Setting-less Protection: Laboratory Testing

Final Project Report

Power Systems Engineering Research Center

Empowering Minds to Engineer
the Future Electric Energy System

Setting-less Protection: Laboratory Testing

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Executive Summary

Present day numerical relays use high end microprocessors for implementing multiple functions of protection, thereby providing many more options for improved protection. At the same time, the additional options have created increased complexity and increased possibilities of human errors, while the basic nature of the problem of selecting settings has remained the same. For example, a transformer protection relay may include all the typical protection functions that traditionally used with electromechanical relays, such as differential, over-current, and voltage over frequency. Since these functions work independently, even if they are implemented on the same relay, they suffer from the same limitations as the common, single relay/single function approach. Each function must be set separately to meet a specified criteria, but the settings must be coordinated with other protective devices. The coordinated settings are typically selected to satisfy requirements that are often conflicting, meaning that compromise settings must be chosen. Because of these compromise settings, occasionally fault conditions can lead to an undesirable relay response. In addition, despite the progress of the last few decades, some protection gaps persist. For instance, we still do not have 100% reliable approaches for certain fault types, such as high impedance faults and faults near neutrals.

Setting-less protection based on dynamic state estimation is an emerging technology that effectively uses advances in numerical relays to improve protection. This protection approach uses simplified settings, just the operating limits of devices. For this reason it was named “setting-less.” A more descriptive name is coordination-less protection. The dynamic state estimation approach requires that complex analytics be performed on data acquired by the data acquisition system of a relay.

The setting-less protection method was inspired from the fact that differential protection is one of the most secure protection schemes that we have and does not require coordination with other protection function. Differential protection simply monitors the validity of Kirchoff's current law in a device, i.e., the weighted sum of the currents going into a device must be equal to zero. This concept can be generalized into monitoring the validity of all other physical laws that the device must satisfy, such as Kirchoff's voltage law and Faraday's law. This monitoring can be done in a systematic way by use of dynamic state estimation.

Dynamic state estimation is used to continuously monitor the dynamic model of the component under protection. Specifically, all the physical laws that a component must obey are expressed in the dynamic model of the component. The monitoring system of the component under protection that continuously measures terminal data (such as the terminal voltage magnitude and angle, the frequency, the rate of frequency change), and other variables (such as temperature and speed), and component status data (such as tap setting and breaker status). The dynamic state estimation processes these measurements and extracts the real time dynamic model of the component and its operating conditions. If any one of the physical laws for the component under protection is violated, dynamic state estimation will identify this condition.

Thus, the dynamic state estimator extracts the real-time dynamic model of the component under protection to determine whether the physical laws for the component are being satisfied. The dynamic model of the component accurately reflects the current operating condition of the component. The decision to trip or not to trip the component is based only on the condition of that component irrespective of the condition (such as faults) of other system components.

In prior research, numerical experiments based on the setting-less protection approach were performed on a number of protection problems, specifically, transmission line protection, capacitor bank protection, transformer protection, reactor protection, induction motor protection and distribution line protection. The research demonstrated that the dynamic state estimation approach provides a secure and dependable protection scheme, and does not require coordination with other devices or protection schemes while, at the same time, addresses many protection gaps.

Following the above positive results from numerical experiments, the method has been refined further and tested in the laboratory. This report summarizes the methodological refinements and the results of further laboratory testing of their application. The analytics of the setting-less protection are complex. To provide a practical approach to setting-less protection, the analytics have been "objectified". Specifically, the basic building blocks of the setting-less protection algorithm have been mathematically abstracted to form objects that are recognizable by a single computational algorithm. The implementation is based on merging unit technology. This report describes:

- Methodology refinements
- Laboratory setup and experiments to test the refinements
- Test results.

The results continue to show that setting-less protection is a viable approach to system protection.

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1. Introduction

Present day numerical relays use high end microprocessors for implementing multiple functions of protection. For example a transformer protection relay may include all the typical protection functions that traditionally used with electromechanical relays, i.e., differential, over-current, V/Hz function, etc. Since these functions work independently, even if they are implemented on the same relay, they suffer from the same limitations as the usual single relay/single function approach. Each function must be set separately but the settings must be coordinated with other protective devices. The coordinated settings are typically selected so that can satisfy requirements that many times are conflicting and therefore a compromise must be selected. For this reason, most of the time the settings represent a compromise and occasionally may exist possible fault conditions that may lead to an undesirable relay response.

It should be understood that numerical relays have provided many more options that have improved the above procedure. At the same time the additional options have created increased complexity and increased possibilities of human errors, while the basic nature of the problem of selecting settings has remained the same: the settings must be selected to satisfy criteria that many times are conflicting. The natural question is whether any new technologies and trends can favorably affect the protection process.

There are technologies that can enable better, integrated approach to the overall protection. Some of these technologies are (technology is in flux with many developments still to occur):

1. Merging units/separation of data acquisition and data processing/protection
2. GPS synchronized measurements (PMUs)
3. Data Concentrator (PDCs, switches, etc.)
4. Smarter sensors
5. Integrated (power system/relay) analysis programs that enable faster and more reliable assessment of settings
6. Data validation/state extraction
7. Other

It is difficult to assess the full impact of these technologies on the future of protection. One thing is clear though: while there is much technology development in hardware and the capabilities of hardware, the development of new approaches to fully utilize the new capabilities is lagging. This is the classical problem of "hardware being ahead of software". This will come with realistic assessment of the new technologies and bold experimentation of new approaches and the eventual emergence of successful approaches.

The existence of the above technologies and the quest for better more reliable (secure and dependable) protection has led to the investigation of dynamic state estimation protection approaches. This investigation has shown that it is possible to develop a protection approach that does not require coordination with other protection functions and at the same time address many protection gaps. The investigation has been carried out with many numerical experiments and has been reported in the previous year report.

Following the above investigation and the positive results, the method has been further refined and also tested in the laboratory. This report summarizes the method refinements and its application in the laboratory.

2. Brief Review of the DSE Based Protection

For secure and reliable protection of power components such as a generator, line, transformer, etc. a new approach has emerged based on component health dynamic monitoring. The proposed method uses dynamic state estimation [4-7], based on the dynamic model of the component, which accurately reflects the nonlinear characteristics of the component as well as the loading and thermal state of the component.

For more secure protection of protection zones such as transmission lines, transformers, capacitor banks, motors, generators, generator/transformer unit, etc., this report proposes a new method. The method has been inspired from the fact that differential protection is one of the most secure protection schemes that we have and it does not require coordination with other protection function. Differential protection simply monitors the validity of Kirchoff's current law in a device, i.e., the weighted sum of the currents going into a device must be equal to zero. This concept can be generalized into monitoring the validity of all other physical laws that the device must satisfy, such as Kirchoff's voltage law, Faraday's law, etc. This monitoring can be done in a systematic way by the use of dynamic state estimation. Specifically, all the physical laws that a component must obey are expressed in the dynamic model of the component. Dynamic state estimation is used to continuously monitor the dynamic model of the component (zone) under protection. If any of the physical laws for the component under protection is violated, the dynamic state estimation will capture this condition. Thus, it is proposed to use dynamic state estimator to extract the dynamic model of the component under protection [2-5] and to determine whether the physical laws for the component are satisfied. The dynamic model of the component accurately reflects the condition of the component and the decision to trip or not to trip the component is based on the condition of the component only irrespectively of the condition (faults, etc.) of other system components. The proposed method requires a monitoring system of the component under protection that continuously measures terminal data (such as the terminal voltage magnitude and angle, the frequency, and the rate of frequency change - this task is identical to present day numerical relays), other variables such as temperature, speed, etc., as appropriate, and component status data (such as the tap setting, breaker status, etc.). The dynamic state estimation processes these measurements and extracts the real time dynamic model of the component and its operating conditions.

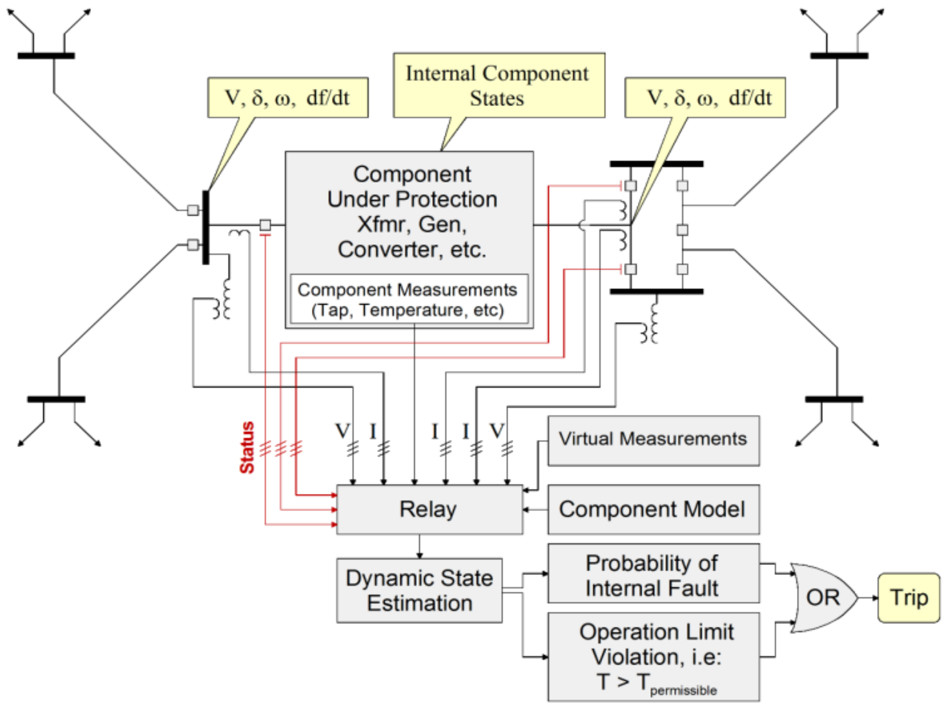

Figure 2.1: Illustration of Setting-less Component Protection Scheme

After estimating the operating conditions, the well-known chi-square test [6] calculates the probability that the measurement data are consistent with the component model, i.e., the physical laws that govern the operation of the component. In other words, this probability, which indicates the confidence level of the goodness of fit of the component model to the measurements, can be used to assess the health of the component. The high confidence level indicates a good fit between the measurements and the model, which indicates that the operating condition of the component is normal. However, if the component has internal faults, the confidence level would be almost zero (i.e., the very poor fit between the measurement and the component model). Figure 2.1 shows the concept of entire proposed protection scheme.

In general, the proposed method can identify any internal abnormality of the component within a cycle and trip the component immediately. Furthermore, it does not degrade the security because a relay does not trip in the event of normal behavior of the component, for example, in case of transformer protection, inrush currents or over excitation currents, since in these cases, as long as the inrush currents are consistent with the transient behavior of the transformer as dictated by the dynamic model, the method will produce a high confidence level that the transients are consistent with the model of the component. Note also that the method does not require any settings or any coordination with other relays.

It is important to note that the proposed scheme will perform best when: (a) the measurements are as accurate as possible - dependent on the type of instrument transformer used, i.e., VT, CT, etc. and the instrumentation channel, i.e., control cable, etc. and (b) the accuracy of the dynamic model of the component under protection. These issues, while important, are beyond the scope of this report. These issues will be addressed in a subsequent report.

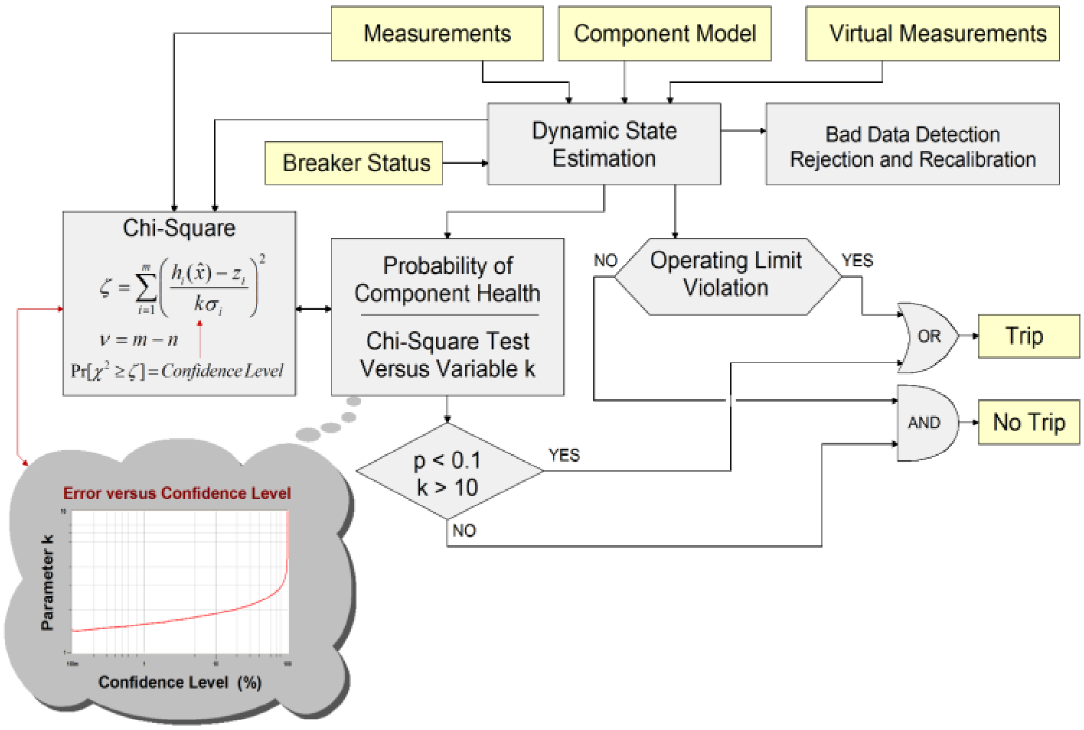

Figure 2.2: Illustration of Setting-less Protection Logic

The proposed protection logic is briefly illustrated in Figure 2.2. The method requires a monitoring system of the component under protection that continuously measures terminal data (such as the terminal voltage magnitude and angle, the frequency, and the rate of frequency change) and component status data (such as tap setting (if transformer) and temperature). The dynamic state estimation processes these measurement data with the dynamic model of the component yielding the operating conditions of the component.

This approach faces some challenges which can be overcome with present technology. A partial list of the challenges is given below:

1. Ability to perform the dynamic state estimation in real time
2. Initialization issues
3. Communications in case of a geographically extended component (i.e., lines)
4. New modeling approaches for components - connects well with the topic of modeling
5. Requirement for GPS synchronized measurements in case of multiple independent data acquisition systems.
6. Other

The modeling issue is fundamental in this approach. For success the model must be high fidelity so that the component state estimator will reliably determine the operating status (health) of the component. For example consider a transformer during energization. The transformer will experience high in-rush current that represent a tolerable operating condition and therefore no relay action should occur. The component state estimator should be able to "track" the in-rush current and determine that they represent a tolerable operating condition. This requires a transformer model that accurately models saturation and in-rush current in the transformer. We can foresee the possibility that a high fidelity model used for protective relaying can be used as the main depository of the model which can provide the appropriate model for other applications. For example for EMS applications, a positive sequence model can be computed from the high fidelity model and send to the EMS data base. The advantage of this approach will be that the EMS model will come from a field validated model (the utilization of the model by the relay in real time provide the validation of the model). This overall approach is shown in Figure 2.3. Since protection is ubiquitous, it makes economic sense to use relays for distributed model data base that provides the capability of perpetual model validation.

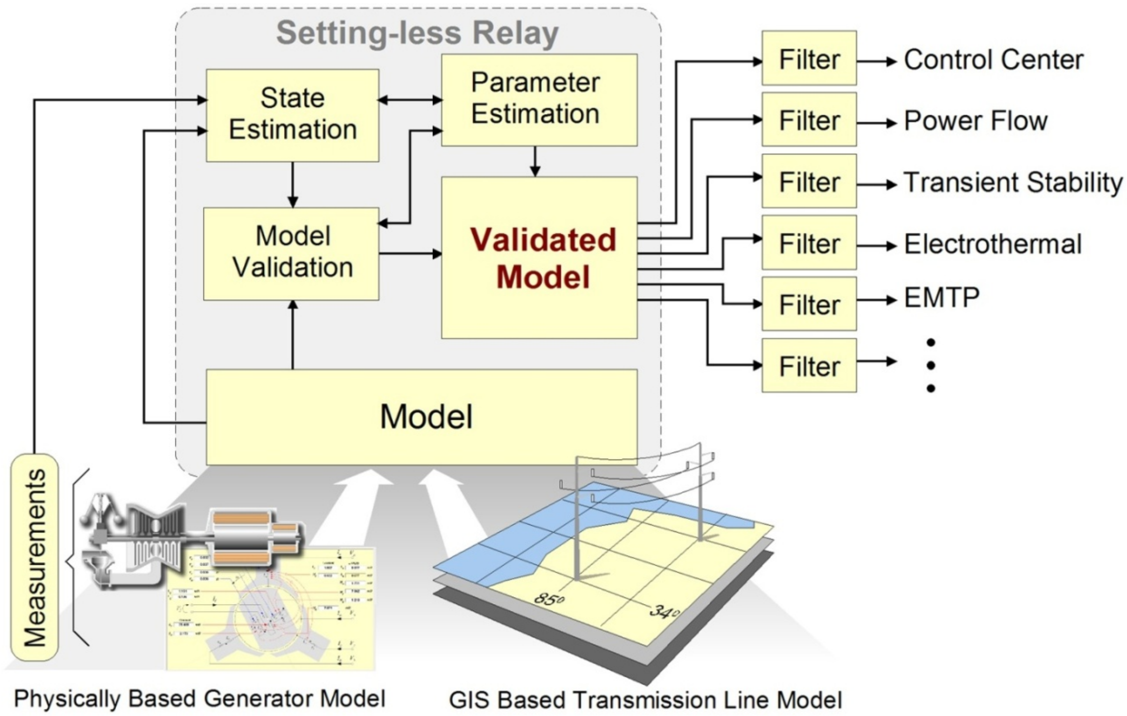

Figure 2.3:Overall Approach for Component Protection

3. Implementation of Setting-less Protection

The implementation of the setting-less protection has been approached from an object orientation point of view. For this purpose the constituent parts of the approach have been evaluated and have been abstracted into a number of objects. Specifically, the setting-less approach requires the following objects:

1. the mathematical model of the protection zone
2. the physical measurements that may consist of analog and digital data
3. the mathematical model of the physical measurements
4. the mathematical model of the virtual measurements
5. the mathematical model of the derived measurements
6. the mathematical model of the pseudo measurements
7. the dynamic state estimation algorithms
8. the bad data detection and identification algorithm
9. the protection logic and trip signals
10. online parameter identification method

The last task has not been addressed in the report but it is an integral part of the overall approach as in many cases it will be necessary to fine tune the model of the protection zone via online parameter identification methods.

