the relaxed problem will reach the same optimal solution as the original problem. If not, the relaxed problem could also find an optimal solution with the negative value of the objective function, which indicates that there is no feasible solution for the original problem. Therefore, by removing constraint (6), the relaxed problem is able to deal with scenarios in which there is overloading in one or more lines which cannot be resolved by the FACTS devices.

## 2.4. Methods

The method used to derive the function in (1) is regression analysis which allows setting up the problem formulation to find an input/output dependency for available data. The method used to solve the resulting optimization problem is Orthogonal Matching Pursuit. Both methods are shortly described in this section.

### 2.4.1. Regression Analysis

Regression analysis [9] is a method used to find an appropriate function between a dependent variable and one or more independent variables. If this function is parametric, regression estimates the parameters in this function using the data of the independent variables and the corresponding obtained values for the dependent variables. In the following, the mathematical formulation of finding the function for the considered application of FACTS control by regression analysis is provided.

Let  $z_{l,n}$  ( $z_{l,n} \in \mathcal{K}_n$ ) represent the  $l$ -th measurement and  $\mathcal{K}_n$  the set of possible measurements for the FACTS device  $n$ . Hence, (1) can be written as:

$$x_n = f_n(z_{1,n}, z_{2,n}, \dots, z_{M_n,n}) \quad (10)$$

where  $M_n$  denotes the number of all possible measurements for the FACTS device  $n$ , i.e. the number of measurements in  $\mathcal{K}_n$ . In order to find this function through regression analysis, the structure of this function needs to be known or assumed. Here we consider the function to be a polynomial function and we use a quadratic function as an example to illustrate the fitting process.

By using the notation  $z_n = [z_{1,n}, z_{2,n}, \dots, z_{M_n,n}]^T$  to denote the vector of all possible measurements, the quadratic function  $f_n$  can be formulated as:

$$x_n = f_n(z_{1,n}, z_{2,n}, \dots, z_{M_n,n}) = \frac{1}{2} z_n^T A_n z_n + b_n^T z_n + c_n \quad (11)$$

By employing a new vector  $\hat{z}_n$  whose entries represent all combinations of variables in vector  $z_n$ , i.e.

$$\hat{z}_n = [1 \ z_{1,n} \ \dots \ z_{M_n,n} z_{1,n}^2 z_{1,n} z_{2,n} \ \dots \ z_{1,n} z_{M_n,n} z_{2,n}^2 z_{2,n} z_{3,n} \ \dots \ z_{2,n} z_{M_n,n} \ \dots \ z_{M_n,n}^2]^T \quad (12)$$

the function  $f_n$  can be written as a linear function of  $\hat{z}_n$ :

$$x_n = f_n(\hat{z}_n) = \alpha_n^T \hat{z}_n \quad (13)$$

where each entry in the vector  $\hat{z}_n$  is called a basis function and vector  $\alpha_n$  contains all coefficients in the function  $f_n$ . The problem that needs to be solved via regression analysis is to determine the best values for the coefficients in vector  $\alpha_n$  which minimizes the deviations of the device settings, determined by plugging the values of the measurements into the function  $f_n$ , from the actual optimal setting.

Hence, in order to achieve this, the data obtained from the offline power flow and OPF calculations is used to setup the following matrices:

$$Z_n = [\hat{z}_n^{(1)T} \ \dots \ \hat{z}_n^{(s)T} \ \dots \ \hat{z}_n^{(N_s)T}]^T \quad (14)$$

$$Y_n = [x_n^{(1)T} \ \dots \ x_n^{(s)T} \ \dots \ x_n^{(N_s)T}]^T \quad (15)$$

where row  $s$  in matrix  $Z_n$  corresponds to the values of the basis functions and row  $s$  in matrix  $Y_n$  the corresponding optimal setting of the FACTS device  $n$  in load and generation scenario  $s$ . Therefore, the regression problem can be formulated as:

$$\min_{\alpha_n} \quad \|Z_n \alpha_n - Y_n\|_2^2 \quad (16)$$

### 2.4.2. Orthogonal Matching Pursuit

It is possible that not all the measurements in  $\mathcal{K}_n$  are important to determine the optimal settings of the FACTS device  $n$ . In order to determine the function only with the measurements which provide the most information of the device setting, a subset  $\mathcal{K}_n^I \subseteq \mathcal{K}_n$  containing these important measurements needs to be identified. Therefore, the following problem is formulated:

$$\begin{aligned} \min_{\alpha_n} \quad & \|Z_n \alpha_n - Y_n\|_2^2 \\ \text{s. t.} \quad & M_n^I \leq \lambda \end{aligned} \tag{17}$$

where  $M_n^I$  denotes the number of the measurements included in set  $\mathcal{K}_n^I$  and  $\lambda$  is a predefined constant limiting the number of key measurements to be selected. Hence, the task actually is to find a sparse solution for  $\alpha_n$  and this solution should be the one which best fulfills the task of minimizing the deviations of the value provided by function  $f_n$  from the actual optimal setting. It needs to be sparse because only the entries in the coefficient vector  $\alpha_n$  corresponding to the basis functions generated by the measurements in set  $\mathcal{K}_n^I$  should be non-zero.

Orthogonal Matching Pursuit (OMP) [10] is an efficient numerical algorithm to find an approximate solution of a sparse coefficient vector to the regression problem with a limited number of non-zero entries. The basic idea of the OMP is to identify important coefficients by checking the inner product between the normalized column vector of matrix  $Z_n$  and  $Y_n$ . The larger the absolute value of the inner product of a certain column  $i$  of  $Z_n$ , the more important the corresponding  $i$ -th entry in the coefficient vector  $\alpha_n$  is. Here, we use a modified OMP algorithm to solve the regression problem formed in (17). The procedure of the modified OMP algorithm is described as follows:

*Step 1:* Set  $F = Y_n$ , the index set of the basis functions  $\Omega_B = \{\}$ , the set of the key measurements selected  $\mathcal{K}_n^I = \{\}$  and  $h = 0$ ;

*Step 2:* Calculate

$$\gamma_i = \frac{\langle Z_{n,i}, F \rangle}{\|Z_{n,i}\|_2} \quad \text{for all } i \tag{18}$$

where  $Z_{n,i}$  denotes the  $i$ -th column in  $Z_n$ ;

*Step 3:* Choose  $\gamma_j$  with the largest absolute value;

*Step 4:* Find the corresponding measurement(s) which generate the  $j$ -th basis function in  $\hat{z}_n$  and add them to the key measurements set  $\mathcal{K}_n^I$ ; Let  $p_M$  denote the number of the measurements selected at this step;

*Step 5:* Find all the basis functions generated by the key measurements in set  $\mathcal{K}_n^I$  and add the corresponding indices to the index set  $\Omega_B$ ;

*Step 6:* Find the value for coefficients  $\alpha_{n,i}, i \in \Omega_B$  by

$$\min_{\alpha_{n,i}, i \in \Omega_B} \left\| \sum_{i \in \Omega_B} \alpha_{n,i} Z_{n,i} - Y_n \right\|_2^2 \quad (19)$$

*Step 7:* Calculate the residual

$$F = Y_n - \sum_{i \in \Omega_B} \alpha_{n,i} Z_{n,i} \quad (20)$$

*Step 8:* Update  $h = h + p_M$ . If  $h < \lambda$ , go to step 2; otherwise, stop and set  $\alpha_{n,i} = 0, i \notin \Omega_B$ .

By employing the OMP algorithm, the set of the important measurements and the corresponding coefficients in the function  $f_n$  can be determined simultaneously. The  $\lambda$  value represents the tradeoff between the fitting accuracy and the number of key measurements selected.

The solution of this algorithm will provide the parameters of the regression function corresponding to the features which include the key measurements. Simulation results using the method are provided in 5.1.

### 3. PROBABILISTIC POWER FLOW

The objective in this section is to provide an overview over probabilistic load flow algorithms and provide a means to generate correlated wind generator power output values which reflect the fact that the outputs from two not co-located wind generators are nevertheless correlated. The level of correlation depends on their proximity or more precisely if they are located within the same weather patterns.

#### 3.1. Problem Formulation

In the past, most of the electric power was generated by dispatchable bulk power plants. With the loads being fairly predictable, power flow patterns did not change much from day to day. However, the goal is to transition to a renewable energy future with a significant amount of non-dispatchable generation. The consequence is that power flows become much more variable and less predictable. The traditional deterministic power flow calculations which provided accurate means to determine the resulting power flow and required transmission capacities is not capable of capturing the probabilistic nature of the power flows in a system with significant amounts of wind and/or solar generation. Probabilistic load flow in which it is taken into account that the power injections are represented by probability distribution functions is a significantly better fit for this situation.

Input parameters to probabilistic load flow calculations are probability density functions of the variable resources such as wind and solar generation. The probability density function for wind generation typically has a beta characteristic [11], [12]. The outputs of a probabilistic load flow analysis are probability density functions for the values of interest, e.g., the load flows on transmission lines. These output curves provide an indication of the loading probability of a line or the probability of a specific parameter to take a specific value.

As the output of a wind generator is dependent on the local wind speed, it has to be assumed that the outputs from multiple wind farms are not uncorrelated, i.e. depending on the spatial distance between wind farms the outputs may be heavily correlated. This needs to be taken into account when carrying out probabilistic load flow analysis. Hence, in this section, we will first provide an overview over probabilistic load flow methods and then discuss how correlation between beta distributed functions can be defined.

#### 3.2. Literature Review

Probabilistic load flow was proposed by Borkowskain in 1974 [13]. Prior, only deterministic methods have been known [14], [15]. Since 1974 various techniques for solving the probabilistic load flow problem have been developed and research in this area is still ongoing. At the beginning DC load flow was used whereas using AC load flow followed a bit later. Overviews over various methods are given in [16], [17]. A statistical method similar to probabilistic load flow is stochastic load flow [18]-[21]. A stochastic load flow analysis uses a state estimator-type algorithm. It is an extremely fast technique but it only can handle probability density functions of Gaussian type. An option for non-Gaussian probability distributions is discussed in [22].

Borkowska [13], Allan [23]-[29] and others [30]-[34] used the convolution technique which says that the probability density of the sum of two independent random variables is the convolution of their density function [35]. With this technique the line flow can be calculated by convoluting the density functions of the power injection random variables. The limitation is that the generation and loads have to be independent to be able to use convolution techniques. For density functions with Gaussian shape a solution of the convolution exists in closed form (the result is also a Gaussian distribution [36]) but in general a conversion to a discrete density function and a numerical computation is necessary.

Other techniques are more based on statistics. Two very important elements of such probabilistic power flow methods are known as moments [37]-[39] and cumulants [39], [40] in probability theory and statistics. Statistical moments describe the shape of a distribution. The first four moments are well known. The first moment is the mean, the second one is the variance, the third one is the skewness and the fourth moment is the kurtosis. An alternative to statistical moments are cumulants. Cumulants are a set of quantities to approximate the shape of a distribution. A distribution is better described by its cumulants than by its moments [41]. The cumulants are related to the statistical moments and can be calculated from them using a recursion formula.

Several methods or combinations of methods using moments or cumulants exist. A Taylor series to obtain moments is used in [42]. A combination of moments and cumulants extended by a mathematical method called Von Mises [43] is used in [44] and a mix of method of cumulants and Von Mises is applied to stochastic load flow in [45]. The cumulant method is often combined with the Gram-Charlier series expansion theory. Edgeworth series and Gram-Charlier series are used to approximate a probability distribution [40], [46]. The Gram-Charlier series is used more often to build a density function from cumulants in a probabilistic power flow computation. The interpretation of moments or cumulants by Gram-Charlier series to solve a probabilistic power flow problem is explained and discussed in [47]-[49]. The cumulant method in combination with Gram-Charlier series is applied to an optimal power flow problem in [50] using Gaussian and Gamma distributions. An enhancement of this cumulant method is presented in [51]. For the enhancement discussed in this paper only a Gaussian distribution is used.

Other researchers use the concept of combining cumulants and Gram-Charlier series to rebuild probability density functions. In [52]-[54], network configuration uncertainties or vulnerability assessment [55] as well as reactive power control [56] are discussed based on this method. Transfer capability analysis can also be done through probabilistic load flow using cumulants [57] or moments [58] in combination with Gram-Charlier series expansion. Gram-Charlier series are very good to approximate Gaussian distributions. The more a distribution is different from a normal distribution the higher the order of the moments and cumulants needed for a good approximation. For non-Gaussian distributions this method has some convergence problems.

A new probabilistic load flow approach was proposed in [59]. It uses a point estimation method [60], [61] to achieve a better probability density function fitting from statistical moments. A good summary including different point-estimation methods and their comparison with regards to power flow calculation can be found in [62]. A two-point estimation method was used in

[63], [64]. The point-estimation method was extended in [65] to account for dependencies among random variables.

With Cornish-Fisher expansion series [46], [66] a technique with better convergence properties than Gram-Charlier series for approximation of non-Gaussian density functions was found. The method of combined cumulants and Cornish-Fisher expansion has been applied to power flow problems in [67]-[70]. In [71], [72] Usaola discussed dependencies among input random variables and in [71], he proposed a technique called Enhanced Linear Method. For the wind power injection he uses dependent beta-distributed random variables. Loads are modeled by dependent or independent normal variables with a given correlation matrix. To generate the correlated random variables Usaola uses a method based on the inverse transformation of a uniform distribution. Cornish-Fisher series are usually much better than Gram-Charlier series for fitting non-Gaussian distributions but the convergence properties are difficult to demonstrate and the complex mathematical problem is not yet solved completely [66].

The best known and oldest method for probabilistic load flow is Monte Carlo simulation [73]. Monte Carlo is a numerical technique in which by repeated random sampling of the input variables, the probability distribution of the output variables is determined. Many of the methods described above are using Monte Carlo to verify or to compare their computation results. In [74], [75], Monte Carlo is applied to multilinearized load flow equations. A combination of Markov Chains and Monte Carlo is presented in [76]. For transmission planning and complex power systems Monte Carlo is applied in [77], [78] and in [79] the wind generation cost is considered. Using Monte Carlo, there are no limitations with regards to types of distributions and considered problems. It is a universal technique and can be applied to almost all problems in probabilistic load flow. It can compute a result where other methods are restricted but the disadvantage is that it is a very time consuming process. For complex problems, Monte Carlo simulations might not be feasible due to this reason.

We reviewed many different ways to compute a solution for a probabilistic power flow problem. Most of the methods used Gaussian distributed random variables. Correlation among loads and generation is discussed and considered only in few papers.

### 3.3. Probability Distributions

The distribution functions which will be needed in the further derivations are the gamma and the beta distributions. Hence, we shortly provide the mathematical formulation of these functions in this section. Furthermore, a definition for the correlation of two probability distributions is given.

#### 3.3.1. Gamma Distribution

The standard gamma distribution [80] on interval  $[0, \infty)$  is given by:

$$f(x, \gamma, \lambda) = \frac{\lambda^\gamma}{\Gamma(\gamma)} x^{\gamma-1} e^{-\lambda x}, \quad x \in [0, \infty), \gamma, \lambda > 0$$

with Gamma function

$$\Gamma(\gamma) = (\gamma - 1)!$$

where  $\gamma$  is the shape parameter and  $\lambda$  is the scale parameter.

### 3.3.2. Beta Distribution

The standard beta distribution [81] on interval  $[0,1]$  is given by:

$$f(x, \alpha, \beta) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1-x)^{\beta-1}, \quad x \in [0,1], \alpha, \beta > 0$$

with Beta function

$$B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha + \beta)} = \int_0^1 u^{\alpha-1} (1-u)^{\beta-1} du$$

where  $\alpha$  and  $\beta$  are shape parameters of the distribution. Both shape parameters can be calculated if mean  $\mu$  and variance  $\sigma^2$  are known for a specific distribution by using

$$\alpha = \mu \left( \frac{\mu(1-\mu)}{\sigma^2} - 1 \right)$$

$$\beta = (1-\mu) \left( \frac{\mu(1-\mu)}{\sigma^2} - 1 \right)$$

### 3.3.3. Definition of Correlation of Probability Functions

The coefficient of a linear correlation is defined as the fraction of covariance and the product of the square root of the variances of the random variables. This coefficient is also called Pearson correlation coefficient [37]

$$Corr_{X,Y} = \rho_{X,Y} = \frac{\sigma_{X,Y}}{\sigma_X \sigma_Y}$$

where  $\sigma_X$  is the standard deviation of  $X$  and

$$\sigma_{X,Y} = Cov_{X,Y} = E[(X - E[X])(Y - E[Y])]$$

is the covariance between variables  $X$  and  $Y$  with  $E$  denoting the expectation.

If the correlation is zero, the random variables are uncorrelated but they do not have to be independent.

### 3.4. Correlated Beta-Distributed Random Variables

#### 3.4.1. Purpose

Let's assume that the expected wind probability distributions for two specific locations in the grid are known and that these distributions can be formulated as beta distribution functions. The probability distribution functions do not provide any information about the correlation of these functions but it basically just provides the information how likely a specific output is. If the locations are very close to each other, it can be expected that whenever the output of one wind generator is high, the output of the other wind generator is high, too, i.e., their outputs are highly correlated. However, if they are far apart not observing the same weather patterns, the power outputs are probably not significantly correlated, i.e., if the output at one wind generator is high, it is possible that at the other location the wind power output is very low. The correlation of wind generation probability functions is important when determining the required transmission capacities or determining the expected loading of existing transmission lines. Hence, the question arises, how one can model correlated wind generator outputs.

#### 3.4.2. Existing Methods

Methods which allow generating correlated random variables are:

*Transformation from Uniformly Distributed Correlated Random Variables to Desired Distribution:* The classical way to produce correlated random variables with a required distribution is to start by formulating uniformly distributed random variables over the interval (0,1) with the desired correlation. These random variables are then transformed to random variables of the desired distribution. This method is used and explained in [82], [83] for generating beta distributed random variables. However, the correlation changes depending on the target distribution and the range of the distribution. Adjustments might be necessary to obtain the desired correlation. Also, sometimes there are restrictions for the range of the correlation.

*Based on Dirichlet Distribution:* In [84], a method is described for generating positively correlated random variables each following a beta distribution. It is used that the marginals of a Dirichlet distribution are beta variates, i.e. this method uses the characteristics that a beta distribution may be created by composing two or more gamma distributions. However, this method has restrictions regarding the shape parameters of beta distributions. One limitation is that the sum of the parameters has to be equal for all beta distributions, i.e.

$$\alpha_1 + \beta_1 = \alpha_2 + \beta_2.$$

*Shared Random Variables:* This method uses similar properties of beta-distributions as the Dirichlet method and was proposed in [85]. Composing a beta-distribution from two or more gamma distributions is the first basic idea of his method. The second essential step is to use the well-known additivity of gamma distributed random variables. Combining these two steps, a beta-distribution with parameters  $\alpha$  and  $\beta$  is created where each shape parameter is a sum of two specific sub-parameters. To obtain a certain correlation for two beta-distributions,

one of these two sub-parameters per shape parameter is shared between both compositions. To share a sub-parameter means to share a gamma distributed random variable.

In the derivation of the method, the authors use a first-order Taylor series expansion (for details the reader is referred to [85]). This approximation may cause a bias. The smaller the parameters  $\alpha$  and  $\beta$ , the larger the bias is. For wind output probability density function the parameters for the beta-distributions are such that a bias of more than 10% to 20% occurs.

In the following, we will use the last approach which is based on shared random variables and extend it to counterbalance the bias which occurs for the considered application.

### 3.5. Extensions to Shared Random Variables Method

#### 3.5.1. Original Method

In the shared random variables method, two characteristics of beta distributed functions are utilized to generate correlated distribution functions:

*Characteristic 1:* Let  $U_1$  and  $U_2$  be two gamma distributed random variables and  $V$  a beta distributed function. The composition of  $U_1$  and  $U_2$  according to

$$V = \frac{U_1}{U_1 + U_2}$$

results in

$$\left. \begin{array}{l} U_1 \sim \mathcal{G}(\gamma_1, \lambda) \\ U_2 \sim \mathcal{G}(\gamma_2, \lambda) \end{array} \right\} V \sim \mathcal{B}(\gamma_1, \gamma_2)$$

The symbol  $\sim$  stands for *distributed as*,  $\mathcal{G}(\gamma_1, \lambda)$  denotes a gamma distribution with shape parameter  $\gamma_1$  and scale parameter  $\lambda$  and  $\mathcal{B}(\gamma_1, \gamma_2)$  denotes a beta distribution with shape parameters  $\gamma_1$  and  $\gamma_2$ . Hence, the shape parameter of each gamma distribution turns into a shape parameter for the composed beta distribution.

*Characteristic 2 (Additivity):* The sum of two gamma distributed random variables results in a gamma distributed random variable, i.e.

$$W = U_1 + U_2,$$

with

$$\left. \begin{array}{l} U_1 \sim \mathcal{G}(\gamma_1, \lambda) \\ U_2 \sim \mathcal{G}(\gamma_2, \lambda) \end{array} \right\} W \sim \mathcal{G}(\gamma_1 + \gamma_2, \lambda)$$

The shape parameter of the resulting distribution is the sum of the shape parameters of the summands.

In the method presented in [85], these two characteristics are now combined to form the following relations

$$V = \frac{W_1}{W_1 + W_2} = \frac{U_1 + U_2}{U_1 + U_2 + U_3 + U_4}$$

with

$$\left. \begin{array}{l} U_1 \sim \mathcal{G}(\gamma_1, \lambda) \\ U_2 \sim \mathcal{G}(\gamma_2, \lambda) \\ U_3 \sim \mathcal{G}(\gamma_3, \lambda) \\ U_4 \sim \mathcal{G}(\gamma_4, \lambda) \end{array} \right\} V \sim \mathcal{B}(\gamma_1 + \gamma_2, \gamma_3 + \gamma_4)$$

resulting in  $\alpha = \gamma_1 + \gamma_2$  and  $\beta = \gamma_3 + \gamma_4$ .

To achieve a certain correlation between beta distributed random variables, the authors share gamma distributed random variables to form the random variables  $Y_1$  and  $Y_2$  according to

$$Y_1 = \frac{X_1 + X_a}{X_1 + X_a + X_2 + X_b}$$

$$Y_2 = \frac{X_3 + X_a}{X_3 + X_a + X_4 + X_b}$$

The random variables  $X_a$  and  $X_b$  are shared variables included to obtain a desired correlation. These variables are calculated from the shape parameters and the desired level of correlation. A first-order Taylor series is used to achieve the splitting into  $\alpha$  and  $\beta$  parameters which approximates the covariance between  $Y_1$  and  $Y_2$ . The reader is referred to 85 for more details. The issue is that for low  $\alpha$  and  $\beta$ , a bias results. E.g. we used the shape parameters  $\alpha_1 = 0.87$  and  $\beta_1 = 1.23$  for the first and the same parameters, i.e.  $\alpha_2 = 0.87$  and  $\beta_2 = 1.23$ , for the second beta distribution. The values of the achieved correlation are approximately a bit more than half of what they should be (e.g. 26% instead of the desired 50% or 55% instead of 100%).

### 3.5.2. Adjustments for Improved Correlation

This section focuses on improving the original method as described in the previous section such as to achieve a correlation which is closer to the desired correlation. It focuses on developing a method of how to calculate the shape parameters  $\gamma_a$  and  $\gamma_b$  for the shared random variables  $X_a$  and  $X_b$  to achieve increased accuracy for small values of the shape parameters.

For a positive correlation with correlation factor  $\rho$  between  $Y_a$  and  $Y_b$ , the shape parameters  $\gamma_a$  and  $\gamma_b$  are calculated according to

$$\gamma_a = \rho \cdot \max(\alpha_1, \alpha_2)$$

$$\gamma_b = \rho \cdot \max(\beta_1, \beta_2)$$

For a negative correlation the calculation of these parameters corresponds to

$$\gamma_a = \rho \cdot \max(\alpha_1, \beta_2)$$

$$\gamma_b = \rho \cdot \max(\beta_1, \alpha_2)$$

With these parameters random variables with distributions

$$X_a \sim \mathcal{G}(\gamma_a, \lambda)$$

$$X_b \sim \mathcal{G}(\gamma_b, \lambda)$$

are obtained. The parameter calculation of the non-shared gamma distributed random variables for positive correlation is performed according

$$\gamma_1 = \alpha_1 - \gamma_a$$

$$\gamma_2 = \beta_1 - \gamma_b$$

$$\gamma_3 = \alpha_2 - \gamma_a$$

$$\gamma_4 = \beta_2 - \gamma_b$$

For a negative correlation the parameters are determined by

$$\gamma_1 = \alpha_1 - \gamma_a$$

$$\gamma_2 = \beta_1 - \gamma_b$$

$$\gamma_3 = \alpha_2 - \gamma_b$$

$$\gamma_4 = \beta_2 - \gamma_a$$

The corresponding random variables are then generated as

$$X_1 \sim \mathcal{G}(\gamma_1, \lambda)$$

$$X_2 \sim \mathcal{G}(\gamma_2, \lambda)$$

$$X_3 \sim \mathcal{G}(\gamma_3, \lambda)$$

$$X_4 \sim \mathcal{G}(\gamma_4, \lambda)$$

The beta distributed random variables  $Y_1$  and  $Y_2$  are then given as above. Simulations showing the accuracy of this approach regarding a desired correlation are given in Section 5.3.

## **4. FLEXIBILITY IN THE TRANSMISSION GRID**

### **4.1. The Need for Flexibility**

Transmission constitutes an important part of the power system. It is the sole means of transferring power from the utilities to the consumers. From the advent of second half of nineteenth century till the late nineteenth century investments in the field of electric transmission have declined [86]. However, with the emergence of newer challenges in the field of power systems such as deregulation, integration of renewables, increasing load and retiring older power plants, transmission expansion problem (TEP) has once again become an important aspect of the electricity system. Developments such as opening the transmission lines to the open market and calling inter-regional bodies to interact with each other in order to be able develop inter-regional transmission lines signifies the importance subjected to transmission investments and expansion.

Achieving flexibility in today's transmission is of great importance. The need for flexibility in the system can be attributed to multiple factors; the uncertain physical variables and unknown load pattern to name a few of them. In addition to the obvious variability in the system due to uncertain demand growth and renewable generation availability and geography, deregulation also plays a part by infusing varying dispatch patterns in the system. The inherent uncertainties in the system in tandem with markets induced uncertainties makes the transmission expansion planning problem a complex multi objective optimization problem. Hence high flexibility is required in order to optimally solve this problem. In the past, flexibility in the area of generation has been a much researched field [87]-[89], transmission planning flexibility is catching up pace in recent years; the reason to that being the need to integrate variable resources such as wind and solar into the system and the nature of present deregulated electricity markets.

Flexibility has become an important planning criterion for power systems. As we advance towards a future where we have even less control on our resources, methods have to be developed for quantitative evaluation of flexibility in the system. This would enable accommodating forecasting errors and uncertainty in the planning process. A flexible solution also results in minimizing risk due to uncertainty. Market flexibility, operational flexibility and technical flexibility all contribute towards the overall flexibility of the grid. Deregulation led to research being done in the field of market flexibility [90] but for long term planning one must also consider technical or strategic flexibility to achieve overall system flexibility. The usage of power flow control devices can lead to operational and strategic flexibility in the transmission planning process.

### **4.2. Methods**

The usage of heuristic techniques such as probabilistic power flow and stochastic representation of variables have in the past shown encouraging results in the field of power system planning. Substantial research has been done in these fields [91] with respect to power systems and the techniques and methodologies are well understood. Deterministic methods can be extremely computationally intensive and in some cases even infeasible [92] thus heuristic methods are pursued in this section. An overview of probabilistic load flow methods has been provided in the previous section and the ability of these techniques to deliver positive results has been shown such as in [23]. Monte Carlo Markov Chain simulation has been used to solve multi objective

optimization problem in power systems [93]-[95] and is also the method which will be used in this section.

In this project, an amalgamation of stochastic and deterministic methods is used for our analysis. We first use heuristic techniques to model the various input scenarios which replicate the true distributions of the input parameters to an acceptable accuracy. These generated input scenarios are then used by deterministic methods to run power flows to reach analytical solutions. This kind of analysis is commonly used in case of probabilistic power flow methods.

#### **4.2.1. Monte Carlo Markov Chain Simulation**

Monte Carlo Markov Chain Method [96], [97] is the heuristic technique used in this section to model demand uncertainty along with uncertainty in renewable generation availability in our problem formulation. As already discussed, uncertainties are present in the system due to present market design, introduction of renewables, forecasting errors, government policies and many others. This results in a large number of uncertain parameters in the system. Complete enumeration of the conformational chain that depicts all possible combinations of these parameters is impossible and hence Monte Carlo methods are used to replace the set of all possible confrontations by a subset which consists of a number of finite confrontations. Here, the size of subset of the finite number of confrontations is much lower than that of total number of confrontations.

Monte Carlo Markov Chain (MCMC) methods use a group of algorithms where parameters are sampled from their respective probability distribution by constructing a Markov Chain such that the function obtained has a distribution closer to its equilibrium distribution. Multiple algorithms to construct these Markov Chains of uncertain parameters from their distributions are available. The determination of the sampling algorithm that would create an equivalent distribution which when compared with the stationary distribution results in reasonable accuracy is an important task in the process. These distributions might be Gaussian or non-Gaussian in nature and the algorithm should be flexible enough to sample from either of them.

Mathematical quantification of the accuracy of each of these sampling algorithms for the same number of confrontations is difficult to judge. Kolmogorov-Smirnov (K-S) test [98] can be used to determine if a sample  $X$  drawn at random using random walk algorithm could have close enough accuracy to the hypothesized, continuous cumulative distribution function CDF. Kolmogorov-Smirnov (K-S) test can also be used to choose the right sample size for the analysis. Monte Carlo methods when run for appropriate number of samples represent the distribution of the stochastic variables to an acceptable accuracy.

#### **4.2.2. Metropolis Algorithm**

Random walk algorithms are used to calculate Markov Chains, which then are used in Monte Carlo Simulations. Some of the common random walk algorithms are Metropolis-Hastings algorithm [99]-[101], Gibbs Sampling [102], [103] and Multiple Try Metropolis. In addition to this, more complex algorithms are available which do not use random walks. The accuracy of the estimate from the sampling algorithm directly depends on the quality of the sampled subset. Hence, sampling plays an important role in improving the accuracy of the estimate. Metropolis being the

most common used random walk algorithm has shown reasonable accuracy in our case as tested by K-S test and hence we have used it in our analysis.

An importance aspect of random walk sampler is that the subset of a finite number of confrontations is chosen such that they are not completely at random but in a way that the chosen confrontations are somehow attracted or biased towards samples that are populated significantly at the equilibrium of the distribution from which it samples. The Metropolis algorithm starts with front confrontation  $x(1)$  which is arbitrarily chosen and then it chooses the second step arbitrarily and then either accepts or rejects it. The actual number of sample size is arbitrary and should be chosen such that it resembles actual equilibrium distribution.

The Metropolis algorithm is given by:

1. Assuming the walker is at point  $x(n)$  in the sequence. In order to get the next point  $x(n + 1)$ , the algorithm takes a trial step  $x'(n + 1)$ . This point  $x'(n + 1)$  is chosen uniformly at random.
2. The trial step is then either accepted or rejected based on the ratio:

$$r = \frac{w(x'(n + 1))}{w(x(n))}$$

where  $w$  corresponds to the probability distribution of the variable. If the ratio is greater than 1, then the trial step is accepted or if it is less than 1 then it is accepted with the probability  $r$ .

3. The same process is repeated until the desired size of the Markov Chain is obtained.

Figure 3 gives the results of a sample application for a Gaussian distribution showing that the Metropolis algorithm is able to choose a random set of values which reflect the given distribution.

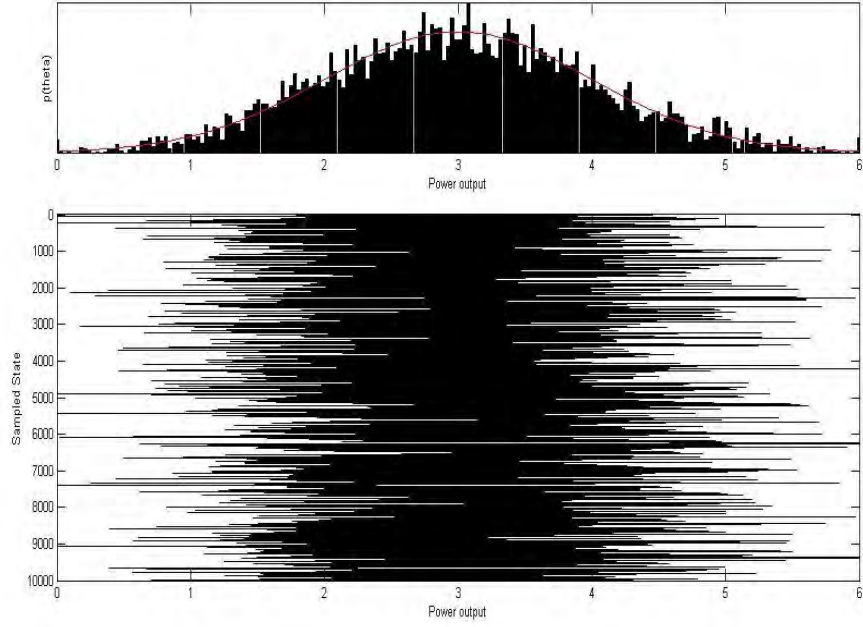


Figure 3: Sampled distribution of stochastic variables using Metropolis algorithm

#### 4.3. Problem Formulation

In order to quantify the benefits with regards to flexibility provided by power flow control devices, power flow calculations are run using the metropolis algorithm to sample the fixed input values. The optimization problem is run for large number of samples such that the sampling of the stochastic variables represents their original distribution. Two different approaches are taken:

1. Maximize power exchange in the system: In this case, one of the generators and the loads are free variable. Then the two cases (with power flow control and without power flow control) are simulated such that no network constraints are invalidated to see which network has a wider operating region letting the non-fixed generator in both networks choose its own operating point. The objective is given as:

$$\max P_{G_k}$$

where  $P_{G_k}$  is the power output of the non-fixed generator subject to the AC power flow equations and the following operational constraints:

$$|S_{ij}| \leq S_{ij}^{max}$$

$$V_i^{min} \leq V_i \leq V_i^{max}$$

which reflect the line flow limits and the limits on the bus voltages. In the case of using power flow control, of course the settings of the power flow control devices are limited. The loads are variables in this case as well. Hence, the resulting total power generation corresponds to the absolute maximum power supply which can be achieved.

2. Economic dispatch: for a large number of samples for changing system parameters (wind and loads), it is determined if there is a feasible solution to the economic dispatch which allows supplying the loads keeping all operational constraints within their limits. This is carried out for the case with and without power flow control devices. Hence, the objective function in this case corresponds to the minimization of the generation costs

$$\min \sum_{i=1}^{N_G} C(P_{G_i})$$

subject to the same constraints as above but with limits on all generator outputs.

#### 4.4. Modeling of VSC-HVDC

The power flow control device considered in the simulation section is a VSC-HVDC line. Hence, in this section, we describe the model used in the simulation to model the VSC-HVDC line. It is represented as a set of four buses: sending bus, receiving bus and two converter buses. The operating mode is dependent on how these buses are represented. Sending and receiving buses represent the AC side of the HVDC line whereas converter buses represent the DC side of the line. In order to model a VSC-HVDC line we need to define a DC line voltage, the line length and also the type of the conductor. DC link voltage ( $V_{dc}$ ) and resistance ( $R_{dc}$ ) is calculated from this information. The losses in the line can also be directly calculated from these parameters.

The convertor voltages and angles can be calculated using the following equations 104:

Active Power Control:

$$P_{sp} = V_{ci}^2 \cdot G_{ci} + V_{ci} \cdot V_k (G_{ci} \cos(\delta_{ci} - \delta_k) + B_{ci} \sin(\delta_{ci} - \delta_k))$$

Reactive Power Control:

$$Q_{sp} = V_{ci}^2 \cdot B_{ci} + V_{ci} \cdot V_k (B_{ci} \cos(\delta_{ci} - \delta_k) - G_{ci} \sin(\delta_{ci} - \delta_k))$$

DC link voltage control:

$$V_{dc} = 2\sqrt{2} \cdot \frac{V_{ci}}{M_{ci}}$$

where,

$P_{sp}$ : Active power through the DC link

$Q_{sp}$ : Reactive power induced into the AC line

$G_{ci}$ : Real part of the converter impedance  
 $B_{ci}$ : Imaginary part of the converter impedance  
 $V_{ci}$ : Converter bus voltage  
 $M_{ci}$ : Modulation index in PWM technique  
 $k$ : bus number AC side

#### 4.4.1. Control Scheme in VSC-HVDC Line

VSC-HVDC gives extra high flexibility in terms of control. Possible operating modes in case of VSC-HVDC are as follows:

Four Modes of Operation		
	AC Side	DC Side
1	Keep AC side voltage constant	Control Active power through the line
2	Keep AC side voltage constant	Keep DC voltage constant
3	Control reactive power into the system	Control Active power through the line
4	Control reactive power into the system	Keep DC voltage constant

One has to choose one of the above mentioned control options. In addition to this, care should be taken such that one converter controls the DC voltage of the line and the other controls the power through it but converter control on the AC side are not mutually exclusive in nature. Although in some of the multi terminal models of VSC-HVDC lines, at least one HVDC AC terminal has to act as a PV bus and other buses act as PQ bus [105].

#### 4.4.2. Modeling

After choosing the control strategy one can decide on how the various buses would behave and should be modeled:

##### 1) Representation of the buses:

AC Side (Sending and Receiving Bus):

1. Control voltage on the bus: PV Bus
2. Control injection of Q: PQ Bus

DC Side (Converter Bus):

1. Control DC line voltage: PV Bus
2. Control active power flow: PQ Bus

2) *Representation of active and reactive power in and out of the line:*

- Sending end is always represented as a load:  $P_L - jQ_L$  (active power going out and reactive power injected into system)
- Receiving end is always represented by a generator:  $P_G - jQ_G$  (active power coming in and reactive power injected into system)
- $P_L$  is the active power out of the sending bus and  $Q_L$  is the reactive power injection into the sending bus if we choose to control the injection of reactive power on the AC side.
- $P_G$  is the active power into the receiving bus minus the losses ( $P_G = P_L - P_{loss}$ ) and  $Q_G$  is the reactive power injection into the receiving bus if we choose to control injection of reactive power on AC side.

3) *Representation of transformer and converter impedances:*

In this model, transformer and converter impedances can easily be represented between the AC buses and the converter buses. These impedances are integrated into the admittance matrix.

The novel part in this model is that even the converter buses are either treated as PQ or PV bus. All the equations can be easily incorporated into the conventional power flow program or even Matpower and hence can easily be solved with minor changes to the existing system. An example model is pictorially represented in Figure 4. The top of the figure shows possible bus configurations whereas the bottom of the figure is a bus configuration for the sending bus controlling the voltage on the AC side and the DC link voltage on the DC side whereas the receiving bus is controlling the power factor on the AC side and the real power transfer on the DC side.

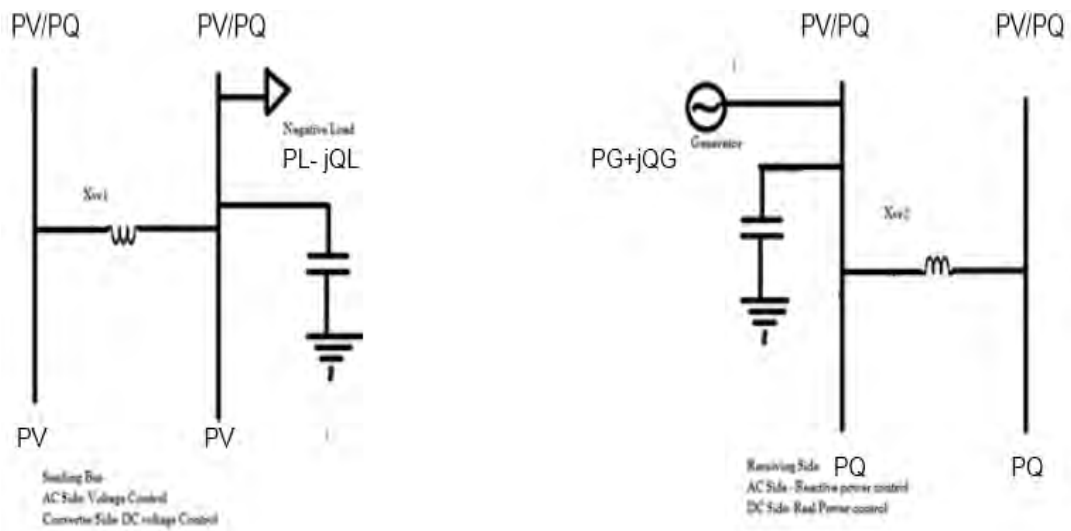


Figure 4: VSC-HVDC Model

## 5. SIMULATION RESULTS

### 5.1. Decentralized FACTS Control

#### 5.1.1. Test System

The proposed two-stage control approach is tested on the IEEE 14-bus system. The system structure is shown in Figure 5. The generator at bus 2 is assumed to be a wind generator. The transmission system needs to transmit the power produced by the generators located in the south to the loads in the north.