An overview of the design of the setting-less protection relay is shown in Figure 3.1.

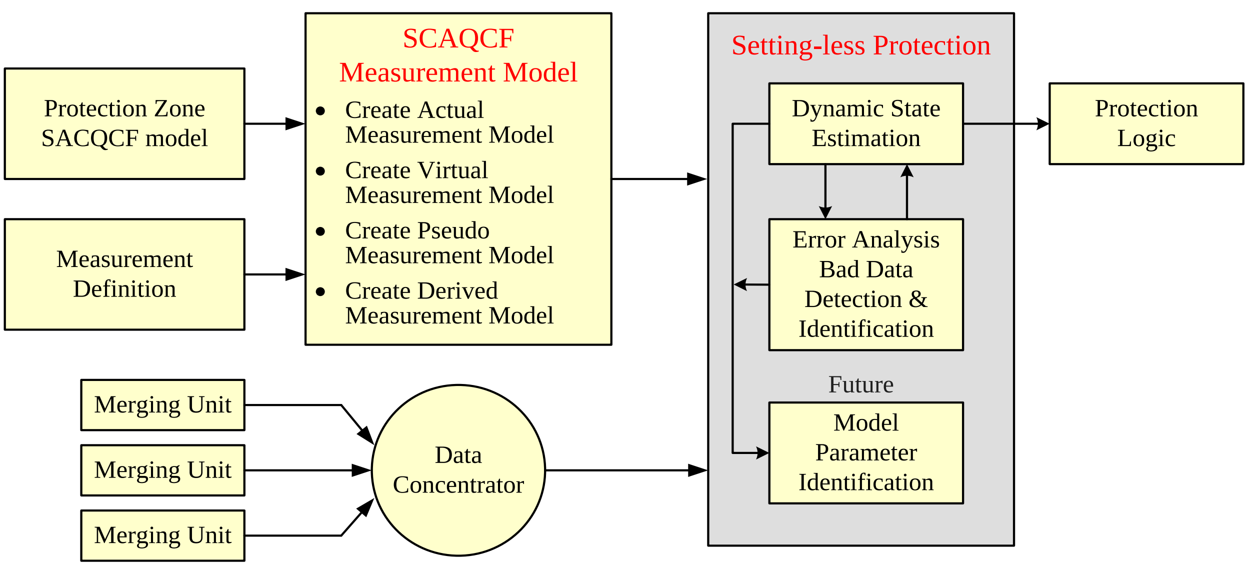

Figure 3.1: Setting-Less Protection Relay Organization

The setting-less protection algorithms have been streamlined for the purpose of increasing efficiency. An object-oriented approach for the DSE based protection algorithm is developed by utilizing the State and Control Algebraic Quadratic Companion Form (SCAQCF). All the mathematical models of the apparatus in the power system are written in SCAQCF format so that the DSE based protection algorithm could be applied to any device. The algorithm automatically formulates the measurement model in the SCAQCF syntax from the SCAQCF device model and the measurement definition file, as illustrated by Figure 3.1. A data concentrator is utilized to align data from multiple merging units with the same time stamp and feed the streaming data into the DSE based protection module. The DSE based protection scheme continuously monitors the SCAQCF model of the component (zone) under protection by fitting the real-time measurement data to the measurement model in the SCAQCF syntax. If any of the physical laws for the component under protection is violated, the dynamic state estimation will capture this condition. Under normal operation, the device estimated measurement data from DSE should be exactly the same as the real measurement data. The mismatch between real measurements and estimated measurements indicates abnormalities in the device, then diagnose or trip decision should be made. Detailed implementation of setting-less protection in an object-oriented way is shown in Appendix A and Appendix B. This section gives a brief introduction of how setting-less protection is implemented.

3.1. Protection Zone Mathematical Model

The mathematical model of the protection zone is required in a standard form. A standard has been defined in the form of the **S**tate and **C**ontrol **A**lgebraic **Q**uadratic **C**ompanion **F**orm (SCAQCF) and in a specified syntax to be defined later. The SCAQCF for a specific protection zone is derived with three computational procedures. Specifically, the dynamic model of a protection zone consists of a set of algebraic and differential equations. We refer to this model as the compact model of the protection zone. Subsequently this model is quadratized, i.e., in case there are nonlinearities of order greater than 2, additional state variables are introduced so that at the end the mathematical model consists of a set of linear and quadratic equations. We refer to this model as the quadratized model. Finally, the quadratized model is integrated using the quadratic integration method which converts the quadratized model of the protection zone into a set of algebraic (quadratic) function. This model is cast into a generalized Norton form. We refer to this model as the Algebraic Quadratic Companion Form. Since the variables in this AQCF contain all states and controls, thus it is named State and Control Algebraic Quadratic Companion Form.

The standard State and Control Algebraic Quadratic Companion Form is obtained with two procedures: (a) model quadratization, and (b) quadratic integration. The model quadratization reduces the model nonlinearities so that the dynamic model will consist of a set of linear and quadratic equations. The quadratic integration is a numerical integration method that is applied to the quadratic model assuming that the functions vary quadratically over the integration time step. The end result is an algebraic companion form that is a set of linear and quadratic algebraic equations that are cast in the following standards form:

$$\begin{Bmatrix} I(\mathbf{x},\mathbf{u}) \\ \vdots \\ 0 \\ \vdots \end{Bmatrix} = Y_{eqx}\mathbf{x} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{eqx}^i \mathbf{x} \\ \vdots \end{Bmatrix} + Y_{equ}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F_{equ}^i \mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{eqxu}^i \mathbf{u} \\ \vdots \end{Bmatrix} - B_{eq} \quad (1)$$

where $I(\mathbf{x},\mathbf{u})$ is the through variable (current) vector, $\mathbf{x}=[\mathbf{x}(t),\mathbf{x}(t_m)]$ is the external and internal state variables, $\mathbf{u}=[\mathbf{u}(t),\mathbf{u}(t_m)]$ is the control variables, t is present time, t_m is the midpoint between the present and previous time, Y_{eq} admittance matrix, F_{eq} nonlinear matrices, and

$$B_{eq}=-N_{eqx}\mathbf{x}(t-h)-N_{equ}\mathbf{u}(t-h)-M_{eq}I(t-h)-K_{eq} \tag{2}$$

The derivation of the standard State and Control Algebraic Quadratic Companion Form for specific protection zones is provided in the appropriate reports that describe the application of the setting-less protection schemes for specific protection zones.

This standardization allows the object oriented handling of measurements in state estimation; in addition it converts the dynamic state estimation into a state estimation that has the form of a static state estimation.

3.2. Object-Oriented Measurements

Any measurement, i.e., current, voltage, temperature, etc. can be viewed as an object that consists of the measured value and a corresponding function that expresses the measurement as a function of the state of the component. This function can be directly obtained (autonomously) from the State and Control Algebraic Quadratic Companion Form of the component. Because the algebraic companion form is quadratic at most, the measurement model will be also quadratic at most. Thus, the object-oriented measurement model can be expressed as the following standard equation:

$$\begin{aligned} z_k(t) = &\sum_i a_{i,t}^k \cdot x_i(t) + \sum_i a_{i,tm}^k \cdot x_i(t_m) \\ &+\sum_{i,j} b_{i,j,t}^k \cdot x_i(t) \cdot x_j(t) \\ &+\sum_{i,j} b_{i,j,tm}^k \cdot x_i(t_m) \cdot x_j(t_m) \\ &+c_k(t) + \eta_k \ , \end{aligned} \quad , \tag{3}$$

where z is the measured value, t the present time, t_m the midpoint between the present and previous time, x the state variables, a the coefficients of linear terms, b the coefficients of nonlinear terms, c the constant term, and η the measurement error.

The measurements can be identified as: (a) actual measurements, (b) virtual measurements, (c) derived measurements and (d) pseudo measurements. The types of measurements will be discussed next.

Actual Measurements: In general the actual measurements can be classified as across and through measurements. Across measurements are measurements of voltages or other physical quantities at the terminals of a protection zone such as speed on the shaft of a generator/model. These quantities are typically states in the model of the component. For this reason, the across measurements has a simple model as follows:

$$z_j = x_i \pm x_j + \eta_j \, . \tag{4}$$

Through measurements are typically currents at the terminals of a device or other quantities at the terminals of a device such as torque on the shaft of a generator/motor. The quantity of a through measurement is typically a function of the state of the device. For this reason, the through measurement model is extracted from the algebraic companion form, i.e., the measurement model is simply one equation of the SCACQF model, as follows:

$$z_j = \sum_i Y_{eqx,i}^{\ k} \cdot x_i + \sum_{i,j} F_{eqx,ij}^{\ k} \cdot x_i \cdot x_j + \sum_i Y_{equ,i}^{\ k} \cdot u_i + \sum_{i,j} F_{equ,ij}^{\ k} \cdot u_i \cdot u_j + \sum_{i,j} F_{eqxu,ij}^{\ k} \cdot x_i \cdot u_j - \sum_i b_{eq,i}^{\ k}\ , \tag{5}$$

where the superscript *k* means the *k*th row of the matrix or the vector.

Virtual Measurements: The virtual measurements represent a physical law that must be satisfied. For example we know that at a node the sum of the currents must be zero by Kirchoff's current law. In this case we can define a measurement (sum of the currents); note that the value of the measurement (zero) is known with certainty. This is a virtual measurement.

The model can provide virtual measurements in the form of equations that must be satisfied. Consider for example the mth SCAQCF model equation below:

$$0 = \sum_i Y_{eqx,i}^{\ k} \cdot x_i + \sum_{i,j} F_{eqx,ij}^{\ k} \cdot x_i \cdot x_j + \sum_i Y_{equ,i}^{\ k} \cdot u_i + \sum_{i,j} F_{equ,ij}^{\ k} \cdot u_i \cdot u_j + \sum_{i,j} F_{eqxu,ij}^{\ k} \cdot x_i \cdot u_j - \sum_i b_{eq,i}^{\ k} \tag{6}$$

This equation is simply a relationship among the states the component that must be satisfied. Therefore we can state that the zero value is a measurement that we know with certainty. We refer to this as a virtual measurement.

Derived Measurements: A derived measurement is a measurement that can be defined for a physical quantity by utilizing physical laws. An example derived measurement is shown in Figure 3.2. The figure illustrates a series compensated power line with actual measurements on the line side only. Then derived measurements are defined for each capacitor section. Note that the derived measurements enable the observation of the voltage across the capacitor sections.

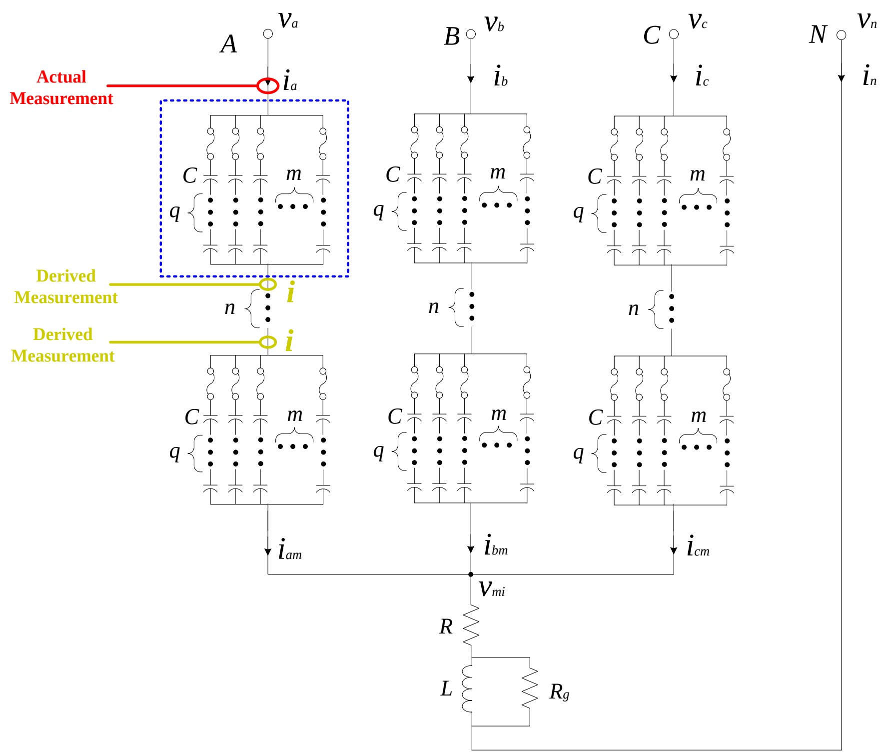

Figure 3.2: Example Derived Measurements

Pseudo Measurements: Pseudo measurements are hypothetical measurements for which we may have an idea of their expected values but we do not have an actual measurement. For example a pseudo measurement can be the voltage at the neutral; we know that this voltage will be very small under normal operating conditions. In this case we can define a measurement of value zero but with a very high uncertainty.

Summary: Eventually, all the measurement objects form the following measurement set:

$$z = h(x,t) + \eta = c + a^T x(t) + b^T x(t_m) + \left[x^T(t) \quad x^T(t_m) \right] F \begin{bmatrix} x(t) \\ x(t_m) \end{bmatrix} + \eta, \tag{7}$$

where z is the measurement vector, x the state vector, h the known function of the model, a, b are constant vectors, F are constant matrices, and η the vector of measurement errors.

3.3. Object-Oriented Dynamic State Estimation

The proposed dynamic state estimation algorithm is the weighted least squares (WLS). The objective function is formulated as follows:

$$\text{Minimize} \;\; J(x,t) = [z - h(x,t)]^T W [z - h(x,t)], \tag{8}$$

where W is the diagonal matrix whose non-zero entries are the inverse of the variance of the measurement errors. The solution is obtained by the iterative method:

$$\hat{x}^{j+1} = \hat{x}^{j} + (H^T W H)^{-1} H^T W (z - h(\hat{x}^{j}, t)), \quad (9)$$

where $\hat{x}$ is the best estimate of states and H the Jacobian matrix of $h(x,t)$.

It is important to note that the dynamic state estimation requires only the mathematical model of all measurements. It should be also noted that for any component, the number of actual measurements and virtual, derived, and pseudo measurements exceed the number of states and they are independent. This makes the system observable and with substantial redundancy.

3.4. Bad Data Detection and Identification

It is possible that the streaming measurements may include bad data. In this case the algorithm must detect the bad data and identify the data. For the case of setting-less protection, it is important to recognize that in case of a component internal fault, all data may appear as bad data. It is important to determine whether any detected bad data are coming from instrumentation and meter errors or from altered component model due to internal faults. This topic is still under investigation as to what the best approach would be.

3.5. Protection Logic / Component Health Index

The solution of the dynamic state estimation provides the best estimate of the dynamic state of the component. The well-known chi-square test provides the probability that the measurements are consistent with the dynamic model of the component. Thus the chi-square test quantifies the goodness of fit between the model and measurements (i.e., confidence level). The goodness of fit is expressed as the probability that the measurement errors are distributed within their expected range (chi-square distribution). The chi-square test requires two parameters: the degree of freedom (v) and the chi-square critical value (ζ). In order to quantify the probability with one single variable, we introduce the variable k in the definition of the chi-square critical value:

$$v = m - n, \qquad \zeta = \sum_{i=1}^{m} \left(\frac{h_i(\hat{x}) - z_i}{k\sigma_i} \right)^2, \quad (10)$$

where m is the number of measurements, n the number of states, and $\hat{x}$ the best estimate of states. Note that since m is always greater than n, the degrees of freedom are always positive. Note also that if k is equal to 1.0 then the standard deviation of the measurement error corresponds to the meter error specifications. If k equals 2.0 then the standard deviation will be twice as much as the meter specifications, and so on. Using this definition, the results of the chi square test can be expressed as a function of the variable k. Specifically, the goodness of fit (confidence level) can be obtained as follows:

$$\Pr[\chi^2 \geq \zeta(k)] = 1.0 - \Pr[\chi^2 \leq \zeta(k)] = 1.0 - \Pr(\zeta(k), v). \quad (11)$$

A sample report of the confidence level function (horizontal axis) versus the chi-square critical value k, (vertical axis) is depicted in Figure 3.3.

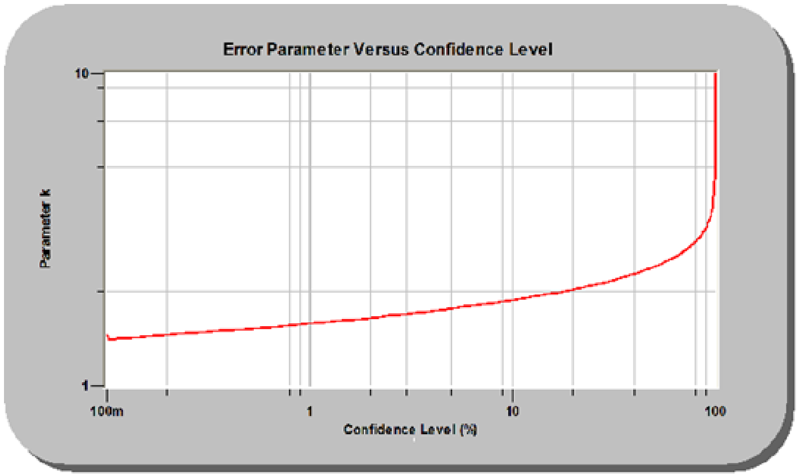

Figure 3.3: Confidence Level (%) vs Parameter k

The proposed method uses the confidence level as the health index of a component. A high confidence level indicates good fit between the measurement and the model, and thus we can conclude that the physical laws of the component are satisfied and the component has no internal fault. A low confidence level, however, implies inconsistency between the measurement and the model; therefore, we can conclude that an abnormality (internal fault) has occurred in the component and has altered the model. The discrepancy is an indication of how different the faulty model of the component is as compared to the model of the component in its healthy status.

It is important to point out that the component protection relay must not trip circuit breakers except when the component itself is faulty (internal fault). For example, in case of a transformer, inrush currents or over-excitation currents, should be considered normal and the protection system should not trip the component. The proposed protection scheme can adaptively differentiate these phenomena from internal faults. Similarly, the relay should not trip for start-up currents in a motor, etc.

3.6. Online Parameter Identification

The dynamic state estimation can be extended to include as states parameters of components. In this case, the parameters of the components can be identified from measurements. This represents a fine tuning of the component model.

This option should not be continuously applied. Instead, it should be exercised only when there is doubt about the correctness of the component parameters. The issue will be addressed in greater detail in the filed applications of the setting-less protective relay.