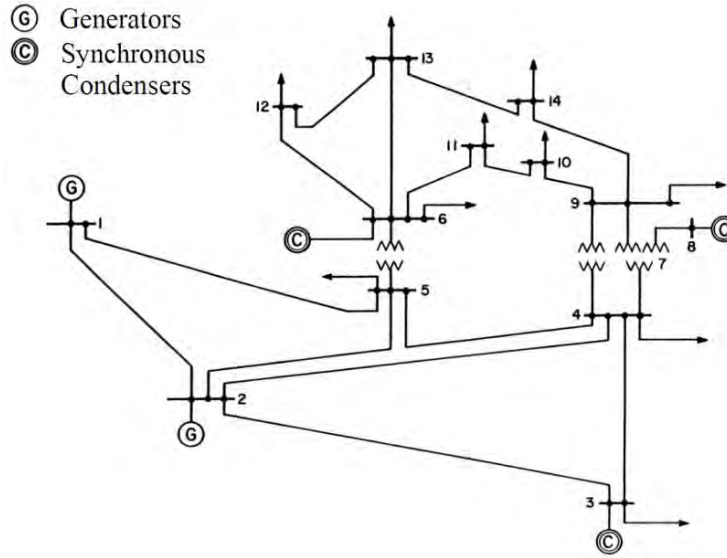


Figure 5: IEEE 14-bus system

1000 different load and generation scenarios are generated. For each load and generation scenario 7 different initial settings of the FACTS devices are considered. These load and generation scenarios cover a wide range of possible system states, from nearly zero loading of the system to around 140% loading. 800 load and generation scenarios are used to determine the coefficients in the fitting function while 200 scenarios are used for accuracy testing.

#### 5.1.2. One TCSC

A single TCSC is placed in Line 1-2. Only the simulation results for the 200 testing load and generation scenarios are shown here and the scenarios are sorted from high loading ones to low loading ones for visualization purposes. Figure 6 shows the objective function values for the following cases: 1) the actual optimal settings of the TCSC obtained from OPF calculations are applied (actual), 2) the estimated values using the quadratic function are applied (estimated) and 3) no TCSC is in the system (without TCSC). Figure 7 shows the errors in the objective function values resulting from applying the regression function to determine the TCSC settings with respect to the actual optimal objective function values.

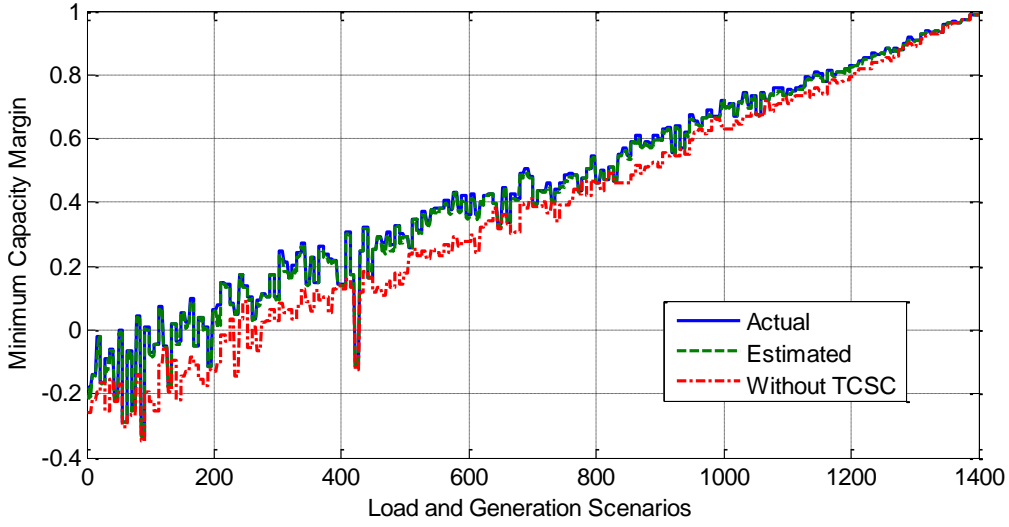


Figure 6: Objective function with one TCSC

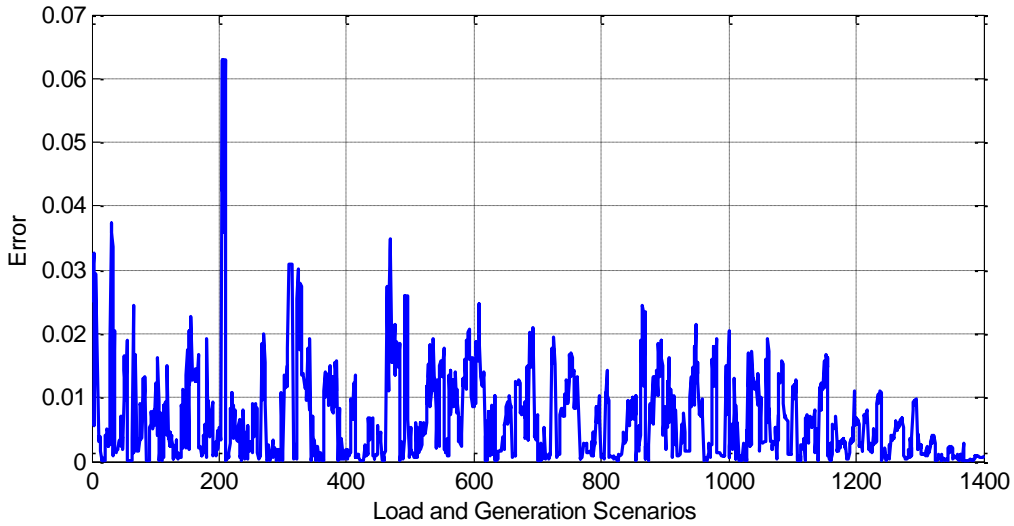


Figure 7: Error in the objective function with one TCSC

The objective function value represents the minimum capacity margin over all transmission lines. A negative value of the minimum capacity margin indicates that one or more transmission lines are overloaded.

Compared to the situations without any TCSC in the system, the minimum capacity margin of the transmission lines is significantly larger with one TCSC in the high loading scenarios while almost the same in the low loading scenarios. In the high loading scenarios, several transmission lines are operating close to their capacity limits and the TCSC is able to push the loadings of these transmission lines away from their capacity limits by redirecting the power flows from the heavy-loaded lines to the light-loaded ones. Therefore, in the high loading scenarios, the TCSC has larger influence on the minimum capacity margin. In the low loading scenarios, the active

power flows through all transmission lines are much lower than their capacity limits, consequently, the capacity margins are all close to one. In these situations, even though the TCSC can redistribute the power flows in the system, the minimum capacity margin is not improved significantly. Hence, the minimum capacity margin is not sensitive with respect to the settings of the TCSC in the light-loaded situations.

As shown in Figure 7, the differences between the objective function values with estimated device settings and those with actual settings are quite small. Therefore, the simulation shows that the regression-based control approach is able to find the close-to-optimal settings of the FACTS device under various load and generation scenarios.

### **5.1.3. Two TCSCs**

Since the settings of all FACTS devices have influence on the system measurements, it is more difficult for the controller of a particular device to determine the regression function with enough accuracy when there are multiple FACTS devices in the system. In order to improve the accuracy for determining the optimal device setting, a piecewise quadratic function is determined with respect to the total power consumption in the system. In the training stage, the load and generation scenarios are partitioned into several subsets according to different ranges of total demand. For each subset, a quadratic function is determined separately. In the operation stage, by getting the information of the total load in the system (which is assumed to be available at current study), the controller of the FACTS device automatically selects the appropriate part of the piecewise function for calculating the optimal device setting.

In the following simulations, one TCSC is placed in Line 1-2 and the other one in Line 2-5. The objective function values with actual optimal settings, with estimated optimal settings and without any TCSC are shown in Figure 8 and the errors of the objective function values using the estimated optimal settings are shown in Figure 9. Similar to the simulation result with only one TCSC, the errors in the objective function values with the estimated optimal settings are small. Consequently, in the situation with multiple FACTS devices in the system, the proposed approach can still determine the optimal settings of each device only based on the information of several local measurements with good accuracy.

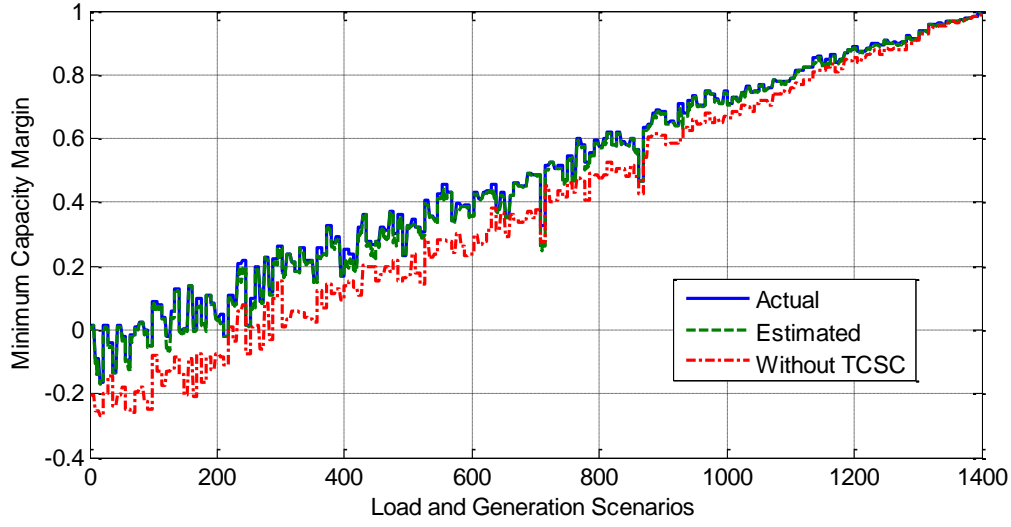


Figure 8: Objective function with two TCSCs

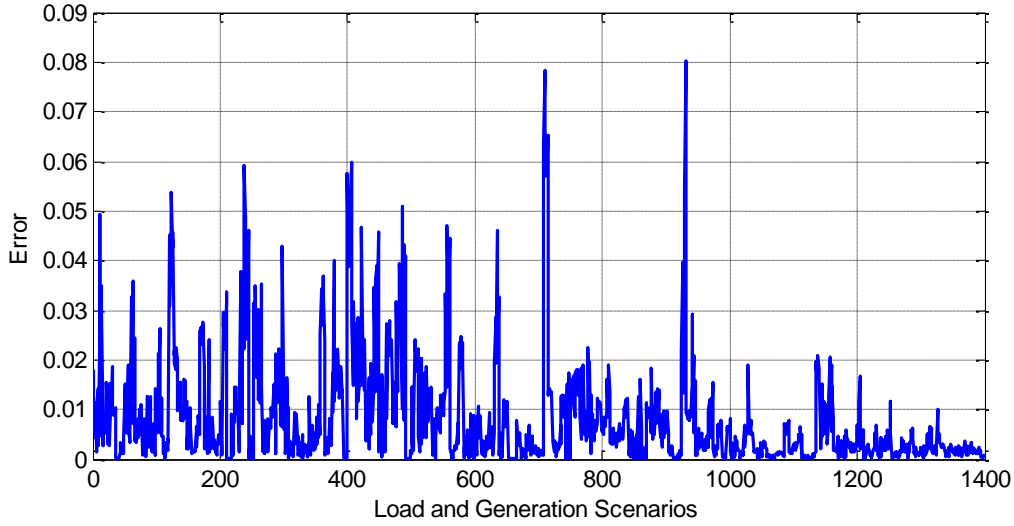


Figure 9: Error in the objective function with two TCSCs

## 5.2. Evaluation of Transmission Flexibility

The two approaches as discussed in Section 4.3 are carried out using Matlab Optimization Toolbox and Tomlab Optimization Toolbox for cases with and without controllable devices. The following analyses are carried out using Monte Carlo Markov Chain simulations which were run for both the cases (with and without controllable devices) for a large sample size. Stochastic parameters such as maximum available generation and load demand changed in every iteration. The stochastic parameters were represented by probability distribution functions and were sampled using Metropolis algorithm to give a good approximate of the actual distribution.

### 5.2.1. Maximization of Power Exchange

For these simulations, the small 5 bus test system shown in Figure 10 is used. The system includes as controllable devices a VSC-HVDC and an SVC. The output of the second generator is maximized in order to maximize the totally supplied load. Figure 11 shows the maximum value for the non-fixed generator 1 for which the power is injected for various values for the fixed generator 2. As the loads are free variables, the sum of the two generators is the absolute maximum power supply which can be achieved if generator 2 is fixed to the given value. It is obvious that the non-fixed generator in the case of a system with controllable devices has a wider operating range and hence it can serve a wider range of uncertain loads as well as variable renewable generation without violating the network constraints. With the problem of uncertain demand growth in the future aggravated by high renewable penetration, it is shown that the system with controllable devices has a better ability to result in feasible operation of the system.

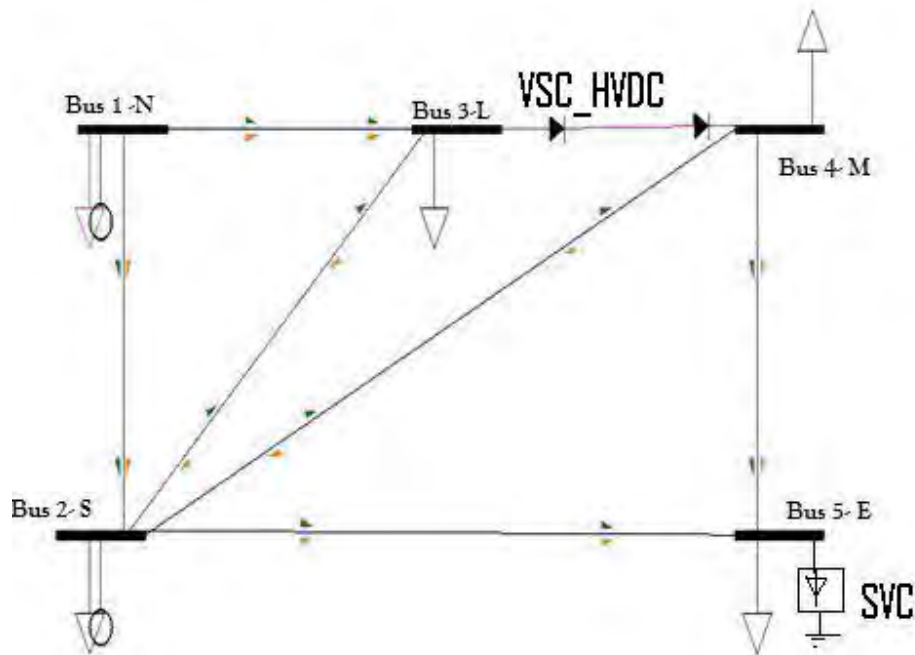


Figure 10: 5-Bus Test System

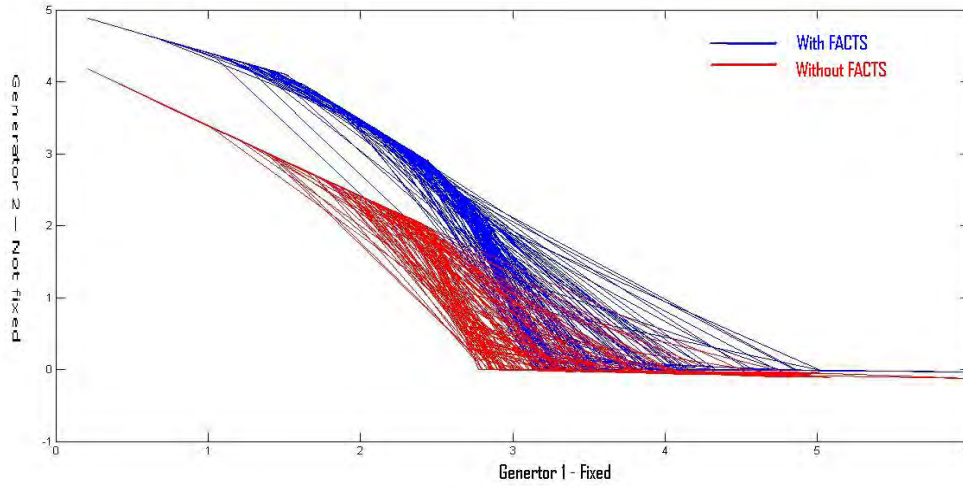


Figure 11: Comparison on the operating limits of the generator in system with and without controllable devices

### 5.2.2. Feasibility of Economic Dispatch

For this simulation a test system consisting of seven buses, three generators, ten lines and five loads is considered for this study (see Figure 12). A VSC-HVDC line (dotted line) and an SVC (modeled as variable reactive power injection) are included in the system. The top two generators are renewable generators for which the maximum value is chosen using the metropolis algorithm. Also, the loads are determined by the metropolis algorithm. Then, an economic dispatch is carried out with linear costs where the cost for the renewable generators is lower than for the conventional generator at the bottom. This results in a situation in which the renewable generators are dispatched first and the conventional generator is used to supply the rest of the load.

The objective of the simulations is to verify whether the presence of power flow control devices would bring in more flexibility into the transmission grid. The number of feasible and non-feasible solutions in the economic dispatch problem for both cases with and without controllable devices is given in Figure 13. It shows that the system containing controllable devices was able to achieve a feasible solution in a much higher number of cases proving the feasibility of the system to operate over a wider range of operating points. On the other hand the system without power flow control devices was unable to reach a feasible solution in a high number of cases depicting its conservative nature and hence unsuitability to address uncertainty. The system including the VSC-HVDC and the SVC over the one without these devices can be attributed to the availability of more control variables in the presence of these controllable devices. If we were to fix one of the generators' maximum output (Wind Max. Output which is uncertain and ever changing), the ability of the other generator to supply power in the case of system with power flow control devices would expand over a broader range when compared to the case of the system without power flow control devices. Such advantages can result in the deferment of investment costs for new transmission infrastructure and also result in the use of cheaper generators while meeting all operational constraints.

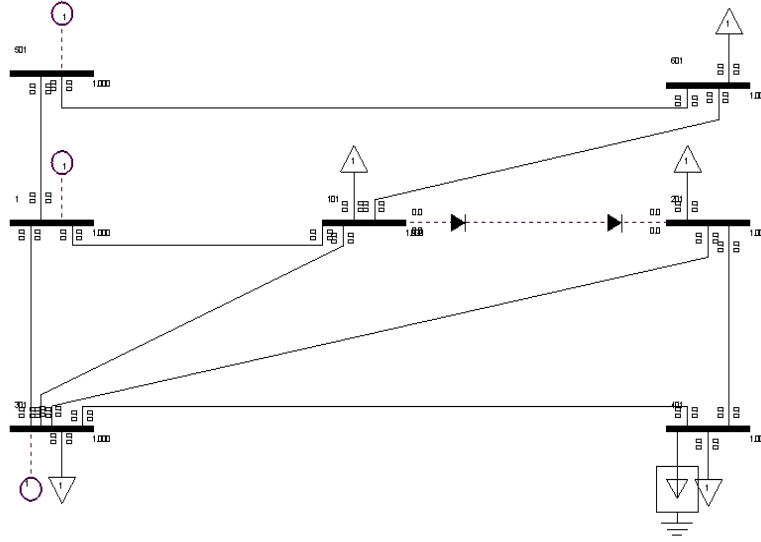


Figure 12: 7-Bus Test System

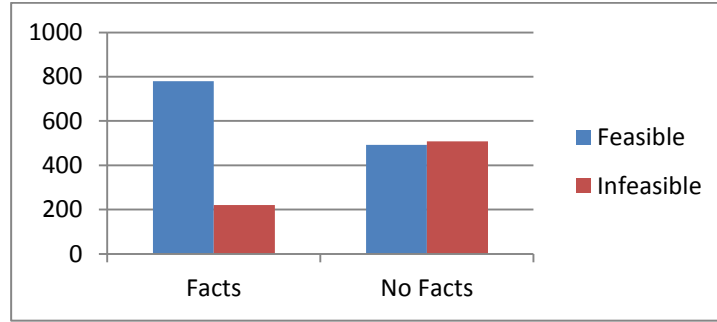


Figure 13: Feasibility of economic dispatch with and without power flow control

### 5.3. Probabilistic Load Flow

#### 5.3.1. Correlation of Beta Distributed Functions

In the first simulation, the method used to create beta distributed functions with a specific correlation is applied to test the accuracy of the achieved correlation. The starting points are two beta distributions with shape parameters  $\alpha_1, \beta_1$  and  $\alpha_2, \beta_2$  each representing a wind generator output. Then the correlation of these two functions is considered in  $\sim 10\%$  increments and the probability distribution for the sum of the power outputs is determined. The result for beta distributions with the same shape parameters is shown in Figure 14 and the result for beta distributions with opposite shape parameters is shown in Figure 15.