3.7. Summary and Comments

The previous subsections have presented the approach for setting-less protection. The method is based on dynamic state estimation. The dynamic state estimation requires a detail model of the component under protection (protection zone) and a data acquisition system that acquires data with sufficient speed, such as 2,000 samples per second or higher. The accuracy of the data acquisition system is important, the more accurate it is the better the selectivity of the relay.

The proposed protection approach has been applied to several types of components (i.e., transformers, transmission lines, capacitor banks, etc.). We present examples in the next section.

4. Laboratory Implementation and Testing

This section presents the laboratory implementation and example applications of setting-less protection for a few typical protection zones. For each one of them the constituent parts of the relay are described. The application has been evaluated by a number of numerical experiments.

4.1. Laboratory Implementation

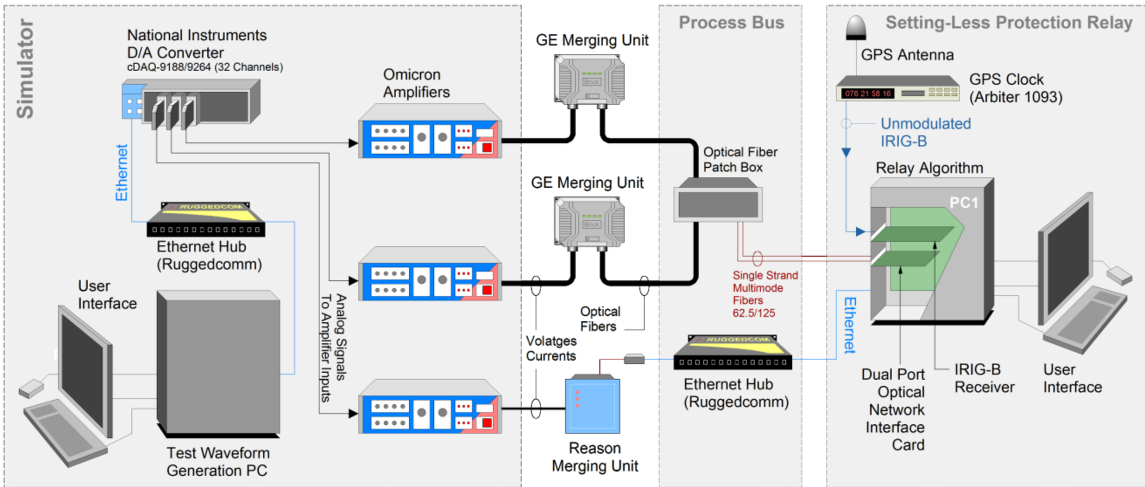

Figure 4.1: Lab Implementation Illustration

The implementation is illustrated in Figure 4.1. Since it is impractical to achieve real measurements from electronic CTs/PTs in the lab, computer simulation based signals are utilized alternatively.

As illustrated by Figure 4.1, the WinXFM program generates digital streaming waveforms representing the terminal voltages and currents of the protection zone (power system component) to the NI 32 channel DC/AC converter. It is emphasized here that the source for the digital streaming waveforms can be simulated voltages and currents of the power system component, or field collected measurements from the CTs/PTs in the COMTRADE format. Omicron amplifiers receive the analog signals from the NI DC/AC converter and amplify these signals to a range which is very similar to real output of electronic CTs/PTs (for voltages around 50V and for currents around 5A). In this manner, the electrical output (voltages and currents) of the Omicron amplifiers are treated as the secondary electrical quantities from the CTs/PTs in this lab implementation. Two merging units from Reason and GE are connected to the output terminals of the Omicron amplifier. The merging units are utilized to collect the analog signals from the amplifier and convert them to digital signals. These digital signals are transmitted to a personal computer which is used as a setting-less relays. The computer communicates with the merging units in accordance to IEC 61850-8-1 and IEC 61850-9-2 and captures the measurements, performs the dynamic state estimation and issues protection logic. The sampling rate of the waveform generated by WinXFM is 5000 samples/sec (similar to the present top merging unit data transmission speed), and the merging units samples the analog data at a rate of 2880 samples/sec, which means the proposed

DSE-based protection approach should determine the protection decision in 400 μs to avoid coming data overlap.

4.2. Testing Procedure – User Interface

The main user interface of setting-less protection relay is shown in Figure 4.2. Module functions are introduced below.

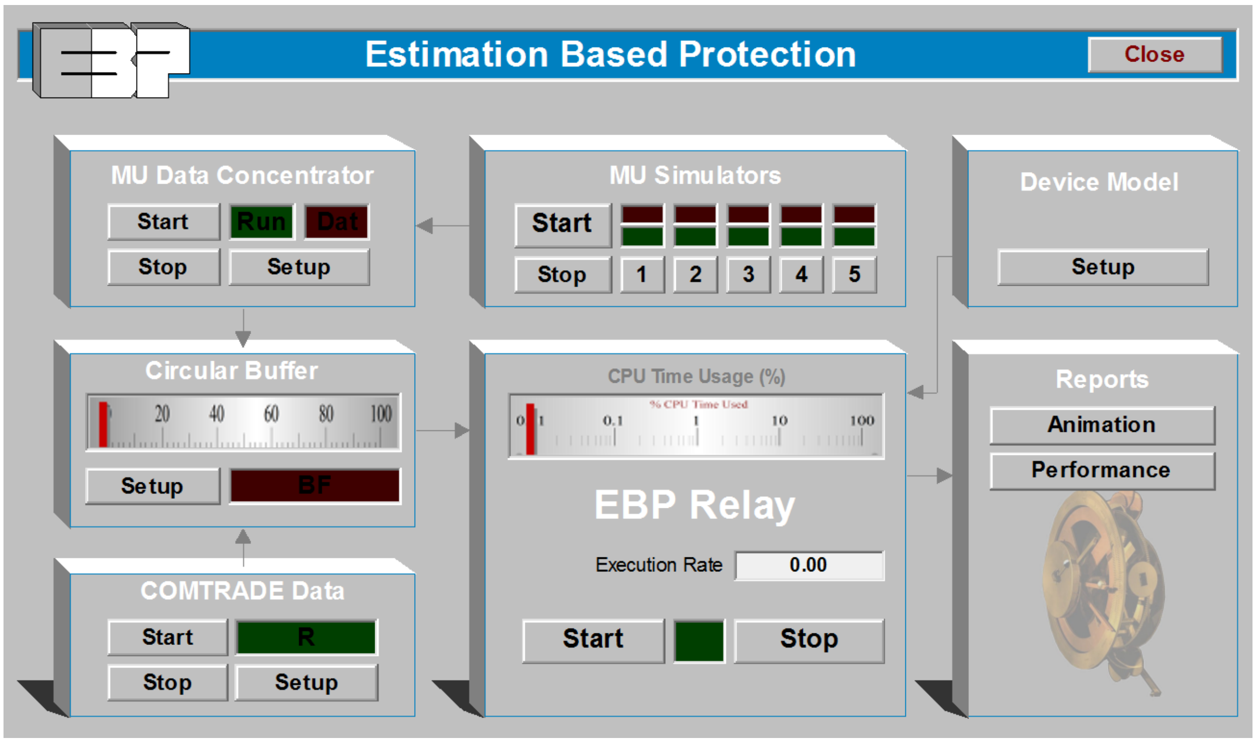

Figure 4.2: Setting-less Protection Relay User Interface

Device Model:
In the device model module, the SACQCF model for the device under protection is selected.

EBP Relay:
EBP stands for estimation based protection which is the fundamental of our proposed setting-less protection. EBP relay module has two inputs and one output. It reads the SCAQCF device model at the initialization step and real-time measurements from circular buffer continuously. The proposed algorithm in section 3 is integrated in this module. It tries to fit the real-time measurements to the SCAQCF model of the device under protection. Any mismatch between the real-time measurements and estimated measurements indicates the measurements not coming from the healthy device model thus the device under protection is in the unhealthy status (internal faults). EBP relay outputs the results of dynamic state estimation and the confidence level of the device under protection. Also the CPU usage is shown in this module.

Circular Buffer:
Circular buffer is the module which provides a buffer for the real-time measurement coming into the setting-less protection module. For test and lab implementation purpose, the real-time data can either come from a simulated COMTRADE file or directly from the Merging Units.

COMTRADE Data:
This module reads the real-time measurements from existing COMTRADE data files from local computer and transfers these data to circular buffer.

Reports:
In the reports module, two functions are established, animation and performance. Animation shows the device terminal voltage and current phasors while performance shows how the dynamic estimation performs.

MU Data Concentrator:
This module indicates the merging units' operation status and concentrates the data from multiple merging units and transfers these data to the circular buffer. The user interface of the MU data concentrator is shown in Figure 4.3. As Figure 4.3 shows, the user interface of the MU data concentrator gives all the measurement channels provided by the multiple merging units communicated with the personal computer and the user can modify the settings for each measurement channel, such as the measurement type, the scaling factor, and the instrument transformer ratio, as illustrated in Figure 4.4. Since this module provides the capability of align the measurements from different merging units with the same time stamp (data concentration), the user interface provides the functionality of setting the maximum latency for each sample at a specific time point, which means if the data concentrator receives a measurement at time T1, it will wait a maximum latency, which is set by the user, for other measurements with the same time T1, and after waits for the maximum latency time, the data concentrator will transfer the measurements with the time stamp T1 to the circular buffer and treat the missing packets as error packets. The user interface also provides other useful information of the data concentrator, such as the average latency of the coming measurements and the percentage of the usage of the circular buffer.

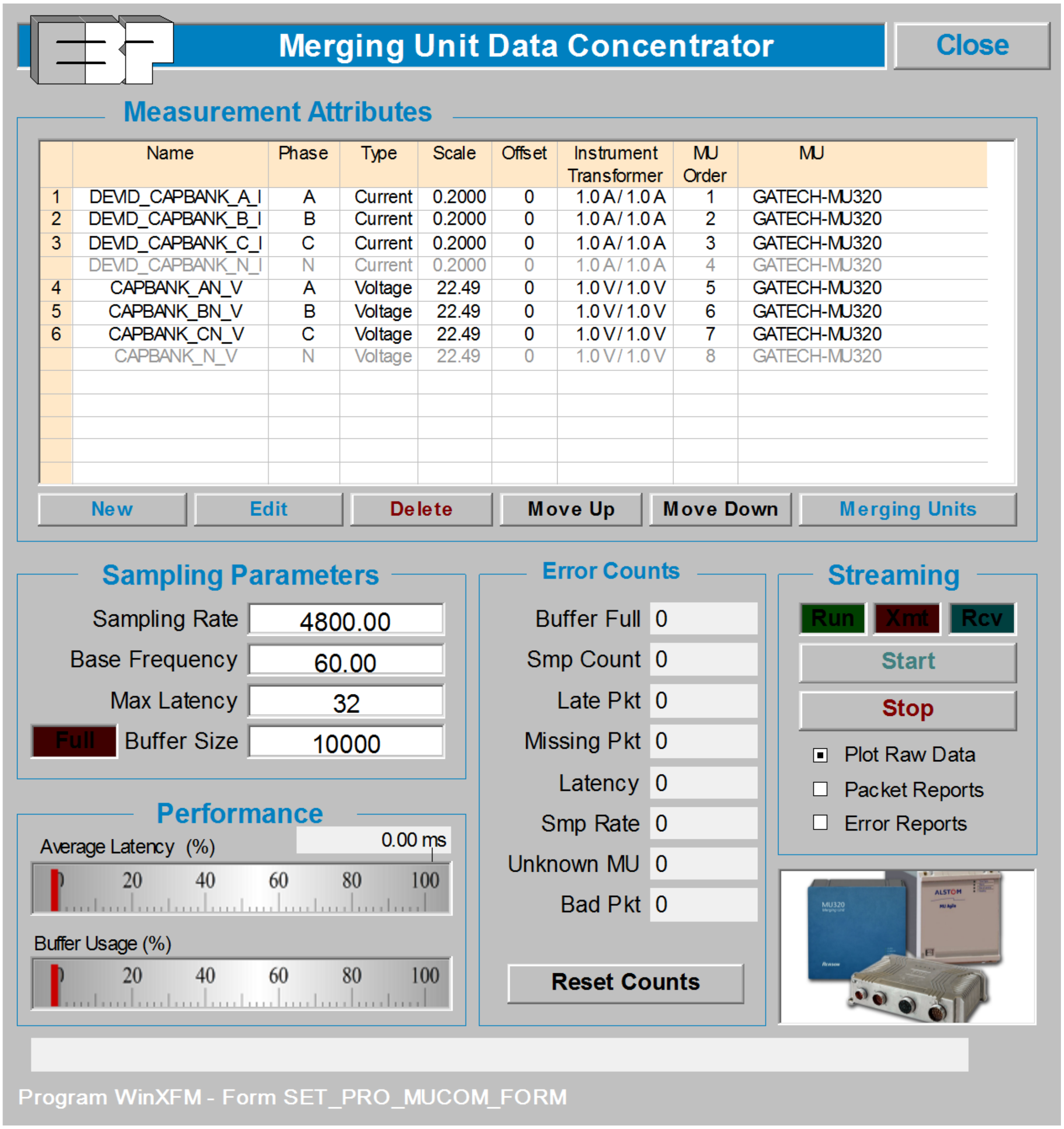

Figure 4.3: MU Data Concentrator User Interface

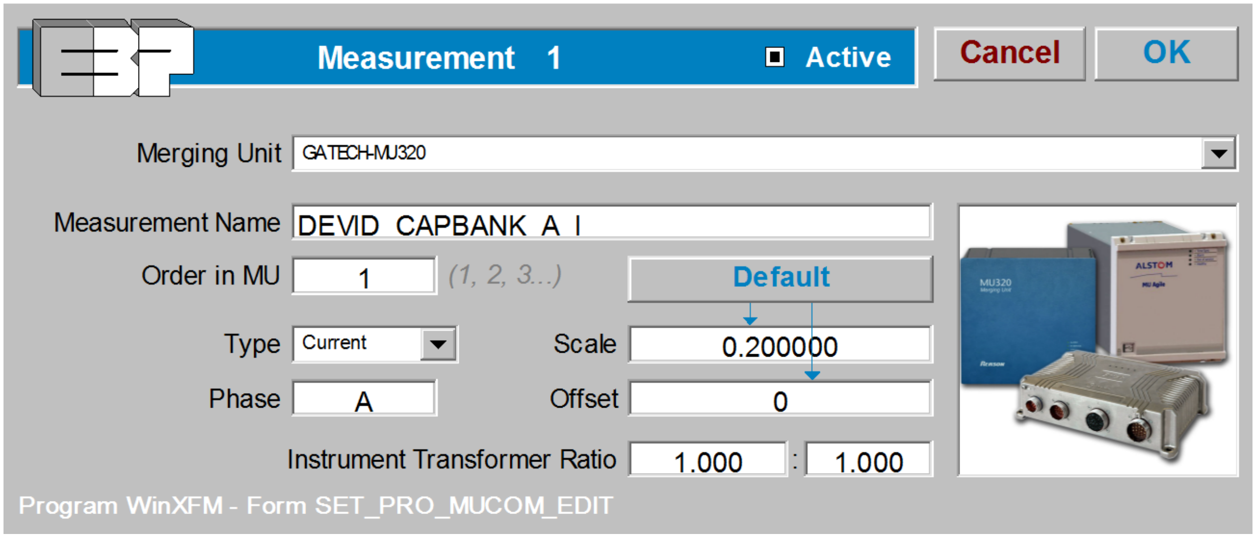

Figure 4.4: User Interface for the Measurement Channel of the Merging Unit

4.3. Application to Capacitor Bank Protection

A capacitor bank typically consists of strings of capacitor cans connected in series and in parallel to form the capacitor bank of the proper ratings. Figure 4.5 shows an example capacitor bank. It is important to point out that the instrumentation of capacitor banks typically includes the terminal voltages and currents as well as other measurements such as neutral voltage and current, current in parallel strings of capacitor cans, etc. All the available measurements should be included in the analytics of the protection function for the capacitor bank. We briefly describe the constituent parts of the setting-less protection for the capacitor bank and then present some example results and execution time result.

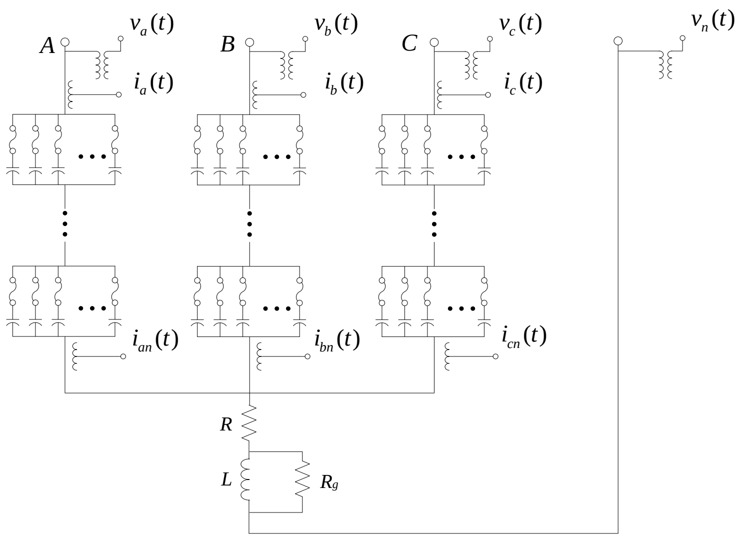

Figure 4.5: Three Phase Capacitor Bank Example

Mathematical Model: The mathematical model is presented for a specific capacitor bank. The example capacitor bank ratings are: voltage rating: 115 kV, reactive power rating 48 MVAr. The protection instrumentation includes the following measurements: terminal voltage measurements (phase to ground), terminal current measurements, neutral voltage, and three phase current measurements before the neutral connection. The simplified three phase capacitor bank circuit model and the above referenced measurements are shown in Figure 4.6.

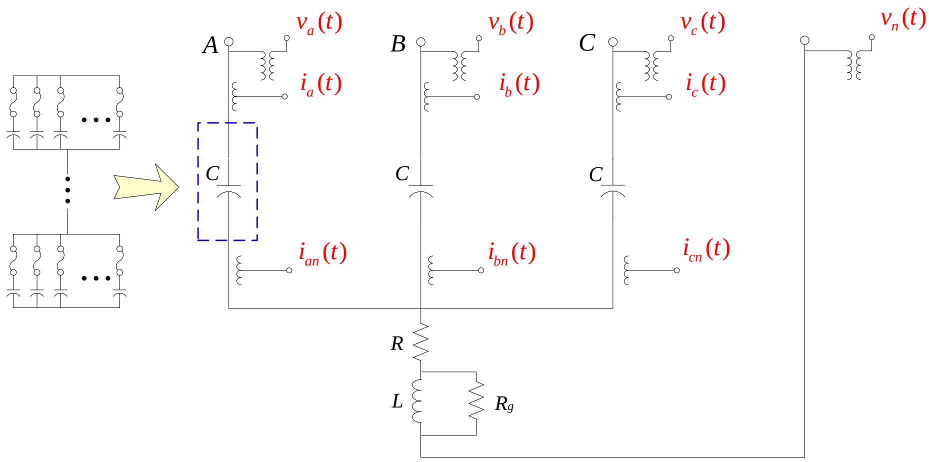

Figure 4.6: Three Phase Capacitor Bank Simplified Model (C is the net capacitance of a phase)

The derivation of the SCAQCF device model for the capacitor bank of Figure 4.6 and associated measurements is provided in Appendix C. The SCAQCF model is shown below in compact form:

$$\begin{Bmatrix} I(\mathbf{x},\mathbf{u}) \\ \vdots \\ 0 \\ \vdots \end{Bmatrix} = Y_{eqx}\mathbf{x} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{eqx}^i \mathbf{x} \\ \vdots \end{Bmatrix} + Y_{equ}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F_{equ}^i \mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{eqxu}^i \mathbf{u} \\ \vdots \end{Bmatrix} - B_{eq}$$

$$B_{eq} = -N_{eqx}\mathbf{x}(t-h) - N_{equ}\mathbf{u}(t-h) - M_{eq} I(t-h) - K_{eq}$$

$$\mathbf{h}(\mathbf{x},\mathbf{u}) = Y_{opx}\mathbf{x} + Y_{opu}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{opx}^i \mathbf{x} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F_{opu}^i \mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{opxu}^i \mathbf{u} \\ \vdots \end{Bmatrix} - B_{op}$$

Scaling factors: *Iscale, Xscale and Uscale*

Connectivity: *TerminalNodeName*

$$\textit{subject to}: \ \mathbf{h}_{\min} \le \mathbf{h}(\mathbf{x},\mathbf{u}) \le \mathbf{h}_{\max}$$

$$\mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max}$$

where:

$$Y_{eqx} = \begin{bmatrix}
\frac{4C}{h} & 0 & 0 & 0 & -\frac{4C}{h} & 0 & -\frac{8C}{h} & 0 & 0 & 0 & \frac{8C}{h} & 0 \\
0 & \frac{4C}{h} & 0 & 0 & -\frac{4C}{h} & 0 & 0 & -\frac{8C}{h} & 0 & 0 & \frac{8C}{h} & 0 \\
0 & 0 & \frac{4C}{h} & 0 & -\frac{4C}{h} & 0 & 0 & 0 & -\frac{8C}{h} & 0 & \frac{8C}{h} & 0 \\
-\frac{4C}{h} & -\frac{4C}{h} & -\frac{4C}{h} & 0 & \frac{12C}{h} & 0 & \frac{8C}{h} & \frac{8C}{h} & \frac{8C}{h} & 0 & -\frac{24C}{h} & 0 \\
-C & -C & -C & -\frac{h\cdot G}{6} & \frac{h\cdot G}{6}+3C & -G\cdot L & 0 & 0 & 0 & -\frac{2h\cdot G}{3} & \frac{2h\cdot G}{3} & 0 \\
-C & -C & -C & 0 & 3C & \frac{h}{6}-g\cdot L & 0 & 0 & 0 & 0 & 0 & \frac{2h}{3} \\
\frac{C}{2h} & 0 & 0 & 0 & -\frac{C}{2h} & 0 & \frac{2C}{h} & 0 & 0 & 0 & -\frac{2C}{h} & 0 \\
0 & \frac{C}{2h} & 0 & 0 & -\frac{C}{2h} & 0 & 0 & \frac{2C}{h} & 0 & 0 & -\frac{2C}{h} & 0 \\
0 & 0 & \frac{C}{2h} & 0 & -\frac{C}{2h} & 0 & 0 & 0 & \frac{2C}{h} & 0 & -\frac{2C}{h} & 0 \\
-\frac{C}{2h} & -\frac{C}{2h} & -\frac{C}{2h} & 0 & \frac{3C}{2h} & 0 & -\frac{2C}{h} & -\frac{2C}{h} & -\frac{2C}{h} & 0 & \frac{6C}{h} & 0 \\
0 & 0 & 0 & \frac{h\cdot G}{24} & -\frac{h\cdot G}{24} & 0 & -C & -C & -C & -\frac{h\cdot G}{3} & \frac{h\cdot G}{3}+3C & -G\cdot L \\
0 & 0 & 0 & 0 & 0 & -\frac{h}{24} & -C & -C & -C & 0 & 3C & \frac{h}{3}-g\cdot L
\end{bmatrix}$$

$$
B_{eq} = \begin{bmatrix}
-i_a(t-h) - \frac{4}{h}\cdot C\cdot[v_a(t-h) - v_m(t-h)] \\
-i_b(t-h) - \frac{4}{h}\cdot C\cdot[v_b(t-h) - v_m(t-h)] \\
-i_c(t-h) - \frac{4}{h}\cdot C\cdot[v_c(t-h) - v_m(t-h)] \\
-i_n(t-h) - \frac{4}{h}\cdot C\cdot[-v_a(t-h) - v_b(t-h) - v_c(t-h) + 3v_m(t-h)] \\
-\frac{h}{6}\cdot G\cdot(v_m(t-h) - v_n(t-h)) - G\cdot L\cdot i_L(t-h) - C\cdot[v_a(t-h) + v_b(t-h) + v_c(t-h) - 3v_m(t-h)] \\
-\frac{h}{6}\cdot i_L(t-h) + g\cdot L\cdot i_L(t-h) - C\cdot[v_a(t-h) + v_b(t-h) + v_c(t-h) - 3v_m(t-h)] \\
\frac{1}{2} i_a(t-h) + \frac{5}{2h}\cdot C\cdot[v_a(t-h) - v_m(t-h)] \\
\frac{1}{2} i_b(t-h) + \frac{5}{2h}\cdot C\cdot[v_b(t-h) - v_m(t-h)] \\
\frac{1}{2} i_c(t-h) + \frac{5}{2h}\cdot C\cdot[v_c(t-h) - v_m(t-h)] \\
\frac{1}{2} i_n(t-h) + \frac{5}{2h}\cdot C\cdot[-v_a(t-h) - v_b(t-h) - v_c(t-h) + 3v_m(t-h)] \\
-\frac{5h}{24}\cdot G\cdot(v_m(t-h) - v_n(t-h)) - G\cdot L\cdot i_L(t-h) - C\cdot[v_a(t-h) + v_b(t-h) + v_c(t-h) - 3\cdot v_m(t-h)] \\
-\frac{5h}{24}\cdot i_L(t-h) + g\cdot L\cdot i_L(t-h) - C\cdot[v_a(t-h) + v_b(t-h) + v_c(t-h) - 3v_m(t-h)]
\end{bmatrix}
$$

$$N_{eqx} = \begin{bmatrix} \frac{4C}{h} & 0 & 0 & 0 & -\frac{4C}{h} & 0 \\ 0 & \frac{4C}{h} & 0 & 0 & -\frac{4C}{h} & 0 \\ 0 & 0 & \frac{4C}{h} & 0 & -\frac{4C}{h} & 0 \\ -\frac{4C}{h} & -\frac{4C}{h} & -\frac{4C}{h} & 0 & \frac{12C}{h} & 0 \\ C & C & C & -\frac{h\cdot G}{6} & \frac{h\cdot G}{6}+3C & G\cdot L \\ C & C & C & 0 & -3C & \frac{h}{6}-g\cdot L \\ -\frac{5C}{2h} & 0 & 0 & 0 & \frac{5C}{2h} & 0 \\ 0 & -\frac{5C}{2h} & 0 & 0 & \frac{5C}{2h} & 0 \\ 0 & 0 & -\frac{5C}{2h} & 0 & \frac{5C}{2h} & 0 \\ \frac{5C}{2h} & \frac{5C}{2h} & \frac{5C}{2h} & 0 & -\frac{15C}{2h} & 0 \\ C & C & C & -\frac{5h\cdot G}{24} & \frac{5h\cdot G}{24}-3C & G\cdot L \\ C & C & C & 0 & -3C & \frac{5h}{24}-g\cdot L \end{bmatrix}$$

$$M_{eq} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The scaling factors are:

$$X_{scale} = \begin{bmatrix} 115000 & 115000 & 115000 & 115000 & 115000 & 80 & 115000 & 115000 & 115000 & 115000 & 115000 & 80 \end{bmatrix}^T$$

$$I_{scale} = \begin{bmatrix} 230 & 230 & 230 & 230 & 1 & 1 & 230 & 230 & 230 & 230 & 1 & 1 \end{bmatrix}^T$$

The terminal nodes are:

CAPBANK_A
CAPBANK_B
CAPBANK_C
CAPBANK_N

For this model no inequalities are imposed.

A number of example events and the performance of the setting-less protective relay for these events are discussed below.

4.3.1. Capacitor Bank: Test case 1 – Internal fault 1

The test system is shown in Figure 4.7. The capacitor bank is shown in the middle lower part of the diagram. It is a 115 kV, 48 MVAr capacitor bank as described above. This case involves an internal fault in the capacitor bank. The internal construction of the capacitor bank is shown in Figure 4.8. Figure 4.8 also illustrates the location of the measurements. This capacitor bank has the SCAQCF model described earlier.

The test case 1 event involves the following. The system operates under normal conditions when an internal fault occurs inside the capacitor bank which changes the capacitance of phase C. The net capacitance of phase C changes from 4.8μF to 2.4μF because of the internal fault. The fault occurs at time 35.0 sec. The entire simulation lasts 50 seconds. Several other events occur during this period but in this example we focus on the internal fault at time 35.0 seconds. The long simulation time was used for testing the setting-less protective relay in the laboratory. Here we are focused on the performance of the relay before, during the fault and after the fault at time 35.0 seconds. Therefore we present the results over the period 33.5 seconds to 37.5 seconds.

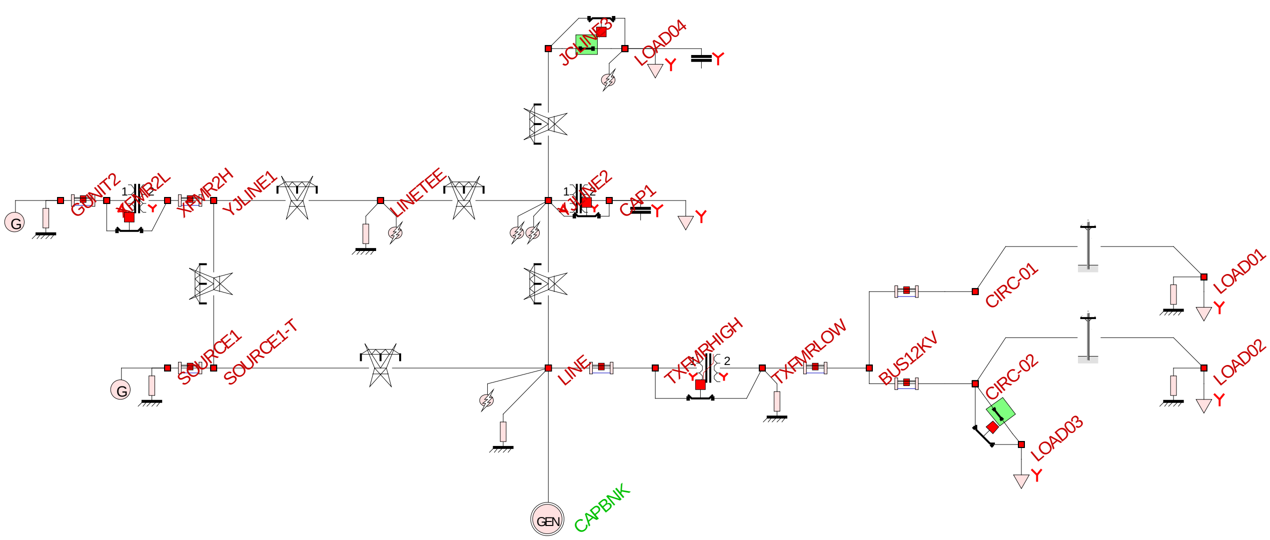

Figure 4.7: Test System for Capacitor Bank Simulations

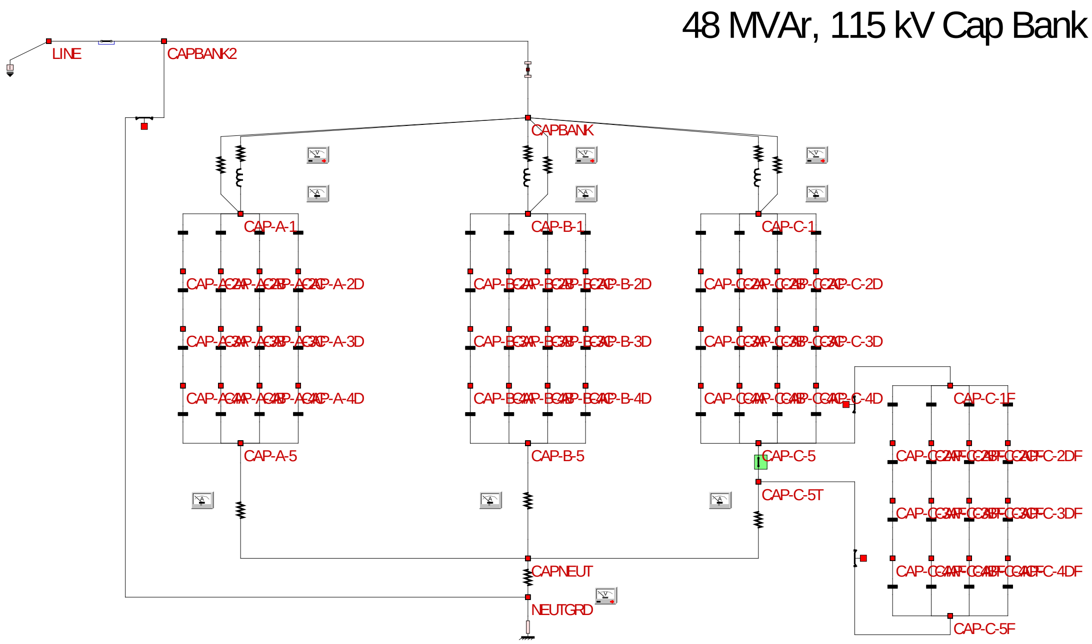

Figure 4.8: Topology of the Capacitor Bank and Location of Measurements

The results of DSE based protection for the period 33.5 to 37.5 seconds are shown in Figure 4.9. Specifically the figure shows seven sets of traces. The upper set of traces shows the actual and estimated values of the capacitor bank voltage of phase C to ground. The second set of traces shows the actual and estimated values of the capacitor bank current of phase C. The third trace shows the residual of the capacitor bank voltage of phase C. The forth trace shows the same residual in its normal rating. The fifth trace shows the sum of the normalized residuals (chi-square variable). The sixth trace shows the confidence level computed by the state estimator analysis of the residuals. Note that the confidence level clearly shows an internal abnormality of the capacitor bank during the internal fault. The seventh trace shows the execution time of the state estimator. Note that the execution time is less than 29 microseconds.

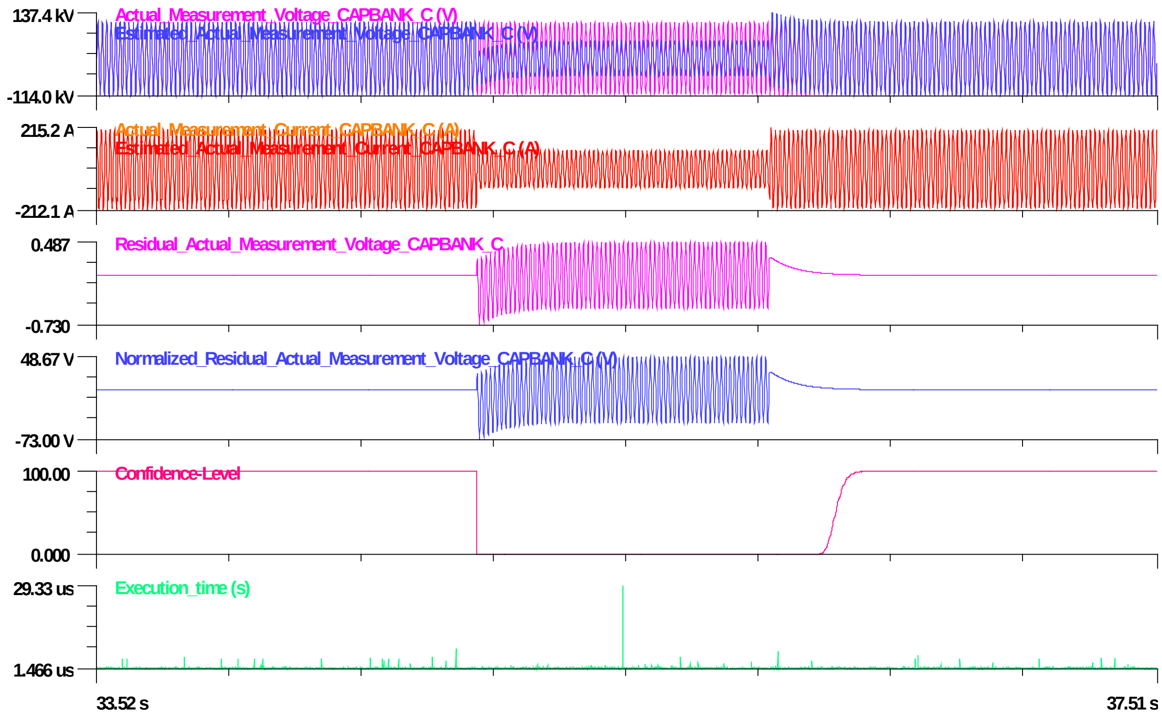

Figure 4.9: Results of the Capacitor Bank Setting-less Protection

4.3.2 Capacitor Bank: Test case 2 – Internal fault 2

This case involves another internal fault in the capacitor bank. The only difference between this case and the previous case is that the internal fault in this case only slightly changes the net capacitance of phase C. Specifically, another capacitor element is accidently added to one string in phase C capacitor can which changes the string capacitance from 1.2μF to 0.96μF. As a result, the net capacitance of phase C is 4.56μF instead of the normal value 4.8μF. The net capacitance change caused by the internal fault is really small compared with the test case 1. The entire system is the same as Figure 4.7. The construction of the capacitor bank is shown if Figure 4.10. Figure 4.10 also illustrates the location of the measurements.

The test case 2 event also involves the same external events as described in case 1. The internal fault still occurs at 35.0 second. Again, since we are focused on the performance of the relay before, during the fault and after the fault, we only present the results over the period 33.5 seconds to 37.5 seconds.