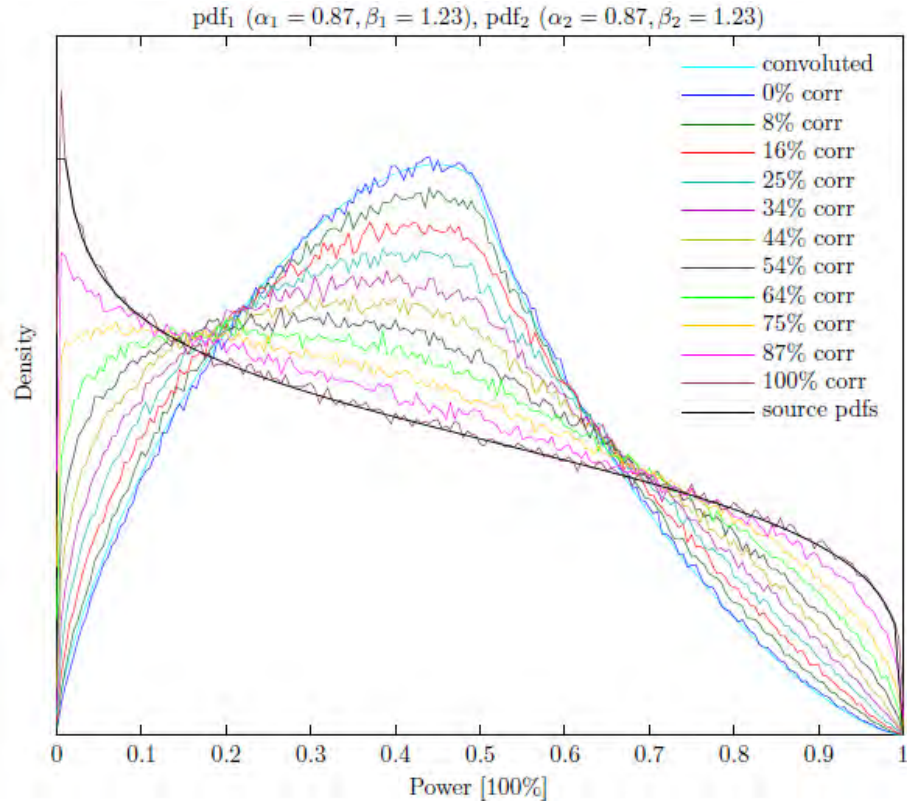


Figure 14: Positive correlations for distributions with same shape parameters

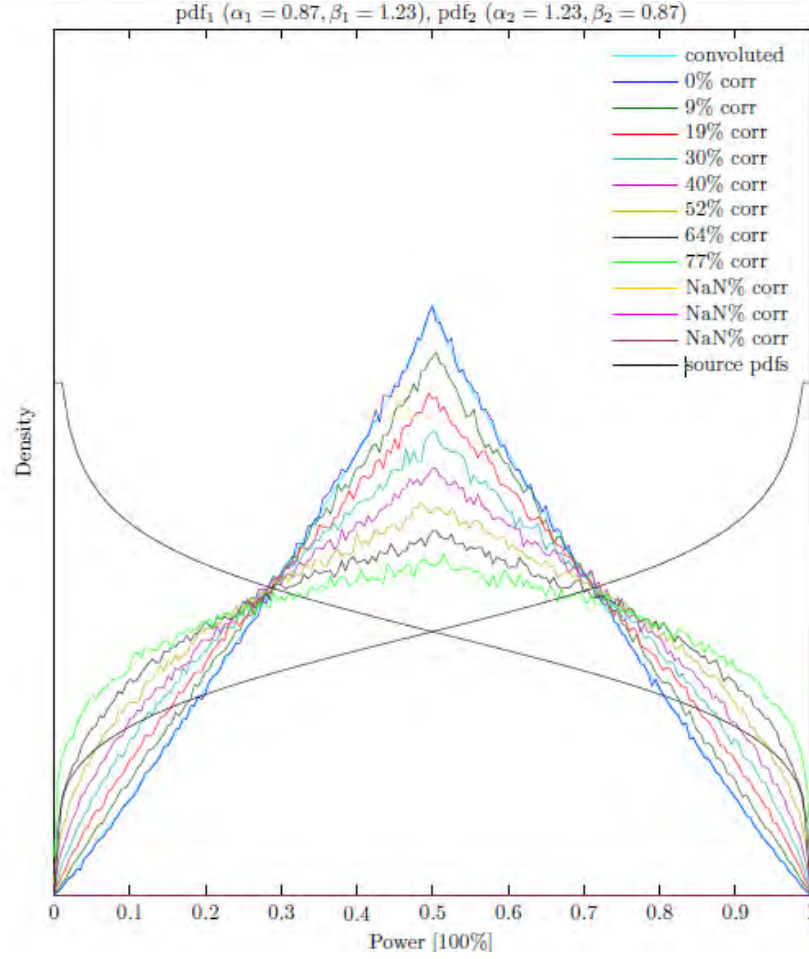


Figure 15: Positive correlations for distributions with opposite shape parameters

Back calculating the correlation of the generated random variables is used to test the accuracy of the method to achieve a specific level of correlation. Hence, the achieved correlation vs. the desired correlation is shown in Figure 16. It can be seen that the bias is very small.

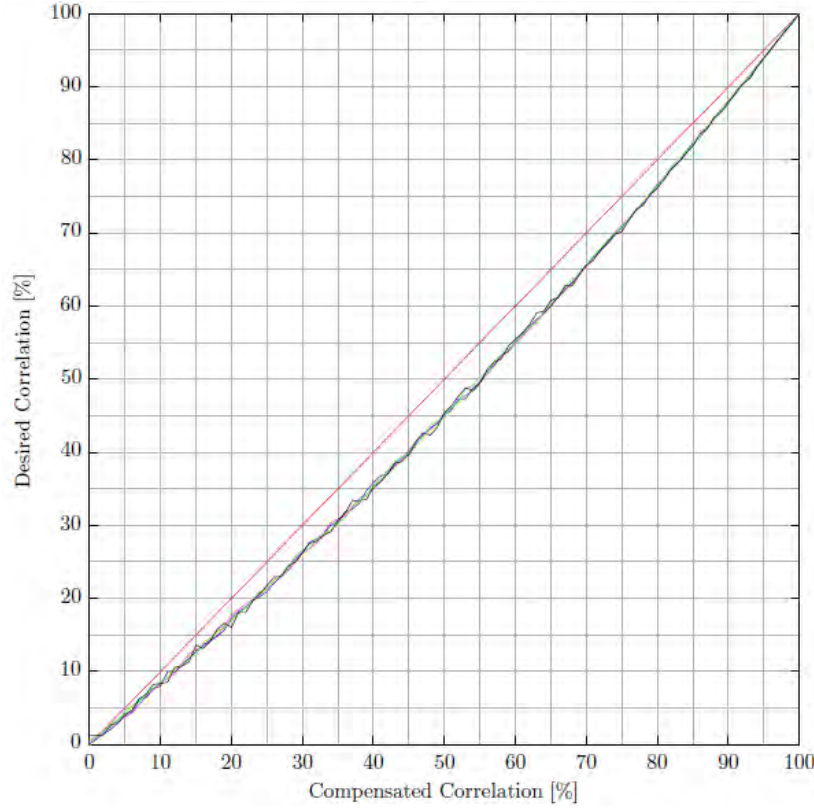


Figure 16: Achieved correlation vs. desired correlation

### 5.3.2. Load Flow for Correlated Wind Generation Plants

In this section, the method for the determination of correlated beta distributions is used to generate random power outputs for wind generators with a desired correlation. The simulations are supposed to provide a concept of proof and show the impacts of correlated power injections. The small system shown in Figure 17 is used as test system. There are two wind generators each located at a separate bus and the load collocated with the slack generator at a separate bus. The slack generator makes up for any missing power to fully supply the load. The probability distributions used for the wind generators and the loads are given in Figure 18 and Figure 19, respectively. The shape parameters for the wind generator outputs are chosen according to [12]. The load profile is generated based on data from 106. The best fit for this load profile is a Weibull distribution with scale parameter  $\lambda = 1.75$  and shape parameter  $k = 2.6$ . Using these probability distributions and the method for the generation of correlated beta distributed variables, Monte Carlo simulation are carried out using DC power flow to find the lines flows on lines 1, 2 and 3 and based on these values generate the probability distributions for these variables.

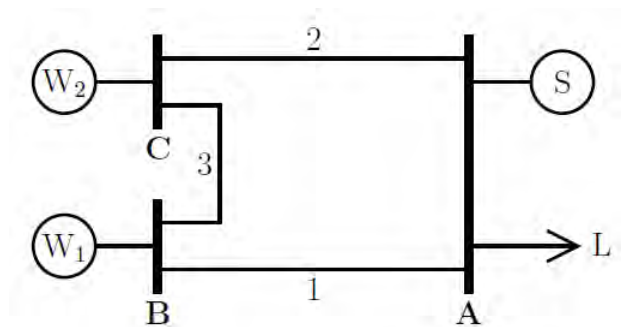


Figure 17: Three bus test system

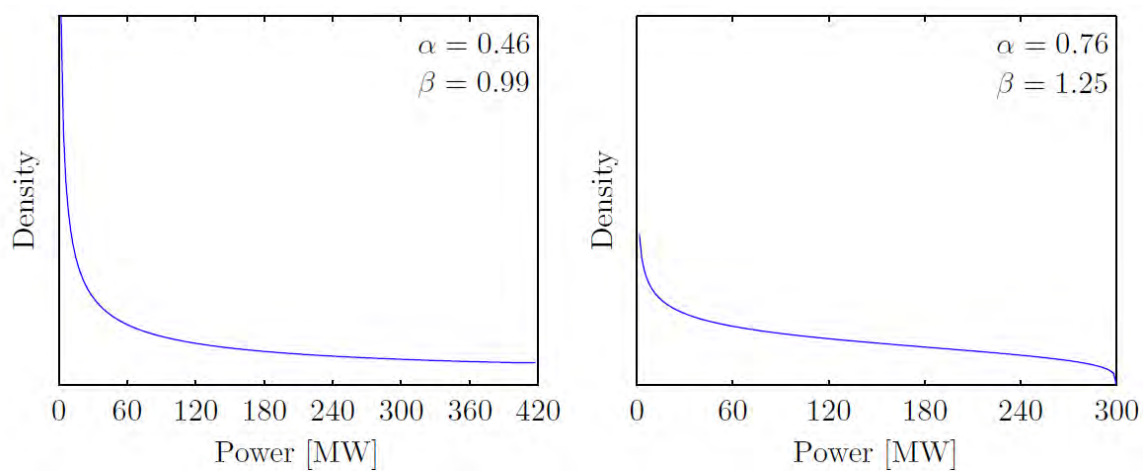


Figure 18: Distributions for wind generator  $W_1$  and  $W_2$

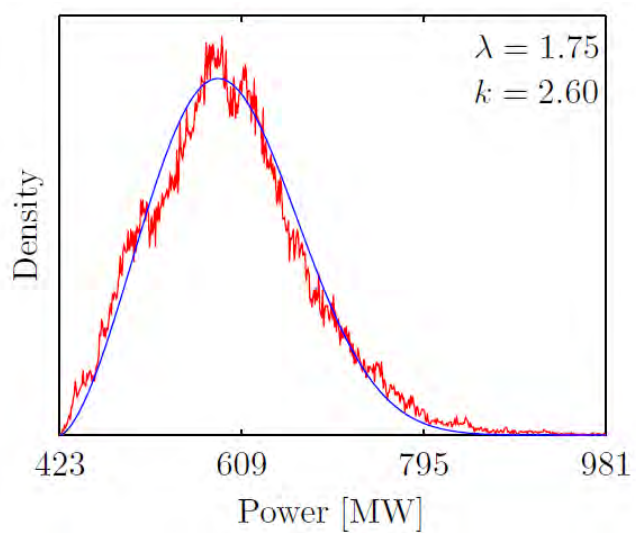


Figure 19: Distribution for load

In Figure 20, Figure 21 and Figure 22, the probability distributions are shown for various levels of correlation between the two wind generators. It can be seen that the correlation has significant influence on the distribution which indicates that this is something which needs to be considered when carrying out probabilistic load flow studies.

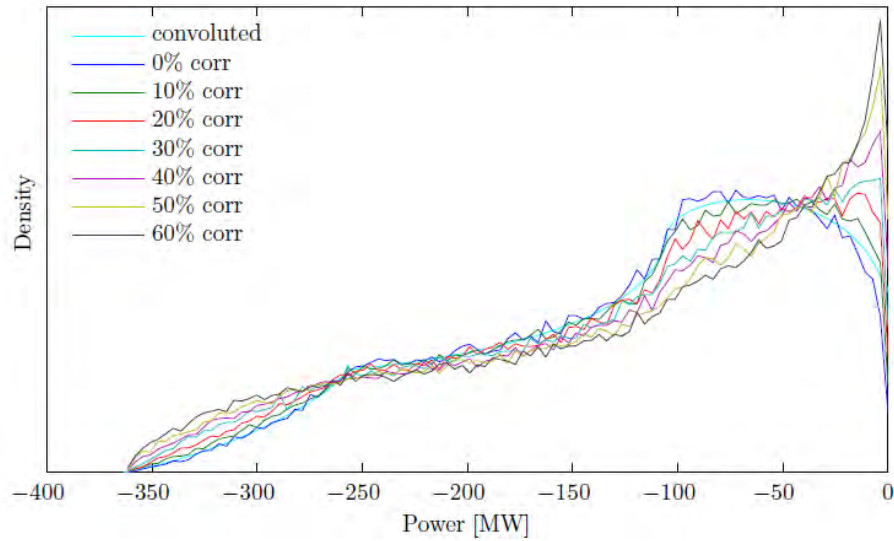


Figure 20: Power flow on line 1

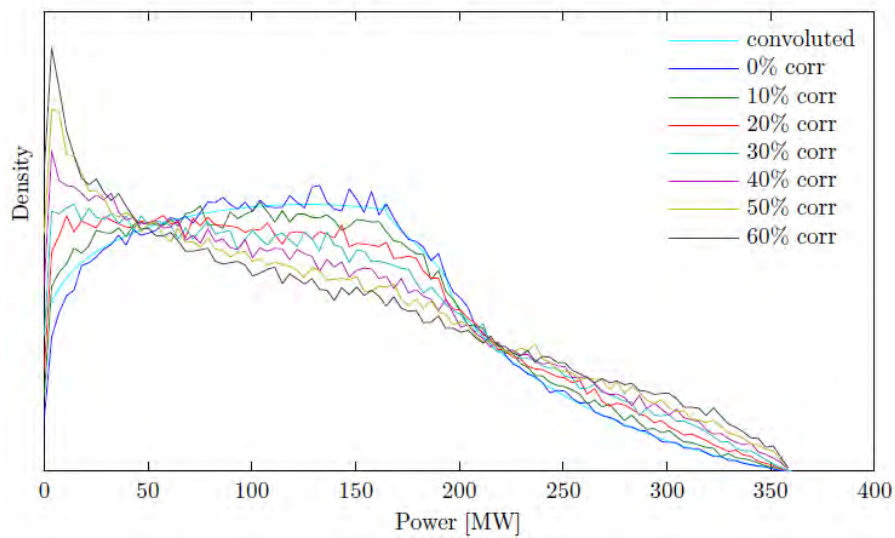


Figure 21: Power flow on line 2

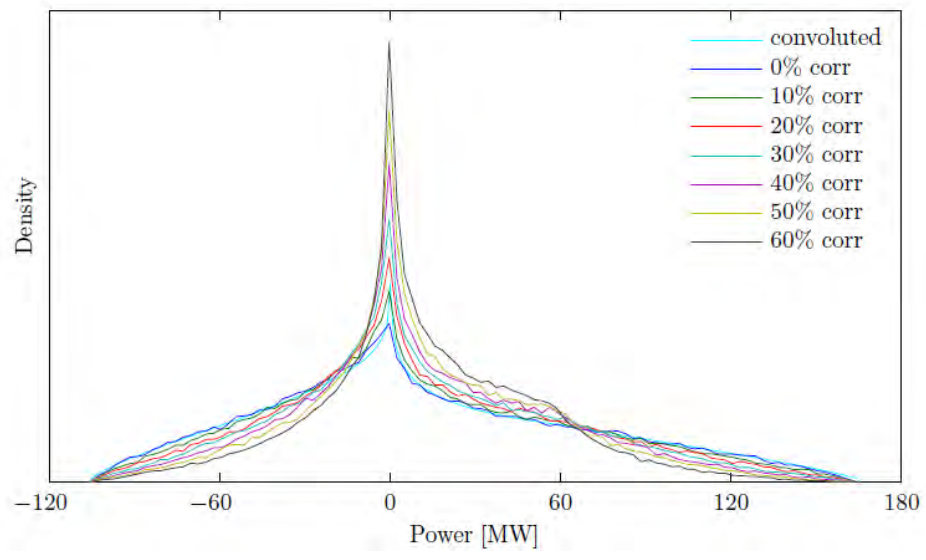


Figure 22: Power flow on line 3

## 6. CONCLUSION AND FUTURE WORK

In this part of the project, we have provided multiple tools for the operation, planning and analysis of flexible power flow control devices in an environment with uncertain power injections:

- The decentralized control concept allows determining the optimal settings of the devices without the need for online optimal power flow calculations. The objective function corresponds to maximizing the minimum transfer margin defined as the difference between line loading and actual flow on the line. Regression analysis has been used to find a regression function to identify the measurements which provide the most information regarding the optimal settings of the devices and to determine the regression function which maps the optimal settings to the values of these key measurements. The simulations showed good accuracy for the considered test case.
- The review of probabilistic load flow methods showed a wide range of methods which are employed for this purpose. The focus of this part of the work was on deriving a tool which allows generating beta distributed random variables with a certain given correlation. Such a method is needed due to the fact that wind generation outputs of wind generators within the same region are correlated. In order to study the implications on the power grid by the means of probabilistic load flow calculations, it is important to take such correlation into account. Simulations showed that the desired correlation can be achieved with fairly good accuracy.
- The final part of the work focused on analyzing the increased flexibility of the power grid if power flow control devices are located in the system. Such flexibility is important to accommodate highly varying renewable generation. A Monte Carlo Markov Chain method was employed to generate scenarios and to study the benefits provided by power flow control devices and compare the outcomes to the case without any power flow control devices. The simulations showed that the range of possible generator dispatch can be increased significantly when using power flow control devices which again favors the integration of renewable generation.

Simulations in small scale systems have been used to give a proof of concept. Future work will include employing these tools in larger systems and using these tools for a quantitative analysis of the benefits achieved by power flow control devices. Furthermore, the fact that the system is operated fulfilling the constraints imposed by N-1 security will be integrated.