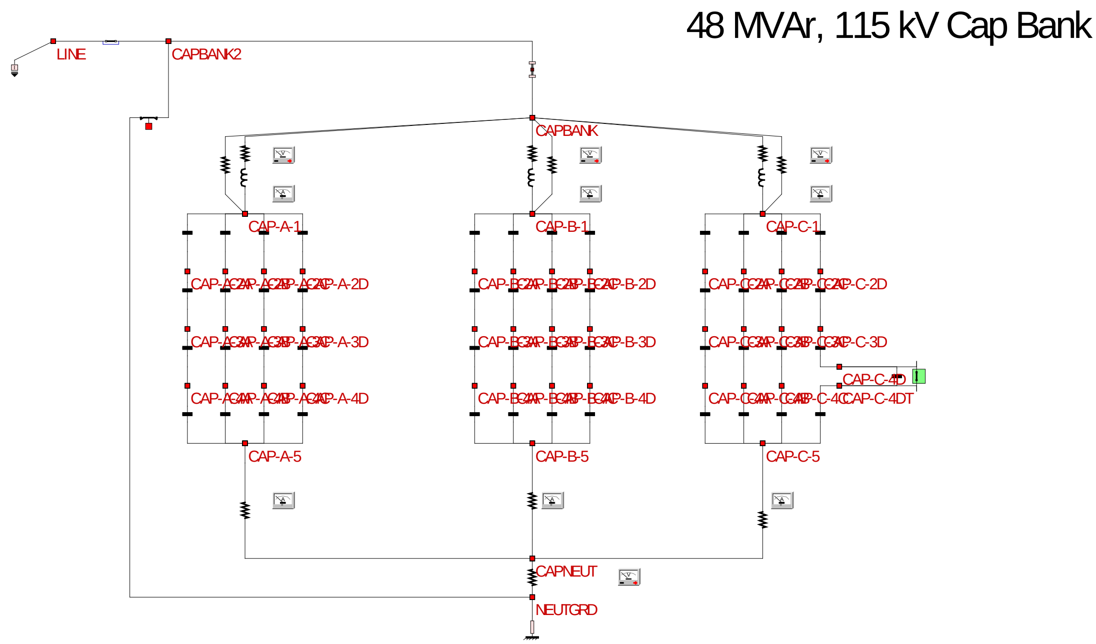

Figure 4.10: Topology of the Capacitor Bank and Location of Measurements

The results of DSE based protection for the period 33.5 to 37.5 seconds are shown below in Figure 4.11. Same as the test case 1 results, the same seven sets of traces are shown in Figure 5.11. Different from case 1 which confidence level is all zero during the internal fault period, in this case the confidence level has some oscillations during the internal fault. The reason for this phenomenon is because the net capacitance change caused by the fault is very slight, i.e., from 4.8μF to 4.56μF. An integral function could be applied to smooth the waveform so that the confidence level will stay low when the internal fault happens. Note that the execution time is very similar to the previous test case 1 and it is less than 41 microseconds.

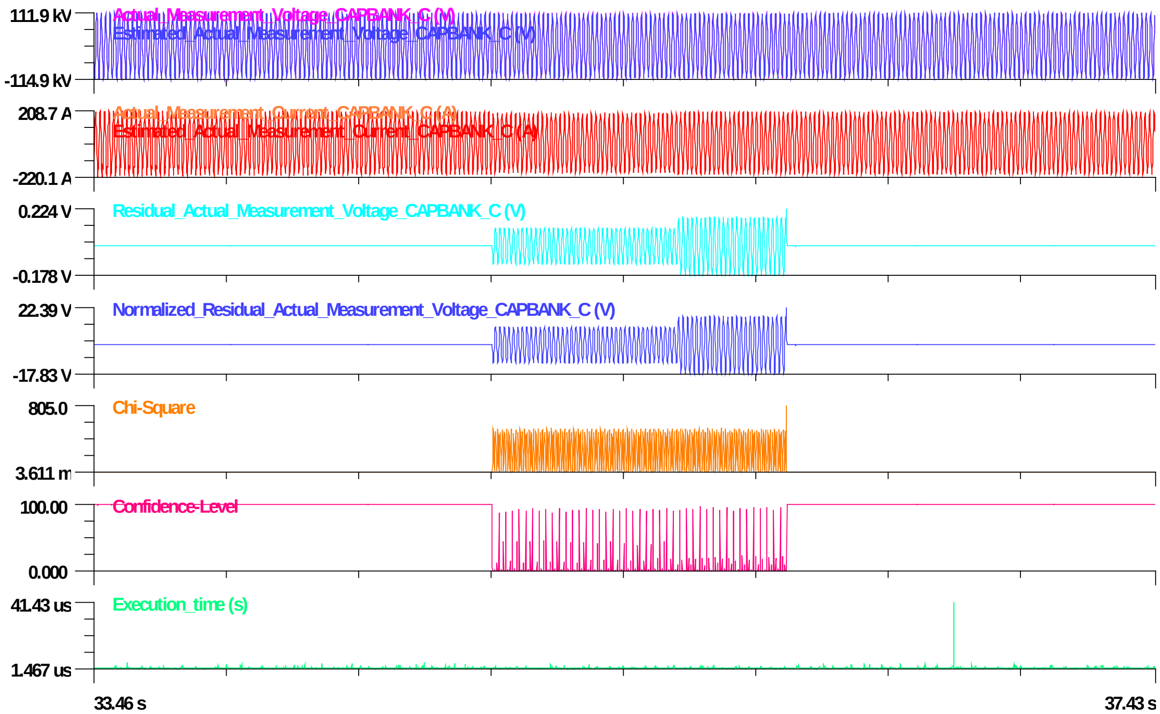

Figure 4.11: Results of the Capacitor Bank Setting-less Protection

4.4. Application to Single Section Transmission Line Protection

The setting-less protection algorithm was applied to a short line as well as a long line. A short line is represented with a single pi equivalent transmission line model shown in Figure 4.12. The figure also shows the typical data acquisition system for relaying applications. It is recommended (as implemented in laboratory) that merging units be used to reduce signal degradation and associated process bus. In this way the setting-less protective relay can be simply connected to the process bus. Note that since the terminals of the line are remote from each other, the measurements must be transmitted through communication lines. All the available measurements should be included in the analytics of the protection function for the transmission line. We briefly describe the constituent parts of the setting-less protection for the transmission line and then present an example results and execution time result.

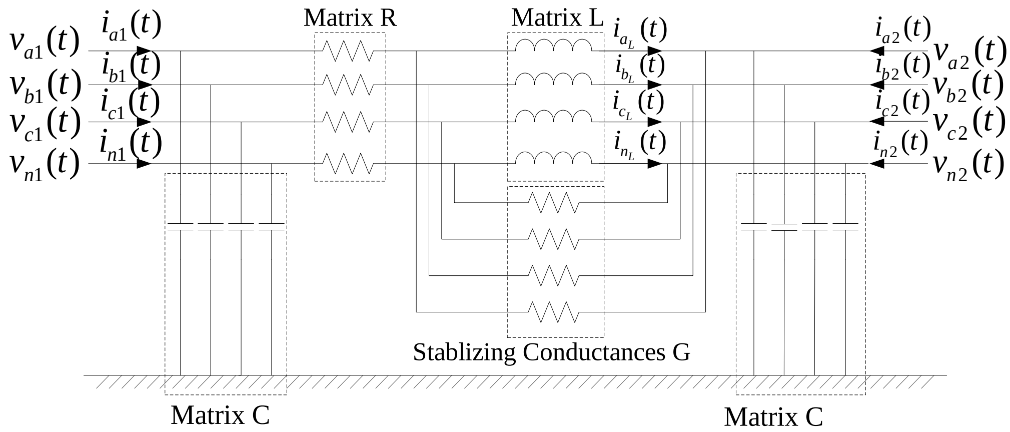

Figure 4.12: Three Phase Short Transmission Line Compact Model

Mathematical Model: The mathematical model is presented for a specific transmission line. The example transmission line ratings are: 115 kV, active power rating 100 MW. The protection instrumentation includes the following measurements: terminal voltage measurements (phase to neutral), terminal current measurements.

The derivation of the SCAQCF device model for the single section transmission line of Figure 4.12 and associated measurements is provided in Appendix D. The SCAQCF model is shown below in compact form:

$$\begin{Bmatrix} I(\mathbf{x},\mathbf{u}) \\ \vdots \\ 0 \\ \vdots \end{Bmatrix} = Y_{eqx}\mathbf{x} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{eqx}^i \mathbf{x} \\ \vdots \end{Bmatrix} + Y_{equ}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F_{equ}^i \mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{eqxu}^i \mathbf{u} \\ \vdots \end{Bmatrix} - B_{eq}$$

$$B_{eq} = -N_{eqx}\mathbf{x}(t-h) - N_{equ}\mathbf{u}(t-h) - M_{eq}I(t-h) - K_{eq}$$

$$\mathbf{h}(\mathbf{x},\mathbf{u}) = Y_{opx}\mathbf{x} + Y_{opu}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{opx}^i \mathbf{x} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F_{opu}^i \mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{opxu}^i \mathbf{u} \\ \vdots \end{Bmatrix} - B_{op}$$

Scaling factors: $Iscale,\ Xscale\ and\ Uscale$

Connectivity: $TerminalNodeName$

$$subject\ to: \quad \mathbf{h}_{\min} \le \mathbf{h}(\mathbf{x},\mathbf{u}) \le \mathbf{h}_{\max}$$
$$\mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max}$$

where:

$$
Y_{eqx} = \begin{bmatrix}
\frac{4}{h}C & 0 & I_4 + \frac{4}{h}GL & -\frac{8}{h}C & 0 & -\frac{8}{h}GL \\
0 & \frac{4}{h}C & -I_4 - \frac{4}{h}GL & 0 & -\frac{8}{h}C & \frac{8}{h}GL \\
-I_4 & I_4 & R + \frac{4}{h}(RGL + L) & 0 & 0 & -\frac{8}{h}(RGL + L) \\
\frac{1}{2h}C & 0 & \frac{1}{2h}GL & \frac{2}{h}C & 0 & I_4 + \frac{2}{h}GL \\
0 & \frac{1}{2h}C & -\frac{1}{2h}GL & 0 & \frac{2}{h}C & -I_4 - \frac{2}{h}GL \\
0 & 0 & \frac{1}{2h}(RGL + L) & -I_4 & I_4 & R + \frac{2}{h}(RGL + L)
\end{bmatrix}
$$

$$
N_{eqx} = \begin{bmatrix}
\frac{8}{h}C & 0 & I_4 - \frac{4}{h}GL \\
0 & \frac{8}{h}C & -I_4 + \frac{4}{h}GL \\
-I_4 & I_4 & R - \frac{4}{h}(RGL + L) \\
\frac{5}{2h}C & 0 & -\frac{1}{2}I_4 + \frac{5}{2h}GL \\
0 & \frac{5}{2h}C & \frac{1}{2}I_4 - \frac{5}{2h}GL \\
\frac{1}{2}I_4 & -\frac{1}{2}I_4 & -\frac{1}{2}R + \frac{5}{2h}(RGL + L)
\end{bmatrix}
$$

$$
M_{eq} = \begin{bmatrix}
I_4 & 0 & 0 \\
0 & I_4 & 0 \\
0 & 0 & 0 \\
-\frac{1}{2}I_4 & 0 & 0 \\
0 & -\frac{1}{2}I_4 & 0 \\
0 & 0 & 0
\end{bmatrix}
$$

The scaling factors are:

$$
X_{scale} = \begin{bmatrix}
115000 & 115000 & 115000 & 115000 & 115000 & 115000 & 115000 & 115000 & 500 & 500 & 500 & 500 \\
115000 & 115000 & 115000 & 115000 & 115000 & 115000 & 115000 & 115000 & 500 & 500 & 500 & 500
\end{bmatrix}^T
$$

$$
I_{scale} = \begin{bmatrix}
500 & 500 & 500 & 500 & 500 & 500 & 500 & 500 & 115 & 115 & 115 & 115 \\
500 & 500 & 500 & 500 & 500 & 500 & 500 & 500 & 115 & 115 & 115 & 115
\end{bmatrix}^T
$$

The terminal nodes are:

YJLINE1_A
YJLINE1_B
YJLINE1_C
YJLINE1_N
YJLINE2_A
YJLINE2_B
YJLINE2_C
YJLINE2_N

For this model no inequalities are imposed.

A number of example events and the performance of the setting-less protective relay for these events are discussed below.

4.4.1. Single Section Transmission Line: Test case - High Impedance Fault, Low Impedance Fault, External Fault, External Breaker Operation

The test system is shown in Figure 4.13. The transmission line is framed by a blue rectangle. It is a 115 kV, 100 MW line as described. This case involves an internal high impedance fault, an internal low impedance fault, an external fault and an external breaker operation on the transmission line. The configuration of the transmission line is shown in Figure 4.14. Figure 4.13 also illustrates the location of the measurements. This transmission line has the SCAQCF model described earlier.

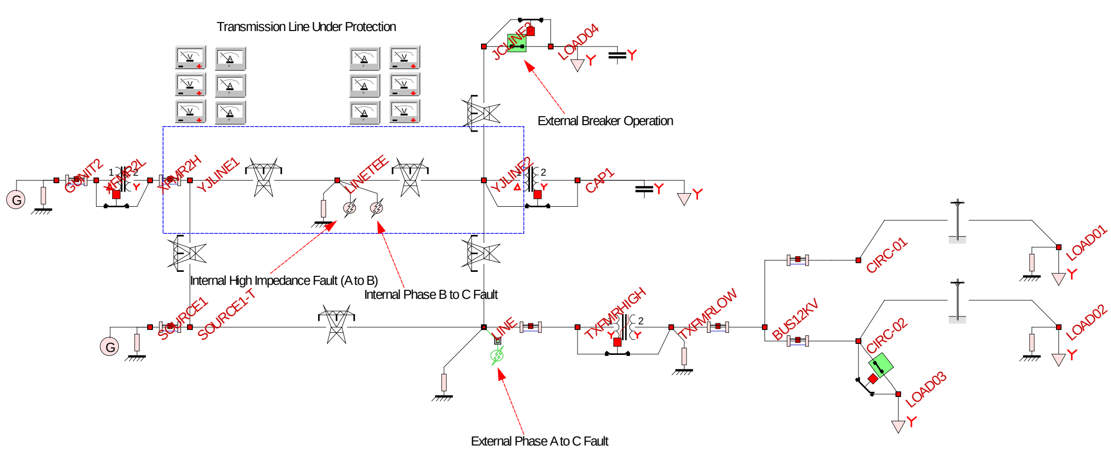

Figure 4.13: Test System for Transmission Line Simulations

The test case event involves the following: The entire simulation lasts 2 seconds. The system operates under normal conditions until the high impedance fault. The high impedance fault (fault conductance is 0.002 mhos) happens at 0.6 sec and cleared at 0.8 sec. The external phase A-C fault happens at 1.0 sec and cleared at 1.2 sec. The low impedance fault (fault conductance is 20 mhos) happens at 1.4 sec and cleared at 1.6 sec. The external breaker operation closes the breaker at 1.8 second.

Figure 4.14: Configuration of the Transmission Line

The results of DSE based protection are shown below in Figure 4.15. Specifically the figure shows six sets of traces. The upper set of traces shows the measured and estimated values of the transmission line voltage of phase A to neutral. The second set of traces shows the measured and estimated values of the transmission line current of phase A. The third trace shows the normalized residual as the difference between measured and estimated values of the transmission line voltage of phase A to neutral. The fourth trace shows the normalized residual as the difference between measured and estimated values of the transmission line current of phase A. The fifth trace shows the confidence level computed by the state estimator analysis of the residuals. Note that the confidence level clearly shows an internal abnormality of the transmission line during the internal faults. The sixth trace shows the execution time of the state estimator. Note that the average time is around 30 microseconds:

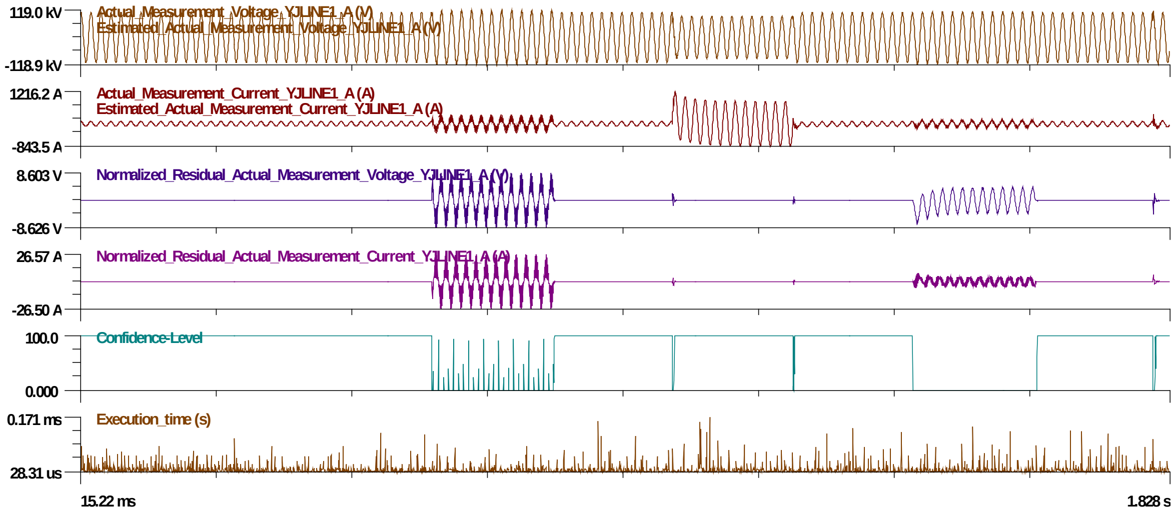

Figure 4.15: Results of the Singe Section Transmission Line Setting-less Protection

Interestingly, since the high impedance internal fault is really small, thus the confidence level has some oscillations during the fault. An integral function could be applied to smooth the wave. On the other hand, the confidence level stays zero for the low impedance internal fault.

4.5. Application to Multi-section Transmission Line Protection

The setting-less protection algorithm was applied to a long line. The long line is connected with some short lines. Each short line is represented with a single pi equivalent transmission line model shown in Figure 4.16. The figure also shows the typical data acquisition system for relaying applications. It is recommended (as implemented in laboratory) that merging units be used to reduce signal degradation and associated process bus. In this way the setting-less protective relay can be simply connected to the process bus. Note that since the terminals of the line are remote from each other, the measurements must be transmitted through communication lines. All the available measurements should be included in the analytics of the protection function for the transmission line. We briefly describe the constituent parts of the setting-less protection for the transmission line and then present an example results and execution time result.

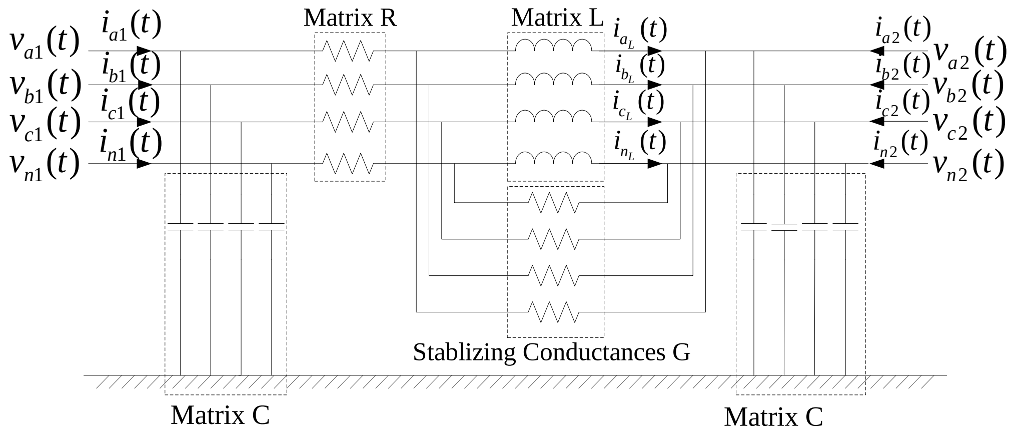

Figure 4.16: Three Phase Short Transmission Line Compact Model

Mathematical Model: The mathematical model is presented for a specific multi-section line, with n sections. The protection instrumentation includes the following measurements: 6 terminal voltage measurements (phase to neutral) and 6 terminal current measurements. The simplified three phase multi-section transmission line model is shown in Figure 4.17. Each section (1~n) represents a single-section line model.

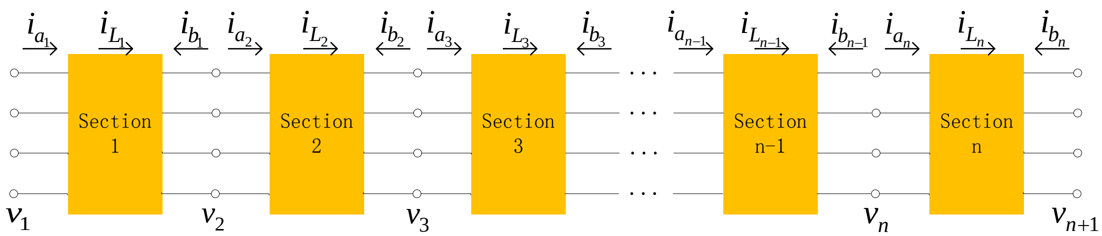

Figure 4.17: Three Phase Multi-section Transmission Line Compact Model

Parameter definition for section i :

Parameters at time t are:

$i_{a_i}(t)$ and $i_{b_i}(t)$ represent 3-phase & neutral current at both sides of section i, at time t.

$v_i(t)$ and $v_{i+1}(t)$ represent 3-phase & neutral voltage at both sides of section i, at time t.

$i_{L_i}(t)$ represents three-phase & neutral current of the inductance in section i, at time t.

Parameters at time t_m are:

$i_{a_i}(t_m)$ and $i_{b_i}(t_m)$ represent 3-phase & neutral current at both sides of section i, at time t_m.

$v_i(t_m)$ and $v_{i+1}(t_m)$ represent 3-phase & neutral voltage at both sides of section i, at time t_m.

$i_{L_i}(t_m)$ represents three-phase & neutral current of the inductance in section i, at time t_m.

The derivation of the SCAQCF device model for the three phase multi-section transmission line of Figure 4.17 and associated measurements is provided in Appendix E. The SCAQCF model is shown below in compact form:

$$\begin{Bmatrix} I(\mathbf{x},\mathbf{u}) \\ \vdots \\ 0 \\ \vdots \end{Bmatrix} = Y_{eqx}\mathbf{x} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{eqx}^i \mathbf{x} \\ \vdots \end{Bmatrix} + Y_{equ}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F_{equ}^i \mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{eqxu}^i \mathbf{u} \\ \vdots \end{Bmatrix} - B_{eq}$$

$$B_{eq} = -N_{eqx}\mathbf{x}(t-h) - N_{equ}\mathbf{u}(t-h) - M_{eq}I(t-h) - K_{eq}$$

$$\mathbf{h}(\mathbf{x},\mathbf{u}) = Y_{opx}\mathbf{x} + Y_{opu}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{opx}^i \mathbf{x} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F_{opu}^i \mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{opxu}^i \mathbf{u} \\ \vdots \end{Bmatrix} - B_{op}$$

Scaling factors: $Iscale,\ Xscale\ and\ Uscale$

Connectivity: $TerminalNodeName$

$$subject\ to: \quad \mathbf{h}_{\min} \le \mathbf{h}(\mathbf{x},\mathbf{u}) \le \mathbf{h}_{\max}$$

$$\mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max}$$

where:

$$Y_{eqx} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \quad N_{eqx} = -\begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \\ N_{31} & N_{32} \\ N_{41} & N_{42} \end{bmatrix} \quad M_{eq} = \begin{bmatrix} I_{8\times8} & 0 \\ 0 & 0 \\ -0.5I_{8\times8} & 0 \\ 0 & 0 \end{bmatrix}$$

And,

$$Y_{11} = \begin{bmatrix} M_{11} & 0 & M_{12} & 0 & 0 & \cdots & 0 & 0 \\ 0 & M_{22} & 0 & 0 & 0 & \cdots & 0 & M_{21} \\ M_{21} & 0 & M_{22}+M_{11} & M_{12} & 0 & \cdots & 0 & 0 \\ 0 & 0 & M_{21} & M_{22}+M_{11} & M_{12} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & M_{12} & 0 & 0 & 0 & \cdots & M_{21} & M_{22}+M_{11} \end{bmatrix}_{(4n+4)\times(4n+4)}$$

$$Y_{13} = \begin{bmatrix} M_{14} & 0 & M_{15} & 0 & 0 & \cdots & 0 & 0 \\ 0 & M_{25} & 0 & 0 & 0 & \cdots & 0 & M_{24} \\ M_{24} & 0 & M_{25}+M_{14} & M_{15} & 0 & \cdots & 0 & 0 \\ 0 & 0 & M_{24} & M_{25}+M_{14} & M_{15} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & M_{15} & 0 & 0 & 0 & \cdots & M_{24} & M_{25}+M_{14} \end{bmatrix}_{(4n+4)\times(4n+4)}$$

$$Y_{12} = \begin{bmatrix} M_{13} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & M_{23} \\ M_{23} & M_{13} & 0 & \cdots & 0 & 0 \\ 0 & M_{23} & M_{13} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & M_{23} & M_{13} \end{bmatrix}_{(4n+4)\times(4n)} \quad Y_{14} = \begin{bmatrix} M_{16} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & M_{26} \\ M_{26} & M_{16} & 0 & \cdots & 0 & 0 \\ 0 & M_{26} & M_{16} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & M_{26} & M_{16} \end{bmatrix}_{(4n+4)\times(4n)}$$

$$Y_{21}=\begin{bmatrix} M_{31} & 0 & M_{32} & 0 & \cdots & 0 \\ 0 & 0 & M_{31} & M_{32} & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & M_{32} & 0 & 0 & \cdots & M_{31} \end{bmatrix}_{(4n)\times(4n+4)} \qquad Y_{22}=\begin{bmatrix} M_{33} & 0 & 0 & 0 \\ 0 & M_{33} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & M_{33} \end{bmatrix}_{(4n)\times(4n)}$$

$$Y_{23}=\begin{bmatrix} M_{34} & 0 & M_{35} & 0 & \cdots & 0 \\ 0 & 0 & M_{34} & M_{35} & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & M_{35} & 0 & 0 & \cdots & M_{34} \end{bmatrix}_{(4n)\times(4n+4)} \qquad Y_{24}=\begin{bmatrix} M_{36} & 0 & 0 & 0 \\ 0 & M_{36} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & M_{36} \end{bmatrix}_{(4n)\times(4n)}$$

$$Y_{31}=\begin{bmatrix} M_{41} & 0 & M_{42} & 0 & 0 & \cdots & 0 & 0 \\ 0 & M_{52} & 0 & 0 & 0 & \cdots & 0 & M_{51} \\ M_{51} & 0 & M_{52}+M_{41} & M_{42} & 0 & \cdots & 0 & 0 \\ 0 & 0 & M_{51} & M_{52}+M_{41} & M_{42} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & M_{42} & 0 & 0 & 0 & \cdots & M_{51} & M_{52}+M_{41} \end{bmatrix}_{(4n+4)\times(4n+4)}$$

$$Y_{33}=\begin{bmatrix} M_{44} & 0 & M_{45} & 0 & 0 & \cdots & 0 & 0 \\ 0 & M_{55} & 0 & 0 & 0 & \cdots & 0 & M_{54} \\ M_{54} & 0 & M_{55}+M_{44} & M_{45} & 0 & \cdots & 0 & 0 \\ 0 & 0 & M_{54} & M_{55}+M_{44} & M_{45} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & M_{45} & 0 & 0 & 0 & \cdots & M_{54} & M_{55}+M_{44} \end{bmatrix}_{(4n+4)\times(4n+4)}$$

$$Y_{32}=\begin{bmatrix} M_{43} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & M_{53} \\ M_{53} & M_{43} & 0 & \cdots & 0 & 0 \\ 0 & M_{53} & M_{43} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & M_{53} & M_{43} \end{bmatrix}_{(4n+4)\times(4n)} \qquad Y_{34}=\begin{bmatrix} M_{46} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & M_{56} \\ M_{56} & M_{46} & 0 & \cdots & 0 & 0 \\ 0 & M_{56} & M_{46} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & M_{56} & M_{46} \end{bmatrix}_{(4n+4)\times(4n)}$$

$$Y_{41}=\begin{bmatrix} M_{61} & 0 & M_{62} & 0 & \cdots & 0 \\ 0 & 0 & M_{61} & M_{62} & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & M_{62} & 0 & 0 & \cdots & M_{61} \end{bmatrix}_{(4n)\times(4n+4)} \qquad Y_{42}=\begin{bmatrix} M_{63} & 0 & 0 & 0 \\ 0 & M_{63} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & M_{63} \end{bmatrix}_{(4n)\times(4n)}$$

$$Y_{43}=\begin{bmatrix} M_{64} & 0 & M_{65} & 0 & \cdots & 0 \\ 0 & 0 & M_{64} & M_{65} & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & M_{65} & 0 & 0 & \cdots & M_{64} \end{bmatrix}_{(4n)\times(4n+4)} \qquad Y_{44}=\begin{bmatrix} M_{66} & 0 & 0 & 0 \\ 0 & M_{66} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & M_{66} \end{bmatrix}_{(4n)\times(4n)}$$

$$N_{11}=\begin{bmatrix} P_{11} & 0 & P_{12} & 0 & 0 & \cdots & 0 & 0 \\ 0 & P_{22} & 0 & 0 & 0 & \cdots & 0 & P_{21} \\ P_{21} & 0 & P_{22}+P_{11} & P_{12} & 0 & \cdots & 0 & 0 \\ 0 & 0 & P_{21} & P_{22}+P_{11} & P_{12} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & P_{12} & 0 & 0 & 0 & \cdots & P_{21} & P_{22}+P_{11} \end{bmatrix}_{(4n+4)\times(4n+4)}$$

$$N_{31}=\begin{bmatrix} P_{41} & 0 & P_{42} & 0 & 0 & \cdots & 0 & 0 \\ 0 & P_{52} & 0 & 0 & 0 & \cdots & 0 & P_{51} \\ P_{51} & 0 & P_{52}+P_{41} & P_{42} & 0 & \cdots & 0 & 0 \\ 0 & 0 & P_{51} & P_{52}+P_{41} & P_{42} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & P_{42} & 0 & 0 & 0 & \cdots & P_{51} & P_{52}+P_{41} \end{bmatrix}_{(4n+4)\times(4n+4)}$$

$$N_{12}=\begin{bmatrix} P_{13} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & P_{23} \\ P_{23} & P_{13} & 0 & \cdots & 0 & 0 \\ 0 & P_{23} & P_{13} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & P_{23} & P_{13} \end{bmatrix}_{(4n+4)\times(4n)} \qquad N_{32}=\begin{bmatrix} P_{43} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & P_{53} \\ P_{53} & P_{43} & 0 & \cdots & 0 & 0 \\ 0 & P_{53} & P_{43} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & P_{53} & P_{43} \end{bmatrix}_{(4n+4)\times(4n)}$$

$$N_{21}=\begin{bmatrix} P_{31} & 0 & P_{32} & 0 & \cdots & 0 \\ 0 & 0 & P_{31} & P_{32} & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & P_{32} & 0 & 0 & \cdots & P_{31} \end{bmatrix}_{(4n)\times(4n+4)} \qquad N_{22}=\begin{bmatrix} P_{33} & 0 & 0 & 0 \\ 0 & P_{33} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & P_{33} \end{bmatrix}_{(4n)\times(4n)}$$

$$N_{41}=\begin{bmatrix} P_{61} & 0 & P_{62} & 0 & \cdots & 0 \\ 0 & 0 & P_{61} & P_{62} & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & P_{62} & 0 & 0 & \cdots & P_{61} \end{bmatrix}_{(4n)\times(4n+4)} \qquad N_{42}=\begin{bmatrix} P_{63} & 0 & 0 & 0 \\ 0 & P_{63} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & P_{63} \end{bmatrix}_{(4n)\times(4n)}$$

M_{ij} and P_{ij} are defined as follows:

$$E\cdot F_1=\begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} & M_{16} \\ M_{21} & M_{22} & M_{23} & M_{24} & M_{25} & M_{26} \\ M_{31} & M_{32} & M_{33} & M_{34} & M_{35} & M_{36} \\ M_{41} & M_{42} & M_{43} & M_{44} & M_{45} & M_{46} \\ M_{51} & M_{52} & M_{53} & M_{54} & M_{55} & M_{56} \\ M_{61} & M_{62} & M_{63} & M_{64} & M_{65} & M_{66} \end{bmatrix} \qquad E\cdot F_2=\begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \\ P_{41} & P_{42} & P_{43} \\ P_{51} & P_{52} & P_{53} \\ P_{61} & P_{62} & P_{63} \end{bmatrix}$$

$$E=\begin{bmatrix} \frac{4}{h}I_{12\times12} & -\frac{8}{h}I_{12\times12} \\ \frac{1}{2h}I_{12\times12} & \frac{2}{h}I_{12\times12} \end{bmatrix},\ F_1=\begin{bmatrix} \frac{h}{6}A+B & \frac{2h}{3}A \\ -\frac{h}{24}A & \frac{h}{3}A+B \end{bmatrix},\ F_2=\begin{bmatrix} B-\frac{h}{6}A \\ B-\frac{5h}{24}A \end{bmatrix}$$

$$A=\begin{bmatrix} 0 & 0 & I_{4\times4} \\ 0 & 0 & -I_{4\times4} \\ -I_{4\times4} & I_{4\times4} & R \end{bmatrix} \quad B=\begin{bmatrix} C & 0 & G\cdot L \\ 0 & C & -G\cdot L \\ 0 & 0 & R\cdot G\cdot L+L \end{bmatrix}$$

where R, L, C, G are the resistance, inductance, capacitance and stabilizing conductance matrices of the single-section transmission line (as shown in Figure 4.16).

An example event and the performance of the setting-less protective relay for the event is discussed below.

4.5.1 Multi-section Transmission Line: Test case – Internal fault

Figure 4.18 illustrates a transmission line system. The voltage and current measurements are installed at two terminals of the monitored TABL-VACA line, as shown in Figure 4.19 and 4.20. The ratings of the transmission line are: voltage rating 500 kV, capacity rating 3500 MVA. This case involves an internal fault in the transmission line.

This multi-section transmission line has the SCAQCF model described earlier. Take a 2-section transmission line as an example. The scaling factors are:

$$
\begin{aligned}
X_{scale} = [&500000 \quad 500000 \quad 500000 \quad 500000 \quad 500000 \quad 500000 \quad 500000 \quad 500000 \quad 500000 \quad 500000 \\
&500000 \quad 500000 \quad 4041 \quad 4041 \quad 4041 \quad 4041 \quad 4041 \quad 4041 \quad 4041 \quad 4041 \\
&500000 \quad 500000 \quad 500000 \quad 500000 \quad 500000 \quad 500000 \quad 500000 \quad 500000 \quad 500000 \quad 500000 \\
&500000 \quad 500000 \quad 4041 \quad 4041 \quad 4041 \quad 4041 \quad 4041 \quad 4041 \quad 4041 \quad 4041\ ]^T
\end{aligned}
$$

$$
\begin{aligned}
I_{scale} = [&4041 \quad 4041 \quad 4041 \quad 4041 \quad 4041 \quad 4041 \quad 4041 \quad 4041 \quad 4041 \quad 4041 \\
&4041 \quad 4041 \quad 500000 \quad 500000 \quad 500000 \quad 500000 \quad 500000 \quad 500000 \quad 500000 \quad 500000 \\
&4041 \quad 4041 \quad 4041 \quad 4041 \quad 4041 \quad 4041 \quad 4041 \quad 4041 \quad 4041 \quad 4041 \\
&4041 \quad 4041 \quad 500000 \quad 500000 \quad 500000 \quad 500000 \quad 500000 \quad 500000 \quad 500000 \quad 500000]^T
\end{aligned}
$$

The terminal nodes are:

TABL_A
TABL_B
TABL_C
TABL_N
VACA_A
VACA_B
VACA_C
VACA_N

For this model no inequalities are imposed.