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## **Part 3**

# **Large Scale Energy Storage**

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## Nomenclature

$A$	An ( $m \times n$ ) constraint matrix
$b$	An $m$ -dimensional column vector of right hand side coefficients
$C$	The cost coefficient of the decision variables to be minimized
CAES	Compressed Air Energy Storage
$C_B$	Cost of battery in dollars per Wh
$C_{BT}$	Total cost of battery in dollars
$C_E$	Cost of electronics
$C_{ET}$	Total cost of electronics
$c_i$	The cost of the generator at $i^{\text{th}}$ bus
$C_i$	Total initial investment
$C_W$	Cost of wind turbine in dollars per MW
$C_{WT}$	Total cost of wind turbines in dollars
DCOPF	Direct Current Optimal Power Flow
DFIG	Doubly Fed Induction Generator
DP	Dynamic programming
EES	Electrical energy storage systems
EIA	Energy Information Administration
$E_{sq \max}$	The maximal energy storage at storage $i^{\text{th}}$ bus
$F_{\text{cost}}(k, n)$	The total cost from initial state to hour $k$ state $n$
$h$	The number of interval of hours of a day.
ITMAX	Maximum allowed iterations
LB	Lower bound
LMP	Locational marginal price
LP	Linear programming
$m$	The set of states at hour $t - 1$
$n_b$	The number of buses in the system
$N_D$	Number of days for repay of the original investment
$n_g$	The number of generators
$n_l$	The number of transmission lines
NREL	National Renewable Energy Laboratory
$n_s$	The number of large scale storage system

$P_A$	Generation at bus A
$P_B$	Generation at bus B
$P_{cost}(k,n)$	The production cost for state $(k,n)$
$P_{gi}$	Generation in MW at bus $i$
$P_{gi}$	The real power output at generator bus $i$
$P_{gi\ min}, P_{gi\ max}$	The minimal and maximal real power output at generator $i$
$P_{ij}$	The power flow of transmission line $i$ - $j$
$P_{ij\ min}, P_{ij\ max}$	The minimal and maximal power limits of transmission line $i$ - $j$
$P_L$	Real power load in MW
PSERC	Power Systems Engineering Research Center
PSO	Particle swarm optimization
$P_{sq\ min}, P_{sq\ max}$	The minimal and maximal storage capacity at storage $i$
$Q$	$(n \times n)$ matrix describing the coefficients of quadratic terms
$QP$	Quadratic programming
$S_{cost}(k-1, m, k, n)$	The transition cost from state $(k-1, m)$ to state $(k,n)$
SMES	Superconducting magnetic energy storage
TES	Thermal energy storage
UB	Upper bound
WECC	Western Electricity Coordinating Council
$X$	The $n$ -dimensional column vector of decision variables
$x_{ij}$	Reactance of line $ij$

# **1 Introduction to Wind Energy and Large Scale Electric Energy Storage Systems**

## **1.1 Introduction: Wind Energy Integration**

In the United States most electricity is generated from electric power stations that use coal and natural gas. These two despite being reliable and affordable also have drawbacks. These release greenhouse gases in the atmosphere and besides that are finite and unevenly distributed across the globe. There is an immediate need for some alternative fuels which can overcome the issues pertaining to conventional power stations such as solar power and wind power. These alternatives also have disadvantages as the wind energy resources are intermittent in nature. The same intermittency occurs with solar power. As a consequence of absorbing increasing amounts of wind and solar resources, the electrical power system will need more flexibility to respond to the combined instantaneous fluctuations in both load and renewable generation. Such response would come through proving regulation, load-following, and fast ramping services. Moreover, the system may also need to commit more dispatchable and flexible resources in the day-ahead time frame to meet load net of renewable generation due to inaccurate variable generation forecast. The capacity of generation should always be greater than or equal to the peak demand. This makes intermittent sustainable generation alternatives integration potentially difficult.

Energy storage technology has the capability to ease the inclusion of large-scale variable renewable electricity generation, such as wind and solar. During electricity generation wind and solar power emit no greenhouse gases. Compared to conventional generators, the electrical energy storage systems (EESS) have potentially faster ramping rate which can quickly respond to load fluctuations. This speed is the case for electronically controlled storage systems. Therefore, the EESS can be a spinning reserve source which provides a fast load following and reduces the need for spinning reserve sources from conventional generation.

## **1.2 The Central Objectives of this Research**

The wind generation industry is entering into the range of megawatt-scale production <sup>1</sup> and has been getting increasing attention on account of wind energy being available free of cost and also being a non-polluting source of electricity. But a barrier in wind energy integration to the grid is its intermittency and uncertainty. Upgrade of the transmission system is often necessary to mitigate congestion in the power system with increasing demand. However, transmission expansion solutions may not be effective because cost of building a transmission line is often high and obtaining approvals to install new lines will take time. The energy storage at the load could be a more flexible and economical solution to the planning of power system.

Renewable energy, due to its lower controllability, adds uncertainty in the operation of the power system which is a technical challenge for the existing power system. Uncertainty may require additional control action from the conventional generation units and of renewables themselves thus increasing the cost of integration of the renewable resources [1].

This research focuses on the use of bulk energy storage in power systems for different energy storage capacities with wind energy penetration in the power system, thereby studying the operating cost of generation from conventional generators.

### **1.3 The Contemporary Literature of Wind Energy Resources**

Wind power in the world has seen a substantial growth in the past decade making it one of the fastest growing sources of electricity and one of the fastest growing markets in the world today. The analysis conducted by the NREL estimates that current wind technology could generate 37 trillion kilowatt-hours of electricity per year in U.S. [5]. With the increased wind power penetration and sizes of the wind farms such as over 1000 MW of offshore wind farms, their impact on the power system operation – stability, control, power flow will also increase. For large wind farms these sudden changes can lead to power system instability.

Wind farms produce enough electricity to power all of Virginia, Oklahoma or Tennessee [6]. To illustrate the contemporary importance of wind energy, note that:

- In 2010, 2.3 % of the electric energy generation came from the wind in the U.S.
- The state of Iowa is often cited as a high wind energy state, and existing wind projects could produce 20% of the state electricity [6].
- Minnesota, North Dakota, Oregon, Colorado and Kansas all receive more than 5% of their electricity from wind and other states are following close behind with ever-growing wind power fleets [6].
- According to the Annual Report by NREL [9], in 2007 in terms of nameplate capacity, wind power was the second largest new resource added to U.S. electricity grid behind 7,500 MW of new natural gas plants and ahead of 1,400 MW of new coal.
- New wind plants contributed about 35% of the new nameplate capacity added to the U.S. electrical grid in 2007, compared to 19% in 2006, 12% in 2005, and less than 4% from 2000 through 2004 [7].

The U.S. Energy Information Administration (EIA) predicts that electric utilities plan on installing 72,157 MW of additional wind capacity between 2010 and 2014 [10]. Wind power has a number of benefits. Firstly, its primary energy source, the wind is globally abundant both on land (onshore) and at sea (offshore). Secondly, wind power is the most mature and cost effective renewable energy technology. Wind power also has some challenges. Good potential wind sites are often located far from the cities where electricity is required. This may require improving the contemporary transmission infrastructure to deliver the electricity to the load center.

## 1.4 Bulk Energy Storage

Large scale energy storage uses forms of energy such as chemical, kinetic or potential to store energy later being converted to electricity. The main applications in electric power systems are listed as follows:

### *Cut Down Reserve Margin and Reduce Back-up Power Plants:*

Energy storage technologies can provide an effective method of reducing the need for reserve margin and reserve power plants in order to respond to daily fluctuations in demand. Supplying peak electricity demand by using electricity stored during periods of lower demand, thereby reducing the need for expensive fossil-fired reserve generation plants.

### *Integrating Renewable Energy:*

Electricity storage can smooth out this variability and allow unused electricity to be dispatched at a later time. Balancing electricity supply and demand fluctuations over a period of seconds and minutes.

### *Operating Cost Reduction:*

As a result of aging electricity grid, electricity outages cost the U.S. approximately \$150 billion annually [8]. Electricity storage technologies can provide power to the grid to smooth out short-term fluctuations until backup generation is back to normal.

### *Deferral of Transmission Expansion:*

The increasing demand of electricity requires additional transmission infrastructure. New transmission lines from power plants are a costly and time-consuming process. Storage can help to postpone the need to build new transmission lines [10].

Several wide ranging energy storage media and devices have been proposed for alleviating problematic issues coming from the integration of renewable energy resources. The addressed problematic issues relate to the resource availability and uncertainty. As an example, a possible remedy for volatility of the wind energy the major energy storage technology options are:

### *Pumped Hydro:*

In pumped hydro storage, a body of water at a relatively high elevation represents potential or stored energy. During periods of high electricity demand and high prices, the electrical energy is produced by releasing the water to drop in elevation to flow back down through hydro turbines at a lower elevation and into the lower reservoir. During periods of low demand and low cost electricity water is pumped back from a lower-level reservoir. The potential use of this technology is limited by the availability of suitable geographic locations for pumped hydro facilities near demand centers or generation [4]. Pumped hydro storage is appropriate for load-leveling because it can be constructed at large capacities of hundreds to thousands of megawatts (MW) and discharged over long periods of time up to 4 to 10 hours [14]. The efficiency is about 70% - 80% which varies depending on the plant size [16].

### *Compressed Air:*

Compressed air energy storage (CAES) is a hybrid generation technology in which energy is stored by compressing air within an air reservoir and in some cases injecting air at high pressure into underground geologic formations, using a compressor at off-peak and low-cost electric energy. When demand for electricity is high, the compressed air is released and burnt with fuel to drive the generator such as gas-fired turbines. Thereby, allows the turbines to generate electricity using less natural gas [4]. This is also an appropriate load-leveling because it can be constructed in capacities for few hundred MW and can be discharged over long periods of time (4-24 hours) [14].

### *Batteries:*

Energy storage batteries store the electrical energy in the form of a chemical reaction by creating electrically charged ions inside the battery. The reversal of this reaction will result in the discharge of the battery producing electrical energy from the chemical reaction [14]. There are a number of battery technologies under consideration for large-scale energy storage like lead-acid, lithium-ion, and sodium sulfur. Among these lead-acid batteries are mostly used because of their relatively low cost. Batteries can provide power quality, load-leveling and is easy to install [18]. Table 1.1 shows the comparison of lead-acid, nickel-cadmium and lithium-ion batteries.

Batteries store dc charge, and power conversion is required to interface a battery with an AC system. Small, modular batteries with power electronic converters can provide four-quadrant operation (bidirectional current flow and bidirectional voltage polarity) with rapid response. But there are some technical problems with use of batteries, e.g., the cell will discharge itself so they are only suitable for short-term electricity storage. Also, batteries age resulting in a decreasing storage capacity.

Table 1.1: Specification of Batteries

Battery type Specification	Lead acid	Nickel cadmium	Lithium-ion
Energy density (Wh/kg)	30-50	45-80	150-190
Cell voltage (V)	2	1.2	3.6
Overcharge tolerance	High	Moderate	Low
Cycle life (80% discharge)	200-300	1000	500-1000
Charge time (h)	8-16	1	2-4
Toxicity	Very high	Very high	Low
Cost (\$/Wh)	0.125-0.2	0.4-0.8	0.2-0.36

\*Sources of data: [16]-[18]

### *Thermal Energy Storage:*

Thermal energy storage (TES) can be divided in two different types. Firstly, TES applicable to solar thermal power plants and secondly its end-use [20]. TES for a solar thermal power plant consists of a synthetic oil or molten salt that stores solar energy in the form of heat collected by solar thermal power plants to enable smooth power output during daytime cloudy periods and to extend power production for 1-10 hours past sunset [21]. End-use TES stores electricity from off-peak periods through the use of hot or cold storage in underground aquifers, water or ice tanks, or other storage materials and uses this stored energy to reduce the electricity consumption of building heating or air conditioning systems during times of peak demand [22]. During off-peak periods ice can be made from water using electricity, and the ice can be stored until next day when it is used to cool either the air in a large building, thereby shifting the demand off-peak. Using thermal storage can reduce the size and initial cost of cooling systems, lower energy costs and maintenance costs.

### *Hydrogen:*

Hydrogen storage involves using electricity to split water into hydrogen and oxygen through a process called electrolysis. Compressed hydrogen is the simplest system to conceive. When electricity is needed the hydrogen can be used to generate electricity through a hydrogen powered combustion engine or a fuel cell. Hydrogen fuel cells can be used in power quality applications where 15 seconds or more of ride-through are required. On a life-cycle cost basis for long duration applications, fuel cell technology competes with battery systems at discharge times greater than about 2 hours, depending on cost assumptions, and with hydrogen-fueled engines at discharge times greater than about 4 hours. Typical energy efficiency of a fuel cell is between 40-60%, or up to 85% efficient if waste heat is captured for use [23]-[24].

### *Flywheels:*

A flywheel is an electromechanical storage system in which energy is stored in the form of kinetic energy of rotating mass. The charging or discharging of the flywheel storage system takes place by changing the amount of kinetic energy present in the accelerating or decelerating rotor, respectively [4]. The flywheel is coupled with an electrical machine which acts as a motor to drive the flywheel while charging and acts as a generator to discharge the stored energy by decelerating the rotor to stationary position. During charging, an electric current flows through the motor increasing the speed of the flywheel. During discharge, the generator produces current flow out of the system slowing the wheel down [25].

### *Ultra Capacitors / Super Capacitor:*

Capacitors store their energy in an electrostatic field rather than in chemical form. These consist of two parallel electrode plates which are separated by a dielectric. When the voltage is applied across the terminals the positive and negative charges get accumulated over the electrodes of opposite polarity. The capacitor stores energy by increasing the electric charge accumulation on the metal plates and discharges energy when the electric charges are released by the metal plates. Ultra-capacitors are now available in the range of up to 100 kW with very a short discharge time of up to ten seconds [26]. Ultra-capacitors have temperature independent response, low mainte-

nance and long lifetimes, but they have relatively high cost. These devices also have high loss and they are intended to be operated only for a few seconds.

#### *Superconducting Magnetic Energy Storage (SMES):*

Superconducting magnetic energy storage is an energy storage device that stores electrical energy in magnetic field without conversion to chemical or mechanical form. In SMES, a coil of superconducting material allows DC current to flow through it with virtually no loss at very low temperatures. This current creates the magnetic field that stores the energy. On discharge, switches tap the circulating current and release to serve the load with high power output in short interval of time [25]. Although the SMES device itself is highly efficient and has no moving parts, it must be refrigerated to maintain superconducting properties of the wire materials. Therefore, SMES devices require cryogenic refrigerators and related subsystems, thus increasing maintenance costs [14].

Table 1.2 summarizes some of these storage technologies and their characteristics.

### **1.5 Organization of this Report**

This is Part 2 of a three part final report. Part 2 is organized into five chapters. Chapter 2 presents basic concepts of optimal dispatch including different economic dispatch methodologies. These concepts are used in the formation and solution of the algorithm for optimal energy storage.

Chapter 3 demonstrates the idea of optimal scheduling of energy storage using a small illustrative example. Chapter 4 illustrates application of this algorithm in the state of Arizona as a test bed. The test bed is a subset (equivalent) of the Western Electricity Coordinating Council system. Chapter 5 presents conclusions, contributions from the test beds studied in Chapter 4 and lines of future work regarding the use of large scale energy storage in power systems.

There are two appendices provided. Appendix A shows the corresponding Matlab algorithm for the DC optimal power flow developed during this research. Appendix B describes the quadratic programming algorithm.

Table 2.2: A Comparison of Bulk Energy Storage Technologies

Storage method	Capacity	Capital cost (\$/ MWh)	Weight (kg/MWh)	Efficiency	Maintenance cost (\$/MWh)	Maturity	Discharge time	Power level (MW)	Response time (ms)	Lifetime (years)
Pumped Hydro	22,000 MWh	7,000	3,000	0.8	4	Commercial	12 hours	<2000	30	40
CAES	2,400 MWh	2,000	2.5	0.85	3	Commercial	4-24 hours	100-300	3000-15000	30
Batteries	200 MWh			0.7-0.85		Commercial	1-8 hours	<30	30	2-10
Thermal energy	400 MWh	550	300,000	0.8	15	Commercial	6 hours	260		40
Hydrogen	0.3-2000 kWh	15,000	30	0.45-0.8	10	Commercial				10
Flywheel (low speed)	50 kWh	300,000	7,500	0.9	3	Commercial	min to 1 h	<0.100 (each)	5	20
Flywheel (high speed)	750 kWh	25,000,000	3,000	0.93	4	Recent commercial				20
Ultra capacitor	0.5 kWh	28,000,000	10,000	0.95	5	Commercial	10s	0.200	5	40
SMES	0.8 kWh	10,000	10	0.97	1	Commercial	10s	0.100	5	40

\*Sources of data 12-26

## 2 Optimal Dispatch of Energy Storage Systems

### 2.1 Power System Operation

The operation of power systems involves the best utilization of the available energy resources. The operation generally subjected to various constraints to transfer electrical energy from generating stations to the consumers with maximum safety without interruption of supply.

Prior to restructuring of the power system in the United States, unit commitment (identifying the generators which when dispatched, will give the available least-cost operation of available generation resources to meet the electrical load) [32] and economic dispatch were performed by vertically integrated utilities. This operating strategy is done to minimize the production cost of generation. Occasionally, there are power exchanges or interchanges between utilities to take economical advantage of power interchanges. Power pools were formed by several interconnected utilities to effectuate this exchange. Traditionally, coordinating unit commitment and economic dispatch were performed by a central dispatch office [30].

There are three stages in system control, namely unit commitment, security analysis and economic dispatch [36]:

- Unit commitment involves the hour-by-hour ordering of generator units start-up/shut-down in the system to match the anticipated load.
- With a given power system topology and a given number of generators, security analysis assesses the system response to a set of contingencies and provides a set of constraints that should not be violated if the system is to remain in secure state.
- Economic dispatch orders the minute-to-minute loading of the connected generating plants so that the cost of generation is minimum subject to constraints. Figure 2.1 illustrates the operation and data flow in a modern power system.

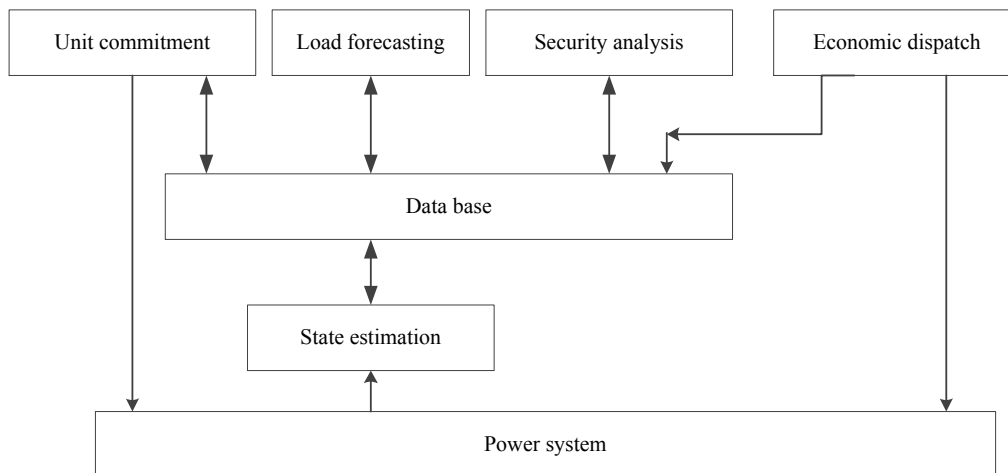


Figure 2.1: Power System Control Activities

## 2.2 The Theory of Optimal Dispatch

The definition of optimal or economic dispatch provided in EPC Act section 1234 is “The operation of generation facilities to produce energy at the lowest cost to reliably serve consumers, recognizing any operational limits of generation and transmission facilities” [27].

The fuel cost (\$/h) of a thermal unit is often expressed as an approximately quadratic function of the power output (MW) of the unit. Therefore the incremental cost (\$/MWh) is almost linear with respect to the unit power output. Without considering other parameters (e.g., transmission losses, reactive losses, line constraints, unit output power constraints), the most economical generation levels occur when the incremental costs of all available units are equal. This simple rule is known as the ‘equal incremental cost rule’ and this is a result of elementary analysis and formulation of the problem as a Lagrange multiplier optimization [28]. If a unit has a higher incremental cost at an output level than other units, it would be cheaper to generate the MW from another unit with a lower incremental cost. The ‘equal incremental cost rule’ needs to be modified when the generator output limits and the transmission losses are taken into consideration. When the MW output level of a unit reaches its upper limit, the unit output is fixed at the upper limit even if the system load increases. Other units which have not reached their maximum limits would share the load increase based on the ‘equal incremental cost’ rule. To account for the transmission losses, the incremental costs are modified with a ‘penalty factor’. The penalty factor is a measure of additional transmission losses due to an incremental increase in the unit output [30].

There are many conventional methods that are used to solve the economic dispatch problem such as the Lagrange multiplier method, lambda iteration. These methods need to compute the economic dispatch each time load changes. As a result, long computation times may result.

## 2.3 Economic Dispatch Methodologies

There are various techniques including traditional and modern optimization methods developed for the economic dispatch without security-constrained (i.e. operation of the power system under credible contingencies). These methods can be classified as conventional optimization methods and intelligent search methods [33]. The conventional optimization methods include lambda-iteration, linear programming (LP), quadratic programming (QP), dynamic programming, and mixed integer programming. Among these methods lambda-iteration method is simple, more favorable, and used in many commercial economic dispatch programs. Some of the intelligence search methods are neural network and particle swarm optimization (PSO).

The system incremental fuel cost rate, called system lambda, is the key to find the most economical generation output of all on-line units. However, when the cost function is more complex than a piecewise linear function or a quadratic function, other methods are more suitable than the lambda-iteration method [31].

The conventional optimization methods are discussed below in brief.

### *The Lambda-Iteration Method:*

In the lambda iteration method, lambda is the variable introduced in solving constraint optimization problem and is called Lagrange multiplier. For the following discussion, use the notation that  $P_{gi}$  are generator powers,  $N$  is the number of generators,  $\epsilon$  is a tolerance to which equalities are set,  $P_L$  is the total load power, and  $\lambda$  is the ‘system lambda’ or incremental cost (\$/MWh). All the inequality constraints to be satisfied in each trial, the equations are solved by the iterative method [31]:

- Step 1: Assume a suitable value of  $\lambda^{(0)}$  this value should be more than the largest intercept of the incremental cost characteristic of the various generators.
- Step 2: Compute the individual generations i.e. calculate  $P_{gi}$  for  $i = 1, 2, \dots, N$ .
- Step 3: First iteration, check the equality constraint i.e. tolerance,  $\epsilon = P_L - \sum P_{gi}$  for  $i = 1, 2, \dots, N$ . If not satisfied set a new value of  $\lambda$  and repeat the above steps ( $\epsilon$  is the tolerance,  $P_L$  is the total load power, and the indicated sum is the sum of all the individual load powers,  $N$  is the number of loads)
- Step 4: Check the convergence. If  $\Delta P_{gi}$  in step 3 are below the user-defined tolerance, the solution converges. Otherwise, go to step 2.

### *Linear Programming (LP) Method:*

Linear programming is a mathematical procedure based on linear algebra and the solution of linear simultaneous equations. The method maximizes or minimizes the objective, which is dependent on a finite number of variables. These variables may or may not be independent of each other, and in most cases are subject to certain conditions referred to as constraints. LP method finds a point in the optimization surface where this function has the smallest (or largest) value. Linear programs are problems that can be expressed in canonical form:

$$\text{Minimize} \quad C^T X \quad (2.1)$$

$$\text{subject to} \quad A_{eq} X = b_{eq} \quad (2.2)$$

$$AX \leq b \quad (2.3)$$

where

$X$	the vector of variables to be determined
$C$	the cost coefficients of the decision variables to be minimized
$A, A_{eq}$	an $(m \times n)$ constraint matrix
$B, b_{eq}$	an $m$ -dimensional column vector of right hand side constraints.

The method for solving economic dispatch by LP uses an iterative technique to obtain the optimal solution [33]:

- Step 1: Select the set of initial control variables.

- Step 2: Solve the power flow problem to obtain a feasible solution that satisfies the power balance equality constraint.
- Step 3: Linearize the objective function and inequality constraints around the power flow solution and formulate the LP problem.
- Step 4: Solve the LP problem and obtain optimal incremental control variables  $\Delta P_{gi}$ .
- Step 5: Update and form the new control variables  $P_{gi\ new}=P_{gi\ old} + \Delta P_{gi}$ . Note that  $P_{gi}$  are generator power levels.
- Step 6: Obtain the power flow solution with updated control variables.
- Step 7: Check the convergence. If  $\Delta P_{gi}$  in step 4 are below the user-defined tolerance, the solution converges. Otherwise, go to step 3.

*Quadratic Programming Method:*

Quadratic programming is a special form of nonlinear programming whose objective function is quadratic and constraints are linear. The most often used objective function in power system optimization is the generator cost function, which generally is a quadratic. The linear programming method can also be used in the quadratic programming model of economic dispatch (see Appendix B).

*Dynamic Programming (DP) Method:*

The basic idea of the theory of DP is that of viewing an optimal policy as one determining the decision required at each time in terms of the current state of the system. This absolute problem is normally solved by discretization of the entire dispatch period into a number of small time intervals over which the load is assumed to be constant and the system is considered to be in steady-state [37]. There are two DP algorithms. They are forward and backward dynamic programming. The start-up cost of a unit is a function of the time. The forward approach is often adopted since the initial condition is known. The backward DP algorithm is appropriate when the terminal condition is known. Suppose a system has  $n$  units. There are  $2^n - 1$  combinations. The recursive algorithm is used to compute the minimum cost in hour  $k$  with state  $n$  is [31],

$$F_{\text{cost}}(k,n) = \min[P_{\text{cost}}(k,n) + S_{\text{cost}}(k-1, m:k,n) + F_{\text{cost}}(k-1,m)] \quad (2.4)$$

where

$F_{\text{cost}}(k,n)$	The total <i>cost</i> from initial state to hour $k$ state $n$ . The term ‘state’ here refers to an operating condition.
$\text{Min}$	Denotes the minimum
$S_{\text{cost}}(k-1, m:k,n)$	The transition cost from state $(k-1, m)$ to state $(k,n)$
$M$	The set of states at hour $t-1$
$P_{\text{cost}}(k,n)$	The production cost for state $(k,n)$

The examples shown in this report use the quadratic programming method to a minimization problem. The QP method is a very powerful solution algorithm because of their rapid conver-

gence near the solution. This property is especially useful for the power system application because an initial guess near the solution is easily attained.

## 2.4 Formulation of the Optimal Bulk Storage Problem

A general minimization problem can be written in the following form:

$$\text{Minimize} \quad f(X) \quad (\text{the objective function}) \quad (2.5)$$

$$\text{subject to:} \quad h_i(X) = 0 \quad i = 1, 2, \dots, m \quad (\text{equality constraints}) \quad (2.6)$$

$$g_j(X) \leq 0 \quad j = 1, 2, \dots, n \quad (\text{inequality constraints}) \quad (2.7)$$

In these expressions,  $f$  is the objective function and  $X$  are the system states;  $h_i$  are the equality constraints and  $g_j$  are the inequality constraints. There are  $m$  equality constraints and  $n$  inequality constraints and the number of variables is equal to the dimension of the vector  $X$ . The system described has constraints that capture line ratings, generator ratings, bus power conservation and the Kirchhoff laws.

The mathematical model of real power economic dispatch with security constraints can be written as follows:

$$\text{Minimize} \quad f(X) = c_i P_{gi} + P_{gi}^T Q P_{gi} \quad i \in n_g \quad (2.8)$$

$$\text{subject to} \quad A_{eq} X = b_{eq} \quad (2.9)$$

$$AX \leq B \quad (2.10)$$

$$\text{such that} \quad \sum P_{gi} = \sum P_{Lk} \quad i \in n_g; k \in n_l$$

$$P_{ij \min} \leq P_{ij} \leq P_{ij \max} \quad ij \in n_t$$

$$0 \leq P_{gi} \leq P_{gi \max} \quad i \in n_g$$

$$P_{sq \min} \leq P_s \leq P_{sq \max} \quad q \in n_s$$

$$0 \leq E_s \leq E_{sq \max} \quad q \in n_s$$

where

$f(X)$	A cost function of design variables $X$
$A, A_{eq}$	Coefficients of inequality constraints, and equality ('eq') constraints
$B, b_{eq}$	A vector of inequality constraints, and equality ('eq') constraints
$A$	Coefficients of inequality constraints
$P_s$	Power in the distributed storage devices
$E_s$	Energy stored

$P_L$	The real power load in MW, $P_{Lk}$ denotes load power at bus $k$
$P_{ij}$	The power flow of transmission line $ij$ in MW
$P_{ij \min}, P_{ij \max}$	The minimal and maximal power limits of transmission line $ij$ in MW
$P_{gi}$	The real power output at generator bus $i$ in MW
$P_{gi \min}, P_{gi \max}$	The minimal and maximal real power output at generator $i$ in MW
$P_{sq \min}, P_{sq \max}$	The minimal and maximal storage charge and discharge at storage device $i$ in MW
$E_{sq \max}$	The maximal energy storage at storage $i$ in MWh
$c_i$	The cost of the generator $i$
$n_l$	The number of transmission lines
$n_g$	The number of generators
$n_s$	The number of large scale storage system
$Q$	$(n \times n)$ symmetric matrix describing the coefficients of quadratic terms
$X$	The $n$ -dimensional column vector of decision variables (note: $X$ contains: (1) control variables such as generation and storage power levels as well as (2) problem unknowns such as line flows and bus voltage phase angles)

## 2.5 Problem of Dimensionality, Equality, and Inequality Constraints

Generally, the number of unknowns  $X$  increases like  $(n_b + n_s + n_l + n_b - 1)h$ . The number of equality constraints increases like  $(n_b + n_b)h + 1$ . The number of inequality constraints increases like  $(2n_l + n_s)h + n_s(2h - 2)$  where,

$n_b$	The number of buses in the system.
$h$	The number of interval of hours of a day.

Equality constraints are an important part of the formulation. The equality constraints ( $A_{eq}$ ) of the optimal power flow (OPF) reflect the physics of the power system. The following equality constraints are enforced during QP.

- Conservation of power at each bus: The physics of the power system are enforced through the power flow equations which require that the net injection of real power at each bus sum to zero. The corresponding generation limits of individual generator are accommodated in the upper bound (UB) and lower bound (LB) of the programming.
- Line load versus phase angle at each bus: Assumption is the voltage at the nodes is 1 p.u.
- $P_{ij} = (\delta_i - \delta_j) / (x_{ij})$ . The notation  $P_{ij}$  is the line load power where the line terminal voltage phases are  $\delta_i$  and  $\delta_j$ , and the reactance of the line is  $x_{ij}$ .
- Charge /discharge schedule for all the storage elements should sum up to zero.

- $\sum P_{si}=0 \quad i \in n_s$  where  $P_{si}$  and  $n_s$  refer to the power to the  $i$ th storage element and the total number of storage elements respectively.

In addition to the equality constraints, there are inequality constraints as in (2.10) (using coefficients  $A$ ) in the model. The inequality constraints in the OPF reflect the limits on physical devices in the power system as well as the limits created to ensure system security. Physical devices that require enforcement of limits are:

- Line loads
- Conservation of energy (storage)
- Power to storage elements.

### 3 A Small Illustrative Example

#### 3.1 Objectives of a Small Illustrative Example

In this section, a simple three bus power system test bed is used to demonstrate the idea of optimal scheduling of energy storage.

- The basic formulation of the problem is given in this section. It is assumed that the given data are:
  - Loads
  - Wind power
  - LMPs at generation buses
- And the constraints are:
  - Line loads
  - The energy and power ratings of the storage
- And the Kirchhoff's laws are:
  - Conservation of power at each bus
  - Line load versus phase angle at each bus

#### 3.2 Description of the Test Bed

The test bed proposed as a small example is denominated as test bed #1. A 3-bus system was considered of how storage can improve integration of renewable resources was developed and used for preliminary test of calculation technique and proof of concept. The 3-bus system is shown in Figure 3.1. The system data and line data are shown in Table 3.1. The LMP (locational marginal price, incremental cost of energy delivered at a bus) of the day is shown in Figure 3.2. Load and renewable energy generation (wind) at the bus B and C are shown in Figure 3.3 and 3.4 respectively. A 100 MVA base is chosen for calculations.

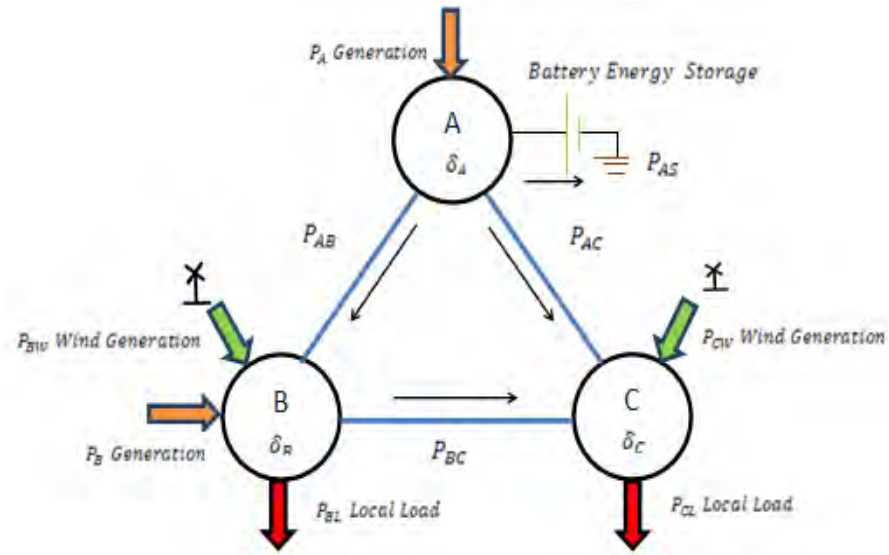


Figure 3.1: Three Bus Test Bed: Test Bed # 1

Table 3.1: Transmission Line Ratings

Transmission line		Reactance ( $\Omega$ )	Thermal rating (MW)
From	To		
A	B	0.01	190
A	C	0.02	100
B	C	0.03	200

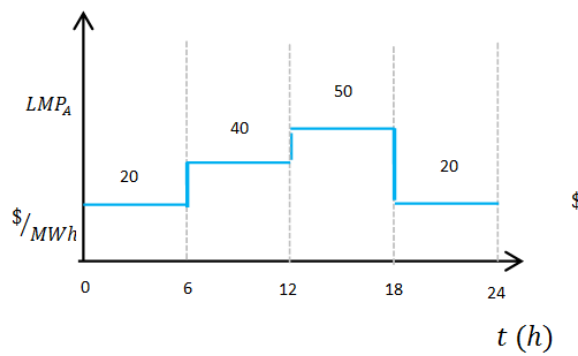


Figure 3.2: LMP at Bus A for Test Bed #1 (\$/MWh)

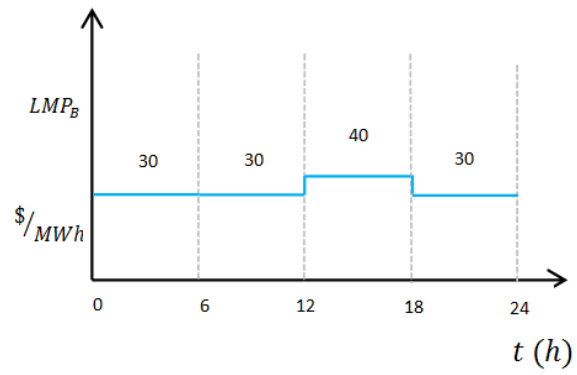


Figure 3.3: LMP at Bus B for Test Bed #1 (\$/MWh)

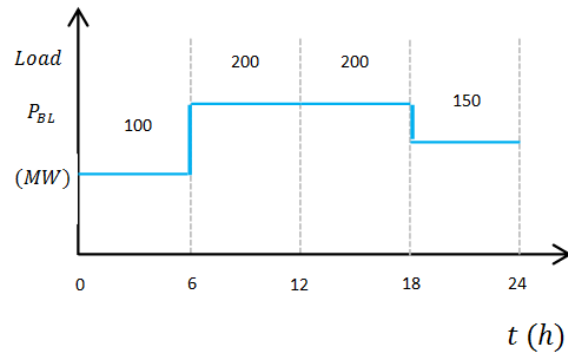


Figure 3.4: Load at Bus B for Test Bed #1 (MW)

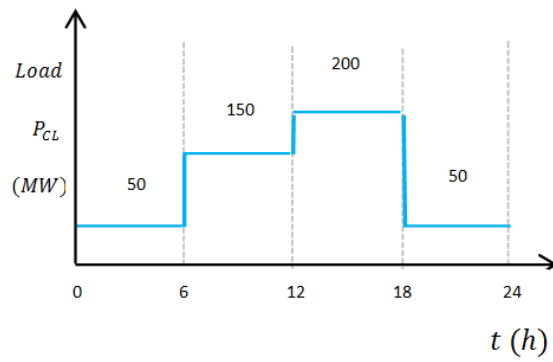


Figure 3.5: Load at Bus C for Test Bed #1 (MW)

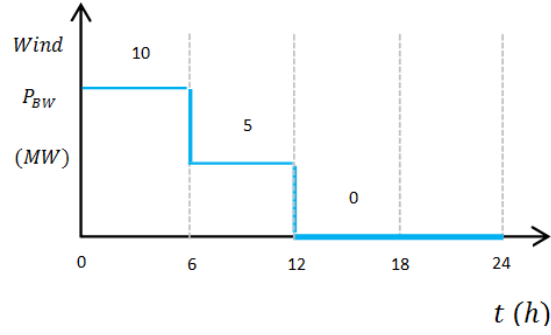


Figure 3.6: Wind Generation at Bus B for Test Bed # 1 (MW)

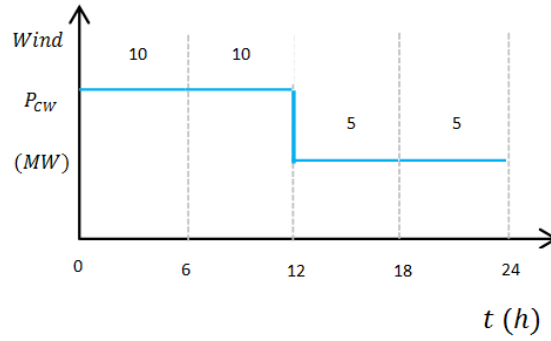


Figure 3.7: Wind Generation at Bus C for Test Bed # 1 (MW)

### 3.3 Formulation of the Problem

The main objective is to maximize beneficial impacts of storage, mainly reflected as minimizing generation dispatch cost. A storage facility is considered to be present at Bus A in Figure 3.1. A QP based algorithm is carried out to optimize generation and storage scheduling with maximal use of renewable generation. For the tests reported, all the wind generation is used. The unknowns, optimum generation schedule and storage (store/discharge) schedule are calculated, minimizing the purchase price of energy for one day. The information used and constraints considered are:

#### *Given Information*

- Loads
- Wind power
- LMPs at generation buses

#### *Constraints*

- Line loads
- Energy and power of storage
- Conservation of power at each bus
- Voltage phase angle at each bus

Bus A is assumed to be the reference bus. The voltage phase angle at each bus constrained to lie between  $-30^\circ \leq \delta \leq 30^\circ$ . Three cases are studied calculating the economic dispatch of generation at bus A ( $P_A$ ) and bus B ( $P_B$ ) for minimum cost with:

- No storage and no constraints line ratings.
- Constraint on line ratings and one storage unit at bus A.
- Constraint on line ratings and two storage units at bus A and B.

All the three cases are studied for a one day time horizon broken into 4 intervals each have a span of 6 hours.

### 3.4 Study of Case 1 (base case)

In this case study, the system is initially assumed without energy storage and without constraints on line ratings are considered. After executing the economic dispatch considering the limits on generation, the output of generating units  $P_A$  and  $P_B$  computed is listed in Table 3.2. The minimum generation cost of the system without energy storage using QP in Matlab is 181,520 dollars per day. At interval 1 and 4, the load is being supplied by the cheap unit A. At intervals 2 and 3, the cheap unit B has to supply power.

Table 3.2: Case 1 Study Results, Test Bed #1

Operational data \ Interval (each 6 hours)		1	2	3	4
Generation (MW)	Bus A	130	0	0	195
	Bus B	0	335	395	0
Line flows (MW)	Bus A to Bus B	88.33	-70	-97.5	140
	Bus A to Bus C	41.67	70	97.5	55
	Bus B to Bus C	-1.67	70	97.5	-100
Voltage angle (radians)	Bus B	-0.0088	0.007	0.0097	-0.014
	Bus C	-0.0083	-0.014	-0.0195	-0.011

### 3.5 Study of Case 2

In this case study, with the energy storage at bus A having rating of 20 MW and energy capacity of 120 MWh. Economic dispatch of generations  $P_A$  and  $P_B$  is calculated for minimum cost using

QP in Matlab with storage and considering the limits on generations and thermal ratings of the transmission lines. The dispatch results are shown in Table 3.3. At the first low-load hour, the storage is charged by 20 MW which is from the cheap unit A. At intervals 2 and 3, the output from the storage is discharged mitigating the congestion on the line from bus A to bus C. In addition the output from the battery is replacing generation from expensive unit A. The total generation dispatch cost is 179,380 dollars per day which is less than that in Case 1. This saving could be much higher in the large system.

From Case 2 it is observed, the energy is stored during the minimum cost of generation and discharged when the cost of generation is high. At the end of the day the storage element is completely discharged. In other words, the battery (storage element) charges during the first interval and discharges during the second and third interval to minimize the cost of generation.