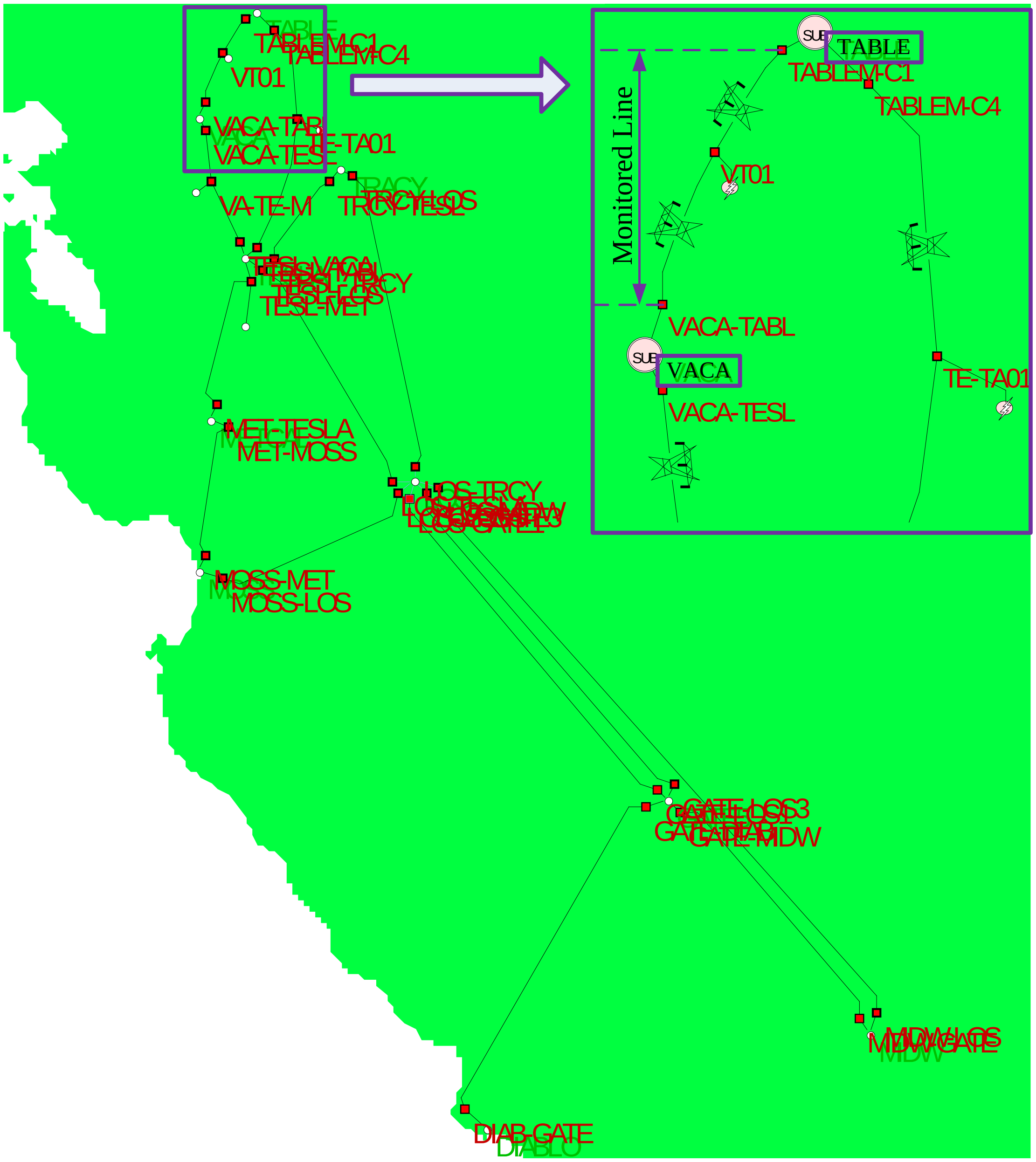

Figure 4.18: Test System for Multi-section Transmission Line

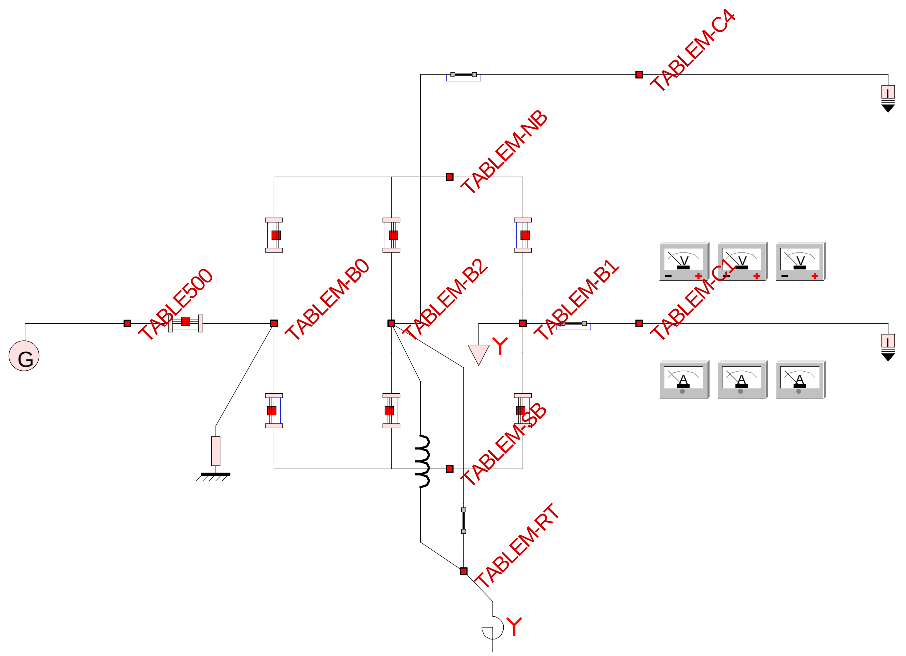

Figure 4.19: Location of Measurements at TABL side

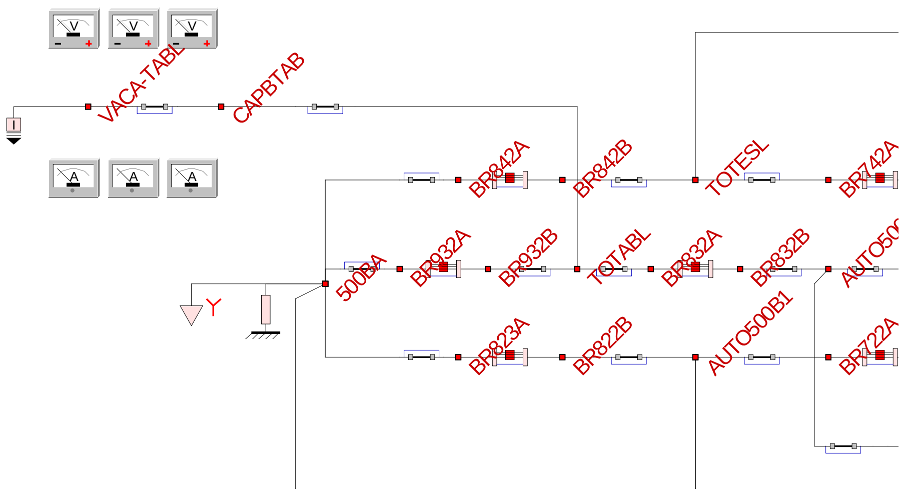

Figure 4.20: Location of Measurements at VACA side

The system operates under normal conditions when an internal fault occurs inside the transmission line. Several other events occur during this period but in this example we focus on the internal fault happens at 25.1 sec and clears at 26.0 sec. The long simulation time was used for testing the setting-less protective relay in the laboratory. Here we are focused on the performance of the relay before, during the fault and after the fault. Therefore we present the results over the period 24.7 seconds to 26.4 seconds.

The results of DSE based protection is shown below in Fig.4.21. Specifically the figure shows seven sets of traces. The upper set of traces show the actual and estimated values of the phase A to ground voltage at TABL side of multi-section transmission line. The second set of traces show the actual and estimated values of phase A current at TABL side. The third and fourth traces show the residuals and normalized residuals of TABL side voltage, respectively. The fifth trace shows the sum of the normalized residuals (chi-square variable). The sixth trace shows the confidence level computed by the state estimator analysis of the residuals. Note that the confidence level clearly shows an internal abnormality of the multi-section transmission line during the internal fault. The seventh trace shows the execution time of the state estimator. Note that the execution time is less than 198 microseconds.

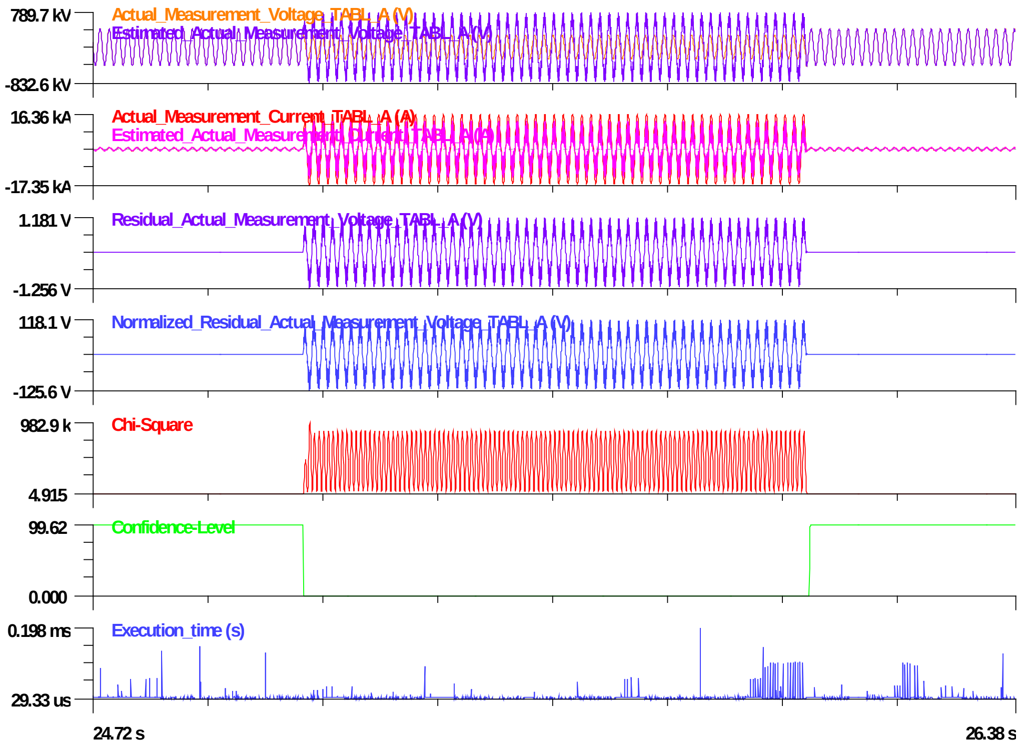

Figure 4.21: Results of the Three Phase Multi-section Transmission Line Setting-less Protection

4.6. Application to Single Phase Saturable Core Variable Tap Transformer Protection

The setting-less protection algorithm has been applied to a single phase saturable core variable tap transformer. The transformer model used for the SCAQCF mathematical model is shown Figure 4.22. The instrumentation of the transformer typically includes the terminal voltages and currents as well as other measurements such as neutral current. All the available measurements should be included in the analytics of the protection function for the transformer. We briefly describe the constituent parts of the setting-less protection for the transformer and then present an example results and execution time result.

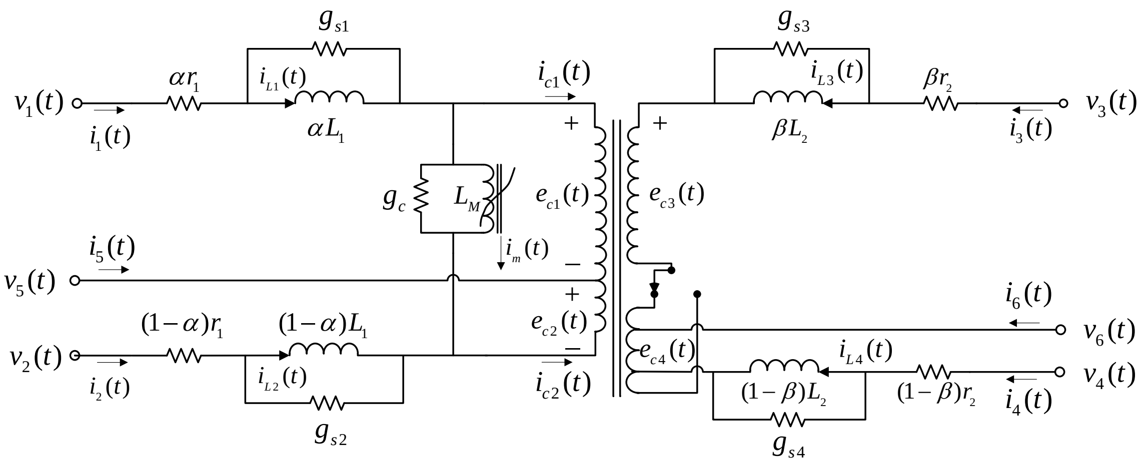

Figure 4.22: Single-Phase Transformer Compact Model

Mathematical Model: The mathematical model is presented for a specific single phase saturable core variable tap transformer. The example transformer ratings are: voltage rating is 25 kV, active power rating 300 kW, the nonlinear exponent is 5. The protection instrumentation includes the following measurements: terminal voltage measurements (phase to ground) and terminal current measurements.

The derivation of the SCAQCF device model for the single phase transformer of Figure 4.22 and associated measurements is provided in Appendix F. The SCAQCF model it is shown below:

$$\begin{Bmatrix} I(\mathbf{x},\mathbf{u}) \\ \vdots \\ 0 \\ \vdots \end{Bmatrix} = Y_{eqx}\mathbf{x} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{eqx}^i \mathbf{x} \\ \vdots \end{Bmatrix} + Y_{equ}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F_{equ}^i \mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{eqxu}^i \mathbf{u} \\ \vdots \end{Bmatrix} - B_{eq}$$

$$B_{eq} = -N_{eqx}\mathbf{x}(t-h) - N_{equ}\mathbf{u}(t-h) - M_{eq}I(t-h) - K_{eq}$$

$$\mathbf{h}(\mathbf{x},\mathbf{u}) = Y_{opx}\mathbf{x} + Y_{opu}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{opx}^i \mathbf{x} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F_{opu}^i \mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{opxu}^i \mathbf{u} \\ \vdots \end{Bmatrix} - B_{op}$$

Scaling factors: $Iscale,\ Xscale\ and\ Uscale$

Connectivity: $TerminalNodeName$

$$subject\ to:\ \mathbf{h}_{\min} \le \mathbf{h}(\mathbf{x},\mathbf{u}) \le \mathbf{h}_{\max}$$
$$\mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max}$$

where:

$$Y_{eqx} = \begin{bmatrix} Y_{eqx\_11} & Y_{eqx\_12} \\ Y_{eqx\_21} & Y_{eqx\_22} \end{bmatrix}_{52\times 52}$$

$$
Y_{eqx\_11}=\begin{bmatrix}
0&0&0&0&0&0&1&0&0&0&0&\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&-1&-1&0&0&0&-\frac{h}{5}&-\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&-1&-1&0&0&0&-\frac{h}{5}&-\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&-\frac{h}{6}&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&-\frac{h}{6}&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&-\frac{h}{6}&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&-\frac{h}{6}&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&-\frac{h}{6}&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&-1&0&-\alpha r_1&0&0&0&0&-\alpha L_1-\frac{\alpha r_1 h}{5}&0&0&0&0&-1&0&0&0&0&0&0&0&0&0\\
0&1&0&0&-1&0&0&(1-\alpha)r_1&0&0&0&0&(1-\alpha)(L_1+\frac{r_1 h}{5})&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&1&0&0&-1&0&0&-\beta r_2&0&0&0&0&-\beta L_2-\frac{\beta r_2 h}{5}&0&0&0&0&-1&0&0&0&0&0&0&0\\
0&0&0&1&0&-1&0&0&0&(1-\beta)r_2&0&0&0&0&(1-\beta)(L_2+\frac{r_2 h}{5})&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&\beta N_2&(\beta-1)N_2&0&0&0&\frac{\beta N_2 h}{5}&\frac{(\beta-1)N_2 h}{5}&0&0&0&0&0&\alpha N_1&(\alpha-1)N_1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\alpha&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\alpha-1&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{\beta N_2}{N_1}&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{(\beta-1)N_2}{N_1}&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&-1&0&0&0&0&-\frac{h}{5}&0&0&0&0&g_c&g_c&0&0&1&0&1&0&0&0\\
0&0&0&0&0&0&0&-1&0&0&0&0&-\frac{h}{5}&0&0&0&-g_c&-g_c&0&0&0&1&-1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&-i_0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{bmatrix}
$$

$$
Y_{eqx\_12} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2h}{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2h}{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2h}{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2h}{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2h}{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
Y_{eqx\_21} = \begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&\frac{h}{24}&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&\frac{h}{24}&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{h}{24}&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{h}{24}&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{h}{24}&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}
$$

$$
Y_{eqx\_22} = \begin{bmatrix}
0&0&0&0&0&0&1&0&0&0&0&\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&-1&-1&0&0&0&-\frac{h}{5}&-\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&-1&-1&0&0&0&-\frac{h}{5}&-\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&-\frac{h}{3}&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&-\frac{h}{3}&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&-\frac{h}{3}&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&-\frac{h}{3}&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&-\frac{h}{3}&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&-1&0&-\alpha r_1&0&0&0&0&-\alpha L_1-\frac{\alpha r_1 h}{5}&0&0&0&0&-1&0&0&0&0&0&0&0&0&0\\
0&1&0&0&-1&0&0&(1-\alpha)r_1&0&0&0&0&(1-\alpha)(L_1+\frac{r_1 h}{5})&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&1&0&0&-1&0&0&-\beta r_2&0&0&0&0&-\beta L_2-\frac{\beta r_2 h}{5}&0&0&0&0&-1&0&0&0&0&0&0&0\\
0&0&0&1&0&-1&0&0&0&(1-\beta)r_2&0&0&0&0&(1-\beta)(L_2+\frac{r_2 h}{5})&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&\beta N_2&(\beta-1)N_2&0&0&0&\frac{\beta N_2 h}{5}&\frac{(\beta-1)N_2 h}{5}&0&0&0&0&0&\alpha N_1&(\alpha-1)N_1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\alpha&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\alpha-1&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{\beta N_2}{N_1}&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{(\beta-1)N_2}{N_1}&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&-1&0&0&0&0&-\frac{h}{5}&0&0&0&0&g_c&g_c&0&0&1&0&1&0&0&0\\
0&0&0&0&0&0&0&-1&0&0&0&0&-\frac{h}{5}&0&0&0&-g_c&-g_c&0&0&0&1&-1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&-i_0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{bmatrix}
$$

$$F_{eq0} = \cdots = F_{eq22} = 0_{52\times 52}$$

$$F_{eq23}: f_{eq[10\times 10]} = -\frac{1}{\lambda_0^2}, \qquad other\ elements\ are\ 0$$

$$F_{eq24}: f_{eq[23\times 23]} = -1, \qquad other\ elements\ are\ 0$$

$$F_{eq25}: f_{eq[24\times 20]} = -1, \qquad other\ elements\ are\ 0$$

$$F_{eq26} = \cdots = F_{eq48} = 0_{52\times 52}$$

$$F_{eq49}: f_{eq[36\times 36]} = -\frac{1}{\lambda_0^2}, \qquad other\ elements\ are\ 0$$

$$F_{eq50}: f_{eq[49\times 49]} = -1, \qquad other\ elements\ are\ 0$$

$$F_{eq51}: f_{eq[50\times 36]} = -1, \qquad other\ elements\ are\ 0$$

$$N_{eqx} = \begin{bmatrix} N_{eqx\_1} \\ N_{eqx\_2} \end{bmatrix}_{52\times 26}$$

$$
N_{eqx\_1} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -\frac{h}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -\frac{h}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -\frac{h}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -\frac{h}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -\frac{h}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
N_{eqx\_1} = \begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&-1&0&0&0&0&-\frac{5h}{24}&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
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0&0&0&0&0&0&0&0&-1&0&0&0&0&-\frac{5h}{24}&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&-1&0&0&0&0&-\frac{5h}{24}&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&-\frac{5h}{24}&0&0&0&0&0&0&0&0&0&0 \\
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0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
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0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}
$$

The scaling factors are:

$$X_{scale} = \begin{bmatrix} X_{scale\_1} & X_{scale\_2} & X_{scale\_3} & X_{scale\_4} \end{bmatrix}^T_{52}$$
$$X_{scale\_1} = \begin{bmatrix} 2.5e4 & 2.5e4 & 2.5e4 & 500 & 500 & 500 & 10 & 10 & 500 & 500 & 100 & 4.0e3 & 4.0e3 \end{bmatrix}$$
$$X_{scale\_2} = \begin{bmatrix} 4.0e4 & 4.0e4 & 2.5e4 & 2.5e4 & 2.5e4 & 500 & 500 & 10 & 10 & 1 & 1 & 1 & 1 \end{bmatrix}$$
$$X_{scale\_3} = X_{scale\_1}$$
$$X_{scale\_4} = X_{scale\_2}$$
$$I_{scale} = \begin{bmatrix} I_{scale\_1} & I_{scale\_2} & I_{scale\_3} & I_{scale\_4} \end{bmatrix}^T_{52}$$
$$I_{scale\_1} = \begin{bmatrix} 10 & 10 & 500 & 500 & 10 & 500 & 10 & 10 & 100 & 100 & 100 & 10000 & 10000 \end{bmatrix}$$
$$I_{scale\_2} = \begin{bmatrix} 500 & 500 & 10 & 10000 & 10000 & 500 & 500 & 100 & 100 & 1 & 1 & 1 & 1 \end{bmatrix}$$
$$I_{scale\_3} = I_{scale\_1}$$
$$I_{scale\_4} = I_{scale\_2}$$

The terminal nodes are:
XFMRH_A
XFMRH_N
XFMRL_A
XFMRL_N

A number of example events and the performance of the setting-less protective relay for these events are discussed below.