Table 3.3: Case 2 Study Results, Test Bed #1

Operational data \ Interval (each 6 hours)		1	2	3	4
Generation (MW)	Bus A	150	0	0	195
	Bus B	0	330	380	0
Line flows (MW)	Bus A to Bus B	88.33	-65.83	-85	140
	Bus A to Bus C	41.67	70.83	100	55
	Bus B to Bus C	-1.67	69.17	95	-100
Voltage angle (radians)	Bus B	-0.0088	0.0066	0.0085	-0.014
	Bus C	-0.0083	-0.0142	-0.02	-0.011
Storage (MW)	Bus A	20	-5	-15	0

### 3.6 Study of Case 3

In case 3 the economic dispatch is solved with limits on line ratings and two storage units at bus A and bus B having combined rating of 20 MW and energy capacity of 120 MWh (10 MW and

60 MWh each). The results are shown in Table 3.4. The cost of economic dispatch of generation per day calculated is \$ 179,080.

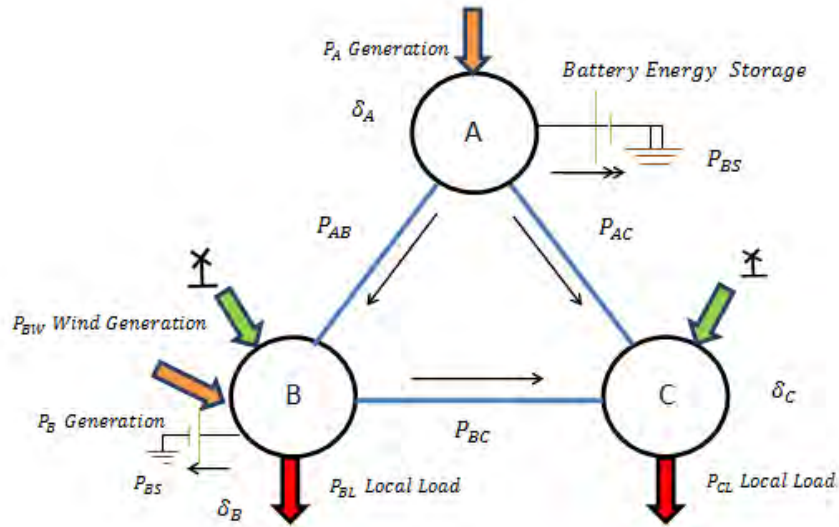


Figure 3.8: Three Bus Test Bed: Test Bed # 1 with Two Storage Units

Table 3.4: Case 3 Study Results, Test Bed #1

Interval (each 6 hours)		1	2	3	4
Operational data	Bus A	150	0	0	195
	Bus B	0	335	375	0
Line flows (MW)	Bus A to Bus B	96.67	-70	-89.7	140
	Bus A to Bus C	43.33	70	99.17	55
	Bus B to Bus C	-3.33	70	95.83	-100
Voltage angle (radians)	Bus B	-0.0097	0.0070	0.0089	-0.014
	Bus C	-0.0087	-0.014	-0.0198	-0.011
Storage (MW)	Bus A	10	0	-10	0
	Bus B	10	0	-10	0

Inference drawn from this case is that the spreading the storage unit reduces the generation production cost. This is because of line rating constraints limiting concentrated energy storage. At the first low-load hour, the storage at bus A and bus B is charged by 10 MW each which is from the cheap unit A. At interval 4, the output from the storage is discharged mitigating the congestion on the line. In addition the output from the battery is replacing generation from expensive unit A and B. The total generation dispatch cost is 179,080 dollars per day which is less than that in case 2.

### **3.7 Impact of Storage: Observations from Test Bed # 1**

The implementation and use of renewable energy may not always be possible due to constraints of transmission and component ratings, when storage is added; these constraints are partially relaxed, this can be observed from cases mentioned above.

High prices are one of the largest barriers facing renewables. During peak demand on the electric grid, electric companies pay more for electricity. Often additional power needs at this time are supplied by natural gas or oil, which has higher fuel costs. The opposite is true during times of low demand, when electricity costs are lower, during this time the energy can be stored and discharged when the demand and fuel cost is high thereby reducing the overall cost of generation per day; this can be observed from the above discussed cases 1, 2 and 3.

The grid needs a consistent, stable supply of energy that can be adjusted during times of peak demand. Black out occurs when supply does not keep up with demand. High demand on the power grid often requires power plants to be fired up to cover short-term electricity demand at a higher price. Large-scale use of renewable energy will require that it can adapt to variable levels of demand on the power grid. Energy storage combined with these renewable energy resources may firm up the power output.

The cost of delivery and generation (fuel) can be minimized by increasing the capacity of storage elements and fuel cost can be further optimized by disbursing the storage unit across the power system. This concept is illustrated in Figure 3.9. Here, corresponding to the total power (MW, across two storage units), six hours of energy storage (MWh) is considered.

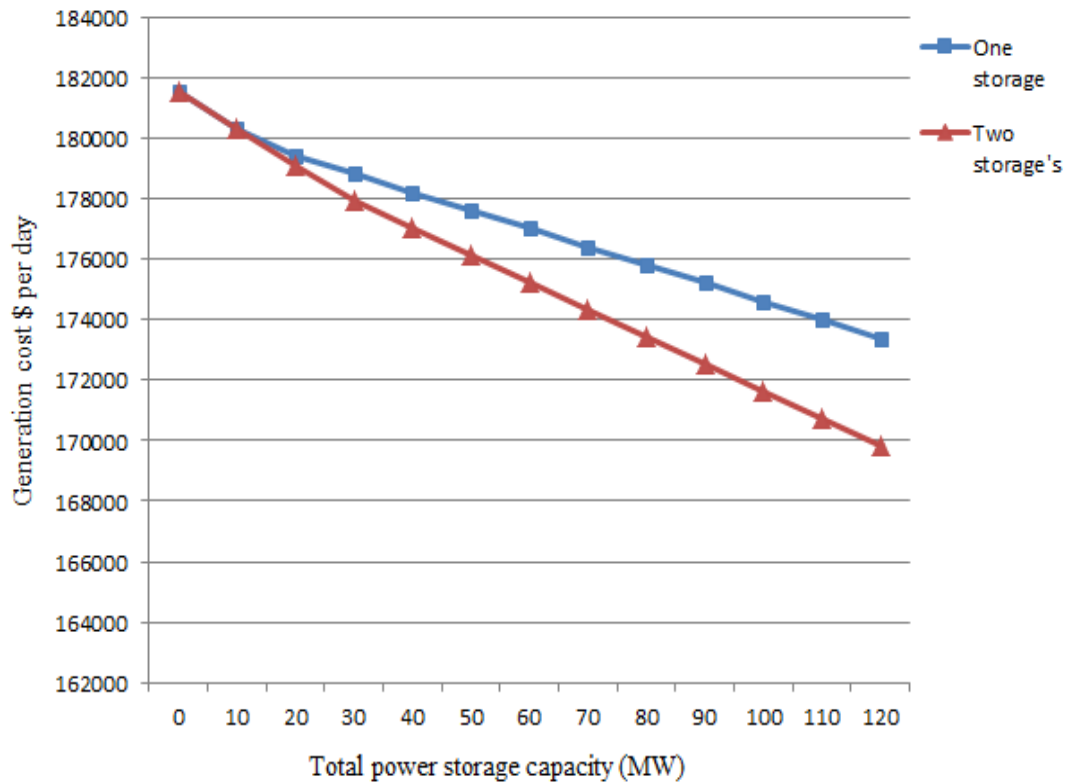


Figure 3.9: Fuel Cost Comparison with One and Two Storage Unit

Energy storage gives additional degrees of freedom in the optimal dispatch problem, thereby potentially allowing the additional use of renewable energy. The simple example of test bed #1 shown has wind penetration in the range of 5 % (Wind peak power / Peak demand power). Much more significant improvements in operating strategies occur at higher storage capacities. This is shown in the Table 3.5.

Table 3.5: Cost Comparison with One and Two Storage Units

Total Storage Capacity (MW)	Fuel cost per day to serve the load (\$/day)	
	One storage unit	Two storage unit
0	181,520	181,520
10	180,320	180,320
20	179,380	179,080
30	178,810	177,910
40	178,210	177,010
50	177,610	176,110
60	177,010	175,210
70	176,410	174,310
80	175,810	173,410
90	175,210	172,510
100	174,610	171,610
110	174,010	170,710
120	173,400	169,810

## 4 Illustrative Example using the State of Arizona as a Test Bed

### 4.1 Description of the Test Bed: State of Arizona

The previous chapter provided an introduction test system to demonstrate the idea of optimal scheduling of energy storage. This chapter looks at more realistic and well-studied example. In this section, the effect of energy storage on the minimization of the objective function using the State of Arizona as a test bed with different storage capacities and wind generation is studied. This benchmark system which represents a portion of the Western Electricity Coordinating Council (WECC) as of April 2009 does not include storage. Therefore, while the use of its network topology, generation bounds as well as transmission line ratings bounds, appropriate values for the storage parameters are added in the profile. The load, wind power and LMPs (assumed, at generation bus) at each bus are also given. The heavy summer case of 2009 is considered (actual load and generation data).

In the test case, an objective function is minimized. Again, this corresponds to minimum operating cost. The constraints and formulation of the problem is the same as provided in the previous chapter. A QP based algorithm is carried out to optimize generation and storage scheduling with maximal use of renewable generation. The optimum generation schedule, storage (store / discharge) schedule, and line flows are control variables, and these quantities are calculated.

A one day time horizon broken into 3 intervals each having a span of 8 hours is studied. The objective of the constrained economic dispatch is to schedule the generation outputs economically including storage over one day. The simplifications made are:

- Reactive power flows are not modeled or considered
- A simple linear relationship is assumed between bus voltage phase angle and line active power flows
- Transmission line losses are neglected.

The portion of the WECC under study is mainly the state of Arizona having the description profile indicated in Table 4.1.

Table 4.1: Description Profile: State of Arizona Power System

Number of buses $n_b$	792	Number of generators	182
Number of lines $n_l$	1079	Number of wind farms *	2

\*Assumed

### 4.2 Case 4

Case 4 is a ‘base case’ study for this test bed. In case 4, no storage units are scheduled and two wind farms are located at Flagstaff and Springerville. Economic dispatch of generations is calculated for the minimum cost using QP in the Matlab optimization toolbox. According to the constraints considered in this work, only active power constraints are considered. Therefore, the re-

spective maximum and minimum operating long term thermal ratings of the transmission lines, generation limits and voltage phase angle limits at each bus is accommodated in the upper (UB) and lower bound (LB) of the program.

Table 4.2 shows the operational data for case 4. The cost of economic dispatch of generation per day calculated is 12.049 million dollars per day (M\$/day).

Table 4.2: Case 4 Study Results, Arizona Test Bed

Wind		Storage			
$P_1$ (MW)	$P_2$ (MW)	$P_1$ (MW)	$W_1$ (MWh)	$P_2$ (MW)	$W_2$ (MWh)
400	300	0	0	0	0

### 4.3 Case 5 – Storage Added

In case 5, the wind power capacity and storage capacity is increased. The case is divided under low, medium and high depending on wind power penetration and storage capacity of the power system. Two wind farms are considered located at Flagstaff and Springerville along with two storage units both at Navajo. The storage units have two ratings, one relating to the power electronic converters (this is the power rating of the unit), and the other as the ultimate energy storage capability (this is the energy rating, e.g., in MWh). For case 5, it is assumed that the power rating (MW) times 6 hours is the energy (MWh) rating.

Table 4.3 tabulates the description of the wind power and storage as well as the solution cost. It is observed that with increasing energy storage, the operating cost reduces. Note that electrical energy is stored during times when generation cost is low and when production exceeds consumption. The stored energy is discharged during the period when the production cost from conventional generating plants is high.

Table 4.3: Case 5 Study Results, Arizona Test Bed

Scenario Operational data		Low 1	Medium 2	High 3
Wind	$P_1$ (MW)	400	600	800
	$P_2$ (MW)	300	500	600
Storage	$P_1$ (MW)	50	100	300
	$P_2$ (MW)	50	150	250
	$W_1$ (MWh)	300	600	1800
	$W_2$ (MWh)	300	900	1500
Cost	QP (Million dollars / day)	11.772	11.453	11.066

#### 4.4 Case 6 – Increase in the Number of Storage Units

In case 6, three scenarios are studied. The number of energy storage units is increased from 2 to 4 to 6. In each scenario, the total power and energy stored is the same, i.e. total power capacity =  $P_{ST} = 700$  MW and total energy  $W_{ST} = 4200$  MWh. These levels are shared among the storage units. The wind power is the same as assumed in case 4. Table 4.4 tabulates the number of storage units and economic dispatch cost in millions of dollars per day obtained using QP for the respective scenarios.

The results indicate that the cost of delivery and generation (fuel) can be minimized by increasing the capacity of storage elements. The fuel cost can be further optimized by selecting optimum locations for the two storage units.

Table 4.4: Case 6 Study Results, Arizona Test Bed

Operational Data \ Scenario		1	2	3
Storage	Number of Units	2	4	6
	$P$ (MW) each unit	350	175	116.67
	$W$ (MW) each unit	2100	1050	700
Cost	QP (M\$/day)	11.616	11.563	11.484

#### 4.5 Case 7 – Large Scale Implementation

This case resembles more of a practical scenario. In other words, a large number of wind machines are accommodated. Note that the 2025 renewable portfolio standard for Arizona is 15%; a higher percentage of wind generation is accommodated in case 7. In case 7, 15% of the total load is derived from wind generation. Also, ten energy storage units are represented having a total capacity of 700 MW with 6 hours of energy storage (i.e., the total energy rating is 6 times 700 or 4200 MWh). Table 4.5 shows the system description.

Table 4.5: Case 7 System Description

Number of buses $n_b$	792	Number of lines $n_l$	1079
-----------------------	-----	-----------------------	------

The wind availability considered throughout the day for case 7, and this is shown in Fig. 4.1. The wind turbines are assumed to generate power at name plate rated capacity. The cost of economic dispatch of generation computed is 10.863 million dollars per day (M\$/day).

The inference made from this case is the optimal location of energy storage units is at the generation buses. This observation is made for storage units such as batteries; however, obviously, the location of pumped-hydro storage is dictated by geography and topography. Storage units can be placed next to wind farms to produce a consistent flow of power. Locations like Bullhead City have a high potential of wind production [5]. Siting wind generation at such locations may be dependent on ratings of the adjacent transmission facilities. Storage unit placed at these locations can store excess wind energy and discharge during later periods. Therefore, use of storage can reduce the cost of upgrade of the electricity link and defer the expansion of the transmission network.

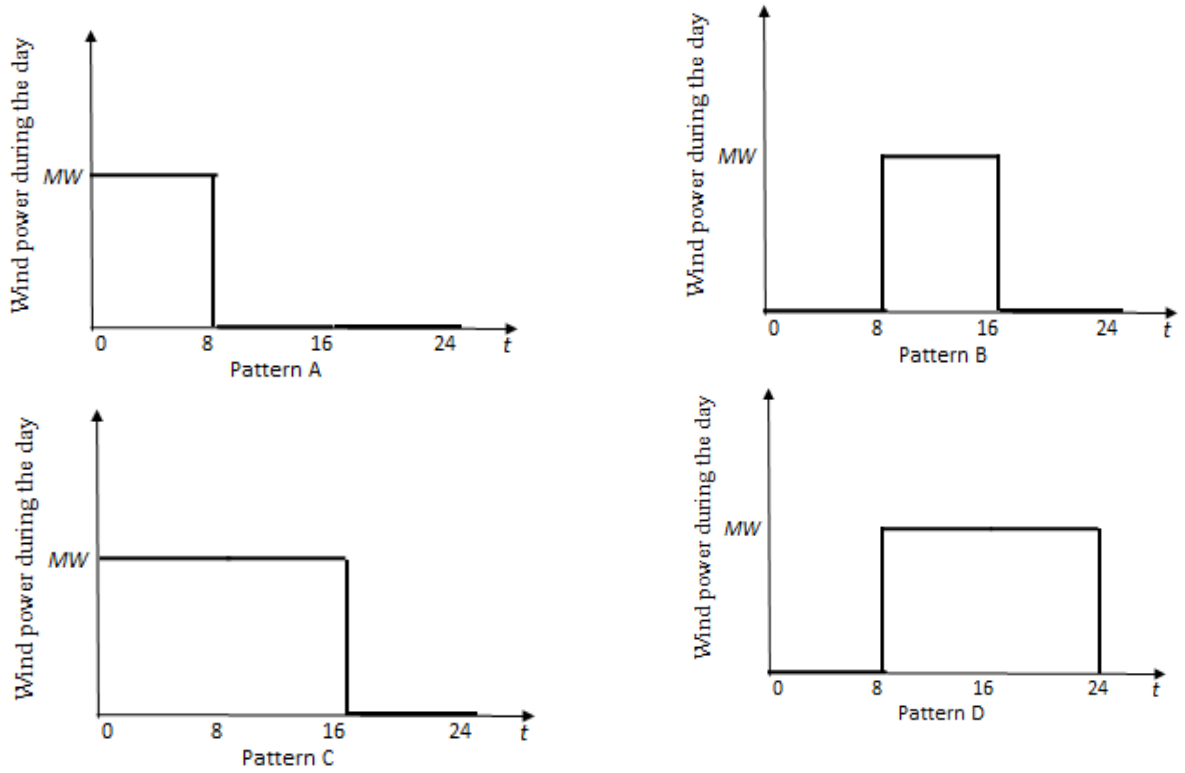


Figure 4.1: Wind Generation Patterns for Case 7,  $t$  is in hours

#### 4.6 Calculation of Payback Period

The payback period in capital budgeting refers to the period of time required to return an investment, to repay the sum of the original investment. An approximate payback period is calculated for the above discussed cases 5, 6 and 7. Mathematically, the length of time required to recover the cost of an investment is calculated as:

$$\text{Cost of Project} = N_{\text{years}} * \text{Annual Cash Inflows}$$

There are two main problems with the payback period method:

- It ignores any benefits that occur after the payback period and, therefore, does not measure profitability.
- It ignores the time value of money.

Annual cash inflows is the savings obtained from cases 5, 6, and 7 when compared with case 4. Following assumption is made: the energy storage system is a lead-acid battery and wind turbine is a doubly fed induction generator (DFIG), type 3 is assumed. The calculation is,

$$\text{Cost of lead-acid battery, } C_B = 0.17 \text{ \$/Wh.}$$

$$\text{Cost of wind turbine, } C_W = 1.2 \text{ to } 2.6 \text{ million \$/MW of name plate capacity.}$$

Cost of electronics (converter),  $C_E = \$ 250$  per kW.

Let,  $N_D$  = Number of days for repay of the original investment. Then the value of  $N_D$  is found for the several cases:

Case 5: The wind power capacity and storage capacity is increased. The case is divided under low, medium and high depending on wind power penetration and storage capacity of the power system. The case 5 test bed has two energy storage systems and two wind turbines with the electronic converters.

*Low case scenario:* Total cost of battery storage is,

$$\begin{aligned} C_{BT} &= \text{Number of units} \times \text{Storage capacity (Wh)} \times \text{Cost of lead acid battery (\$/Wh)} \\ &= 2 \times 300 \times 10^6 \times 0.17 = \$ 102 \text{ million.} \end{aligned}$$

Cost of electronics for the two storage units,

$$C_{ET} = 2 \times 250 \times 50000 = \$ 25 \text{ million.}$$

Cost of wind turbines,

$$C_{WT} = (400 + 300) \times 1.2 = \$ 840 \text{ million.}$$

Total initial investment is,

$$C_i = C_{BT} + C_{ET} + C_{WT} = \$ 967 \text{ million.}$$

Saving's with respect to case 4,

$$S = \$ 0.277 \text{ million /day.}$$

Thus, an approximate payback period is,  $SN_D = C_i$

$$\Rightarrow N_D = C_i / S = 3490.97 \text{ days} = 9.56 \text{ years}$$

Note: The above calculation does not take account of maintenance and battery replacement with inflation rate for the total system. By the same token, the approximate payback period in years for medium case and high case scenario is 7.52 years and 6.63 years respectively.

Case 6: The number of energy storage units is increased from 2 to 4 to 6. In each scenario, the total power and energy stored is kept the same, i.e. total power capacity =  $P_{ST} = 700$  MW and total energy  $W_{ST} = 4200$  MWh. The payback period for 2, 4 and 6 storage units is found to be 10.93 years, 9.74 years and 8.38 years.

Case 7: This case resembles more of a practical scenario. In other words, a large number of wind machines are accommodated. Here, 15% (4400 MW) of the total load is derived from wind generation. Also, ten energy storage units are represented having a total capacity of 700 MW with 6

hours of energy storage (i.e., the total energy rating is 6 times 700 or 4200 MWh). The payback period is 14.25 years. Figure 4.2 represents the payback period of each case.

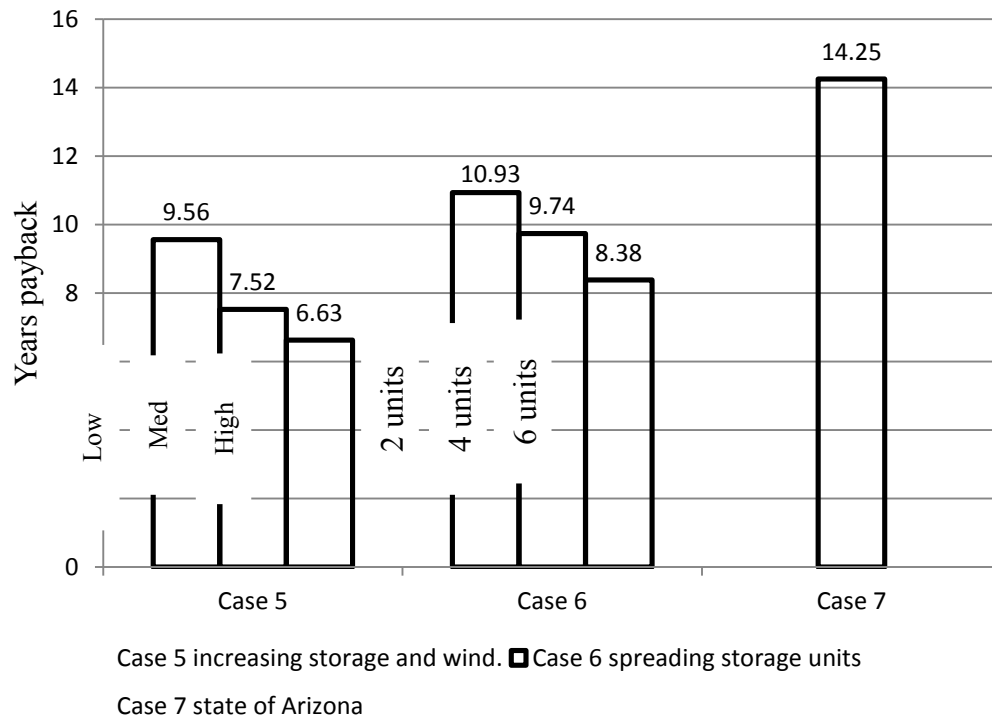


Figure 4.2: Payback Period for Cases 5, 6, 7

## 5 Conclusions and Future Work

### 5.1 Conclusions and Main Contributions

It has been shown in this report that energy storage devices not only facilitate the large scale integration of renewable energy resources into the grid, but also assist in the economic dispatch of generation. In this research, an equivalent section of the WECC system, namely for Arizona, summer peak 2009 was considered. The following main conclusions can be made from the results presented in the previous chapters:

- In the base case (Case 4) without energy storage, the minimum generation dispatch cost of the system is 12.049 million dollars per day and the economic dispatch with the energy storage in system for a comparable case.
- (Case 5) the total generation dispatch cost is 11.772 million dollars per day. The savings increases with an increase in storage capacity. Quantitatively, for the cited case an increase in savings of 0.277 to 0.596 million dollars per day is attained for addition of 900 MWh.
- The test bed state of Arizona with accommodation of 15% (4400 MW) of the total load being served from wind production and 4200 MWh of energy storage the total generation dispatch cost is 10.863 million dollars per day. This figure is observed for the summer 2009 peak period.
- Large scale energy storage can be used to mitigate the overloading of the transmission lines at places where the wind energy potential is high and connection to the grid is expected. For example, in Case 7, wind generation sited at Bullhead City AZ was studied and energy storage at this site is allowed. The addition of 250 MW of wind despite the 140 MW adjacent existing transmission. In this example energy storage is rated at 720 MWh, with a converter rating of 120 MW.
- Defer the upgrade of the transmission systems when renewable resources are added. This results as the usage of energy storage can reduce the power transfer through adjacent lines during peak load periods. Also, the use of storage can decrease the congestion cost as energy storage systems can shift the load from the peak to off-peak load periods. This advantage was illustrated in Case 7 in which up to 116 MW in the period 0800 to 1600 hours is shifted to the period 1600 to 0000 hours. This 116 MW shift was in a line of rating of 398 MVA.
- Disbursing the storage units across the state of Arizona reduces the generation production cost. This is the case because of line rating constraints limiting concentrated energy storage. Case 6 illustrates this point through the comparison of the utilization of 2, 4, and 6 storage units.

Secondary contributions are:

- Quadratic programming has been illustrated as an optimization method for scheduling energy storage.
- Examples have been shown with an actual power system for the state of Arizona.
- Sample code in Matlab has been developed (see Appendix A).

## 5.2 Future Work

In the present work, the objective cost function is based on the power generated from power plants. The thesis mainly focuses on economic dispatch using the state of Arizona as a test bed. This work can be extended by modeling the system external to Arizona and implementing the following:

- Address the dimensionality problem.
- Include the impact of energy storage on the reduction of spinning reserve.
- Model the transmission and storage devices losses occurring in the power system.
- Model the storage technologies characteristics to better represent each of them.
- Include a dynamic response study to study the system stability. Also, check voltage stability in the steady state and in the dynamic case.
- Study the power quality issues in the grid due to the appearance of high levels of DC/AC and AC/DC conversion.
- Perform reactive power studies.

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## APPENDIX A MATLAB CODE

### A.1 Matlab Code Used in this Project

A sampling of the key Matlab scripts used in this work are provided in this appendix.

```
clear;
clc;
tic;

%% Main system data
nb=792; % Number of bus
nl=1079;% Number of lines
h=3; % Intervals of each 24/h hours
ng=nb; % Storage and generation assumed at all buses. Zero storage and
nst=nb; % generation is accommodated in lower and upper bounds
Base=100;

%% Read data
B = xlsread('AZ.xlsx','Line Records','b1:b10000'); % From bus
D = xlsread('AZ.xlsx','Line Records','d1:d10000'); % To bus
J = xlsread('AZ.xlsx','Line Records','k1:k10000'); % Reactance
U=xlsread('AZ.xlsx','Bus Records','n2:n10000'); % Load
V=xlsread('AZ.xlsx','Bus Records','o2:o10000'); % Wind
Y=xlsread('AZ.xlsx','Line Records','m1:m10000'); % Line ratings
S=xlsread('AZ.xlsx','Storage','q1:q10000'); % Energy stored
P=xlsread('AZ.xlsx','Storage','p1:p10000'); % Power Stored
G=xlsread('AZ.xlsx','Storage','r2:r10000'); % Generation ratings

%% Cost function to be minimized
c=zeros(1,ng*h+nl*h+nst*h+(nb-1)*h);
H=zeros(ng*h+nl*h+nst*h+(nb-1)*h,ng*h+nl*h+nst*h+(nb-1)*h);
Q=xlsread('AZ.xlsx','Storage','s2:s10000');
C=xlsread('AZ.xlsx','Bus Records','t2:t10000');
n=0;k=0;
for ro=1:h:ng*h;
    q=Q(ro+k-n)*Base;
    for added=1:h;
        H(ro+added-1,ro+added-1)=2*q;
    end
    k=k+1;
    n=n+h;
end
f=C*Base;
```

```

for i=ng*h+1:ng*h+nl*h+nst*h+(nb-1)*h;
    f(i,1)=0;
end

```

%% Formation of bus dictionary

```

busdict=zeros(nb,1);
busdict(1)=10435; %
running=1;
for iline=1:nl;
    ifrom = B(iline);
    ito = D(iline);
    xline=J(iline);
    ifound=0;
    for i=1:running;
        if busdict(i)==ifrom;
            ifound=i;
            ifromn=i;
        end;
    end;
    if ifound == 0;
        running=running+1;
        busdict(running)=ifrom;
        ifromn=running;
    end;
    ifound=0;
    for i=1:running;
        if busdict(i)==ito;
            ifound=i;
            iton=i;
        end;
    end;
    if ifound == 0
        running = running+1;
        busdict(running)=ito;
        iton=running;
    end;
end;

```

```

busdict1=sort(busdict);

```

%% Formation of equality constraints

% Formation of Aeq

```

aeq=sparse(nb*h+nl*h+1,(3*nb-1+nl)*h);

```

```

inb=sparse(eye(nb*h,nb*h));
aeq(1:nb*h,1:nb*h)=inb; % Generation at buses
aeq(1:nb*h,(nb)*h+1:(nb+nst)*h)=-inb; % Storage at buses

% Stored energy at end of day should be zero
aeq(nb*h+nl*h+1,(nb)*h+1:(nb+nst)*h)=24/h;

% Line injection power
inh=sparse(eye(h,h));
for k=1:nl;
    stb=B(k);
    stbb=0;
    for look=1:nb;
        if busdict1(look)==stb;
            stbb=look;
        end;
    end;
    endb=D(k);
    endbb=0;
    for look=1:nb;
        if busdict1(look)==endb;
            endbb=look;
        end;
    end;
    aeq(h*(stbb-1)+1:h*(stbb-1)+h,nb*h+nst*h+1+h*(k-1):...
        nb*h+nst*h+h*(k-1)+h)=-inh;
    aeq(h*(endbb-1)+1:h*(endbb-1)+h,nb*h+nst*h+1+h*(k-1):...
        nb*h+nst*h+h*(k-1)+h)=inh;
end;
for k=1:nl;
    x=J(k);
    stb=B(k);
    stbb=0;
    for look=1:nb;
        if busdict1(look)==stb;
            stbb=look;
        end;
    end;
    endb=D(k);
    endbb=0;
    for look=1:nb;
        if busdict1(look)==endb;
            endbb=look;
        end;
    end;
end

```

```

end
if stbb~=1;
    aeq(nb*h+1+(k-1)*h:nb*h+(k-1)*h+h,(2*nb*h+nl*h)+1+(stbb-2)*h:...
        (2*nb*h+nl*h)+(stbb-2)*h+h)=-1/x*inh;
end
if endbb~=1;
    aeq(nb*h+1+(k-1)*h:nb*h+(k-1)*h+h,(2*nb*h+nl*h)+1+(endbb-2)*h:...
        (2*nb*h+nl*h)+(endbb-2)*h+h)=1/x*inh;
end
end
inl=sparse(eye(nl*h,nl*h));
aeq(nb*h+1:(nb+nl)*h,nb*h+nst*h+1:(nb+nst+nl)*h)=inl;% Power Flow vs. delta

%Formation of beq
beq=zeros(nb*h+nl*h+1,1);
for k=1:nb*h
    load=U(k)/Base;
    wind=V(k)/Base;
    beq(k,1)=load-wind;
end

%% Formation of inequality constraints

% Formation of A
a=sparse(nl*h^2+nst*h,ng*h+nl*h+nst*h+(nb-1)*h);
a(1:nl*h,(ng+nst)*h+1:(ng+nst+nl)*h)=inl;% Upper line rating
a(nl*h+1:(nl+nl)*h,(ng+nst)*h+1:(ng+nst+nl)*h)=-inl;% Lower line rating

% Maximum energy stored
n=0;m=0;
for i=1:nst;
    a(2*nl*h+1+m:2*nl*h+(h-1)+m,ng*h+1+n:ng*h+h-1+n)=...
        sparse(tril(ones(h-1,h-1)))*24/h;
    k=a(2*nl*h+1+m:2*nl*h+(h-1)+m,ng*h+1+n:ng*h+h-1+n);
    if h==3;
        a(2*nl*h+h+m:2*nl*h+h+m,ng*h+1+n:ng*h+h-1+n)=-k(2:h-1,1:h-1);
        j=m;
    else
        a(2*nl*h+h+m:2*nl*h+h+h-3+m,ng*h+1+n:ng*h+h-1+n)=-k(2:h-1,1:h-1);
        j=m;
    end
    n=n+h;m=2*h+m-3;
end
end

```

```

% Maximum stored power
inb=sparse(eye(nb*h,nb*h));
a(2*nl*h+2*h-2+j:2*nl*h+2*h-3+j+nst*h,(nb)*h+1:(nb+nst)*h)=inb;

% Formation of b
b=zeros(nl*h*2+nst*h,1);
k=0;
for j=1:nl;
    kline=Y(j)/Base;
    b(1+k:h+k,1)=kline;      % Line ratings
    k=k+h;
end
k=0;
for j=1:nl;
    kline=Y(j)/Base;
    b(nl*h+1+k:nl*h+h+k,1)=kline; % Line ratings
    k=k+h;
end
m=0;j=0;n=0;
for i=1:nst;
    s=S(i)/Base;      % Energy stored
    b(2*nl*h+1+m:2*nl*h+h-1+m,1)=ones(h-1,1)*s;
    if h==3;
        b(2*nl*h+h+m:2*nl*h+h+m,1)=ones(h-2,1)*0;
        j=m;
    else
        b(2*nl*h+h+m:2*nl*h+h+h-3+m,1)=ones(h-2,1)*0;
        j=m;
    end
    m=2*h+m-3;
end
k=0;
for i=1:nst;
    p=P(i)/Base;      % Maximum Power stored
    b(2*nl*h+2*h-2+j+k:2*nl*h+3*h-3+j+k,1)=ones(h,1)*p;
    k=k+h;
end

%% Construct lb and ub vectors
lb=zeros((3*nb-1+nl)*h,1);
k=0;n=0;
for ro=nb*h+1:h:2*nb*h;
    lb(ro)=0;
    es=ro-nb*h-h*k+n;

```

```

rate=P(es)/Base;
for added=1:h-1;
    lb(ro+added)=-rate;
end;
k=k+1;n=n+1;
end;
n=0;k=0;
for ro = 2*nb*h+1:h:2*nb*h+nl*h;
    y=-Y(ro-2*nb*h-h*k+n)/Base;
    for added=1:h;
        lb(ro+added-1)=y;
    end
    k=k+1;n=n+1;
end
lb(2*nb*h+nl*h+1:(3*nb-1+nl)*h,1)=-pi/6; % voltage angle within 30 degree
ub=-lb;
n=0;k=0;
for ro=1:h:nb*h;
    g=G(ro+k-n)/Base;
    for added=1:h;
        ub(ro+added-1)=g;
    end
    k=k+1;n=n+h;
end
k=0;n=0;
for ro=nb*h+1:h:2*nb*h;
    ub(ro+h-1)=0;
    es=ro-nb*h-h*k+n;
    %es=((ro-h*(n+2)-1)/h+k);
    rate=P(es)/Base;
    for added=1:h-1;
        ub(ro+added-1)=rate;
    end;
    k=k+1;n=n+1;
end;
options=optimset('Algorithm','interior-point-convex');
[X,fval]=quadprog(H,f,a,b,aeq,beq,lb,ub,[],options);

%Aeq=full(aeq);
toc;

```

## APPENDIX B The Quadratic Programming Algorithm

### B.1 Quadratic Programming

*QP model of economic dispatch*

Let the initial operating point of generator  $i$  be  $P_{geni}^0$ . Expanding the nonlinear objective function using Taylor series 33,

$$\begin{aligned} f_i(P_{geni}) &= f_i(P_{geni}^0) + \left. \frac{df_i(P_{geni})}{dP_{geni}} \right|_{P_{geni}^0} \Delta P_{geni} + \frac{1}{2} \left. \frac{d^2 f_i(P_{geni})}{dP_{geni}^2} \right|_{P_{geni}^0} \Delta P_{geni}^2 + \dots \\ &= a \Delta P_{geni}^2 + b \Delta P_{geni} + c \\ f_i(\Delta P_{geni}) &= a \Delta P_{geni}^2 + b \Delta P_{geni} \end{aligned}$$

where

$$a = \frac{1}{2} \left. \frac{d^2 f_i(P_{geni})}{dP_{geni}^2} \right|_{P_{geni}^0} \quad b = \left. \frac{df_i(P_{geni})}{dP_{geni}} \right|_{P_{geni}^0} \quad c = f_i(P_{geni}^0)$$

are constant and

$$\Delta P_{geni} = P_{geni} - P_{geni}^0$$

*Power balance equation*

Since loads are constant for the given time and using Kirchhoff's law, the following expression of power balance equation obtained,

$$\sum P_{geni} = \sum P_{Lk}$$

*Linearization of branch flow constraints*

The real power flow equation of a branch is,

$$P_{ij} = V_i^2 g_{ij} - V_i V_j (-g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij})$$

where,

- $P_{ij}$       The sending end real power on transmission branch  $ij$
- $V_i$       The node voltage magnitude of bus  $i$
- $\theta_{ij}$       The difference of bus voltage angles between the sending and receiving end of the line  $ij$
- $g_{ij}$       The conductance of transmission branch  $ij$
- $b_{ij}$       The susceptance of transmission branch  $ij$ .

Linearizing the power flow equation and considering a high voltage network, the value of  $\theta_{ij}$  is very small. In addition, assuming the magnitudes of all the bus voltages equal to 1.0 p.u. and the reactance of the line is much bigger than resistance of the line, then  $\Delta P_{ij} = -b_{ij}\Delta\theta_{ij} =$

$$\frac{\Delta\theta_i - \Delta\theta_j}{X_{ij}}$$

*Generator and storage power constraints*

$$\begin{aligned} 0 \leq P_{geni} \leq P_{geni \max} & \quad i \in NG \\ P_{sq \min} \leq P_s \leq P_{sq \max} & \quad q \in n_s \end{aligned}$$

*The QP algorithm*

Quadratic programming is the problem of finding a vector  $X$  that minimizes a quadratic function, subject to linear constraints:

$$\text{Minimize} \quad X^T Q X + C^T X \quad (\text{B.1})$$

$$\text{Such that} \quad A_{eq} X \leq b_{eq} \quad (\text{B.2})$$

$$AX \leq b \quad (\text{B.3})$$

$$lb \leq X \leq ub \quad (\text{B.4})$$

where  $C$  is an  $n$ -dimensional row vector of cost of generation,  $Q$  is an  $(n \times n)$  symmetric matrix describing the coefficients of the quadratic terms, the decision variables are denoted by the  $n$ -dimensional column vector  $X$ , and the constraints are defined by an  $(m \times n)$   $A$ ,  $A_{eq}$  matrix and an  $m$ -dimensional column vector  $b$ ,  $b_{eq}$  of right-hand-side coefficients.

When the objective function  $f(X)$  is convex for all feasible points, the problem has a unique local minimum, which is also the global minimum.

The equation (b.3) can be expressed as [33],

$$g(X) = (AX - b) \leq 0 \quad (\text{B.5})$$

The lagrangian for (B.1) and (B.5) is,

$$L(X, \mu) = CX + X^T Q X + \mu g(X)$$

where  $\mu$  is an  $m$ -dimensional row vector.

According to optimization theory, the Kuhn-Tucker (KT) conditions for a local minimum are given as,

$$\left. \begin{aligned} \frac{dL}{dX_j} &\geq 0, j = 1, \dots, n \\ C + 2X^T Q + \mu A &\geq 0 \end{aligned} \right\} \quad (B.6)$$

$$\left. \begin{aligned} \frac{dL}{d\mu_i} &\leq 0, i = 1, \dots, m \\ AX - B &\leq 0 \end{aligned} \right\} \quad (B.7)$$

$$\left. \begin{aligned} X_j \frac{dL}{dX_j} &= 0, j = 1, \dots, n \\ X^T (C^T + 2QX + A^T \mu) &\geq 0 \end{aligned} \right\} \quad (B.8)$$

$$\left. \begin{aligned} \mu_j g_i(X) &= 0, i = 1, \dots, m \\ \mu(AX - B) &\geq 0 \end{aligned} \right\} \quad (B.9)$$

$$\left. \begin{aligned} X &\geq 0 \\ \mu &\geq 0 \end{aligned} \right\} \quad (B.10)$$

Introduction of nonnegative variables  $y$  to the inequalities in equation (B.6) and nonnegative variables  $v$  to the inequalities in equation (B.7), to obtain,

$$C^T + 2QX + A^T \mu^T - y = 0 \quad (B.11)$$

$$AX - B + v = 0 \quad (B.12)$$

Then, the KT conditions are written as,

$$2QX + A^T \mu^T - y = -C^T \quad (B.13)$$

$$AX + v = B \quad (B.14)$$

$$X \geq 0, \mu \geq 0, y \geq 0, v \geq 0 \quad (B.15)$$

$$y^T X = 0, \mu v = 0 \quad (B.16)$$

The KT conditions in equations (B.13) to (B.16) have a linear form with the variables  $X$ ,  $\mu$ ,  $y$ , and  $v$ . An interior point convex algorithm can be used to solve the equations (B.13) to (B.16). The interior point convex algorithm performs the following steps 35:

- Presolve/Postsolve
- Generate initial point
- Predictor-corrector
- Multiple corrections