4.6.1. Transformer: Test case 1- Energization

This case involves an energization situation in single phase saturable core transformer. The system is the shown in Figure 4.23. The single phase transformer is 25kV, 300 kW as described above. In this example, the protection of single phase saturable core transformer in an energization situation is present. The transformer suddenly connects to the system at t= 0.4s and an energization happens to the transformer. Since the energization is not a fault it should not trip the transformer. Later the load connects the transformer and a transient will occur, but the transformer should not be tripped.

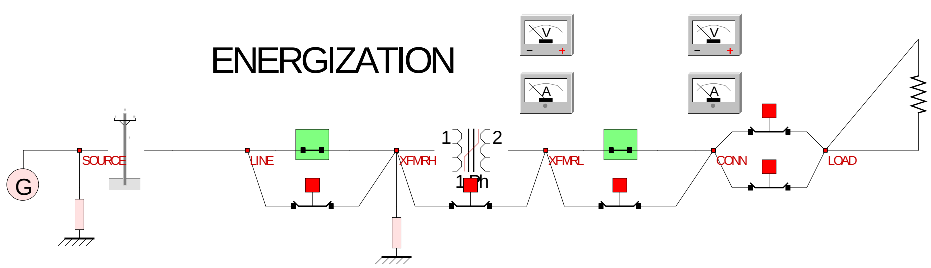

Figure 4.23: Energization Situation for Single Phase Transformer

The results of DSE based protection for the period 0 to 0.9 seconds are shown in Figure 4.24. Specifically the figure shows six sets of traces. The upper trace shows the actual left side terminal voltages of the transformer. The second trace shows the estimated left side terminal voltages of the transformer. The third trace shows the residual of the voltage while the forth trace shows the normalized residual of the left side terminal voltage. The fifth trace shows the sum of the normalized residuals (chi-square variable). The sixth trace shows the confidence level computed by the state estimator analysis of the residuals.

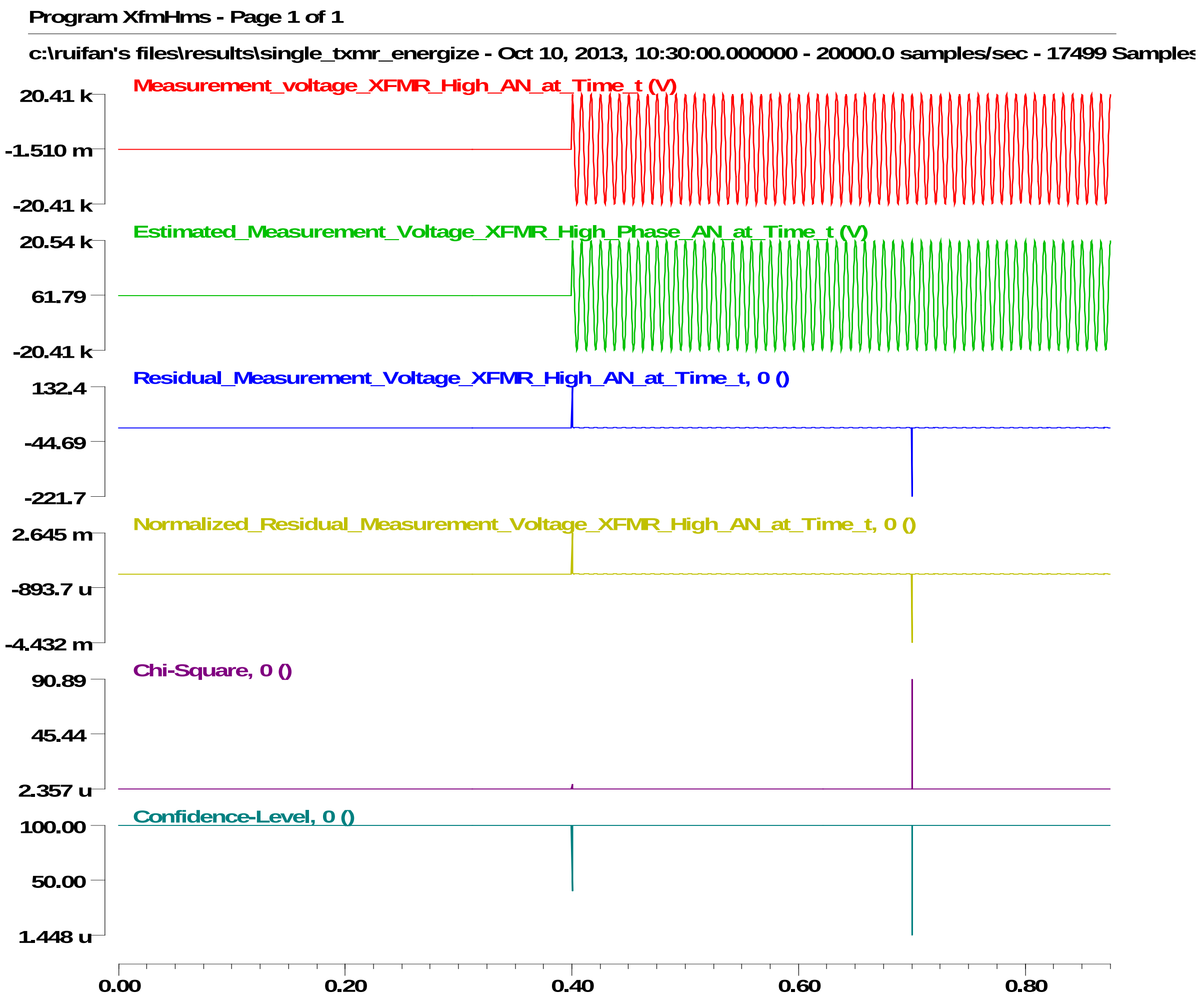

Figure 4.24: Results of Setting-less Protection for Transformer Energization Situation

Seen from the figure, at t=0.4s the confidence level dropped to zero and returned to 100% immediately, which means there is no fault and transformer should not be tripped. This also happened when the load was suddenly connected to system at 0.7 second. The setting-less protection detected it was not an internal fault so that it did not trip the transformer

4.6.2. Transformer: Test case 2- Source Switching

This case involves a source switching situation in single phase saturable core transformer system. The system is the shown in Figure 4.25. The single phase transformer is 25kV, 300 kW as described above. In this example, the protection of single phase saturable core transformer in a source switching situation is present. The transformer connects to one source of the system at t=0s and suddenly it changes to another source at t=0.2s, which has an angle difference with the former one. Since it is not a fault so that transformer should not be tripped.

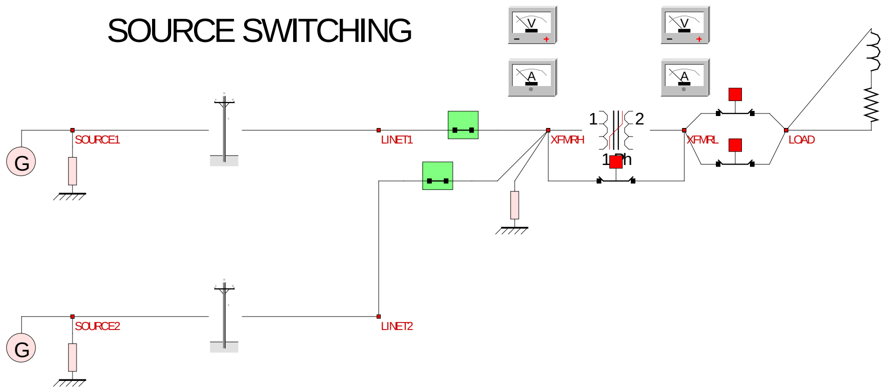

Figure 4.25: Source Switching Situation for Single Phase Transformer

The results of DSE based protection for the period 0 to 0.3 seconds are shown Figure 4.26. Specifically the figure shows six sets of traces. The upper trace shows the actual left side terminal voltages of the transformer. The second trace shows the estimated left side terminal voltages of the transformer. The third trace shows the residual of the voltage while the forth trace shows the normalized residual of the left side terminal voltage. The fifth trace shows the sum of the normalized residuals (chi-square variable). The sixth trace shows the confidence level computed by the state estimator analysis of the residuals.

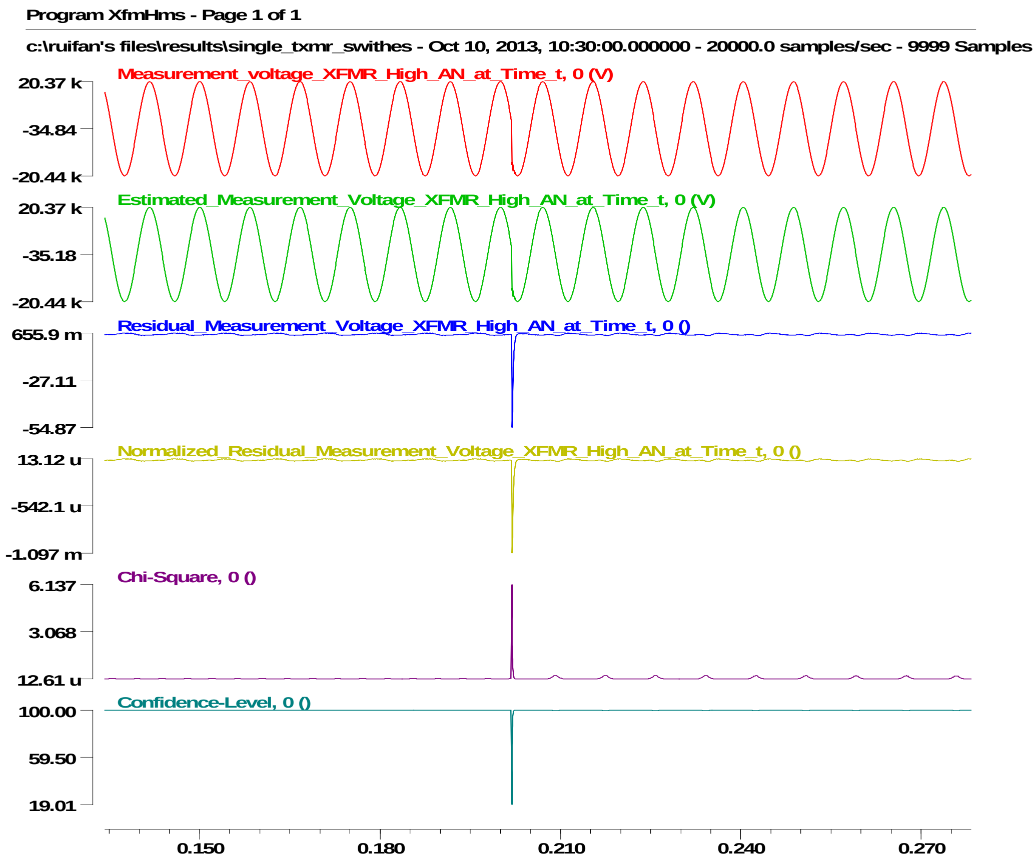

Figure 4.26: Results of Setting-less Protection for Transformer Source Switching Situation

Seen from the figure, at t=0.2 s, the transformer suddenly switched to another source, leading a voltage angle mismatch to the transformer. The confidence level dropped to zero and returned to 100% immediately. The setting-less protection detected it was not an internal fault so that it did not trip the transformer.

4.6.3. Transformer: Test case 3- Internal Fault

This case involves an internal fault situation in single phase saturable core transformer system. The system is the shown in Figure 4.27. The single phase transformer is 25kV, 300 kW as described above. In this example, the protection of single phase saturable core transformer in 10% inter-turn fault situation is present. The entire simulation lasts 10 seconds. Several other events occur during this period but in this example we focus on the internal fault at time 1.5 seconds. The transformer connects to the system at t=0s and suddenly an inter-turn fault occurs inside the transformer at t=1.5s. Since it is an internal fault so that transformer should be tripped.

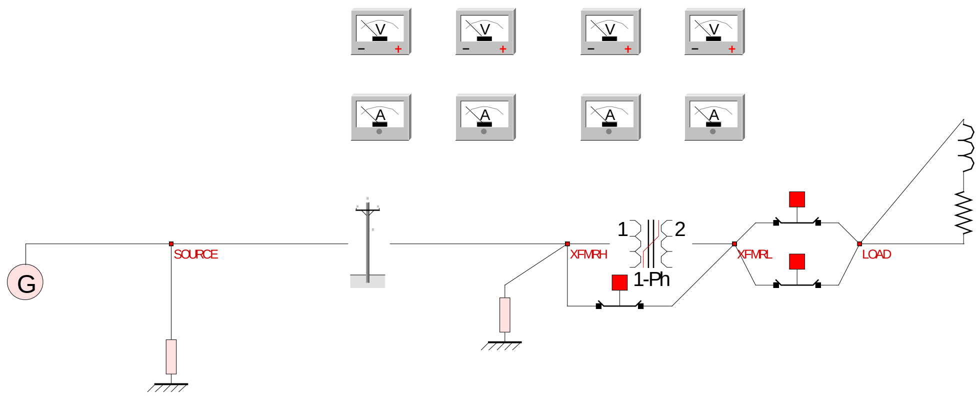

Figure 4.27: Internal Fault Situation for Single Phase Transformer

The results of DSE based protection for the period 1.1 to 2.1 seconds are shown in Figure 4.28. Specifically the figure shows eight sets of traces. The upper set of traces shows the actual and estimated left side terminal voltages of the transformer. The second set of traces shows the actual and estimated left side terminal currents of the transformer. The third trace shows the residual of the voltage while the forth trace shows the residual of the left side terminal current. The fifth trace shows the normalized residual of the voltage while the sixth trace shows the normalized residual of the left side terminal current. The seventh trace shows the sum of the normalized residuals (chi-square variable). The eighth trace shows the confidence level computed by the state estimator analysis of the residuals.

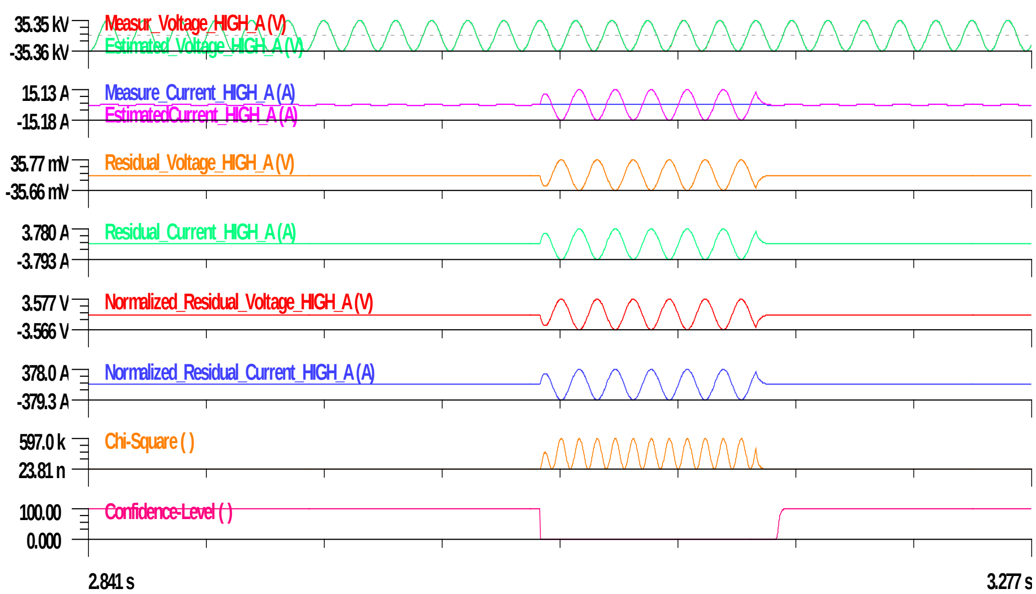

Figure 4.28: Results of Setting-less Protection for Single Phase Transformer

Seen from the result, an internal fault happens at 1.5s and clears at 1.7s. Because of the fault, the residual becomes large, the confidence level drops to zero very quickly and it stays there, which means the relays should send a trip signal to the breakers to prevent the damage of transformer.

4.7. Application to Three Phase Saturable Core Reactor Protection

Magnetic (saturable) core reactors are used for compensation of long transmission lines, current limiting applications as well as in filters to limit harmonics. The circuit model of a three phase reactor is shown in Figure 4.29. Note that in Figure 4.29, g_L is the conductance that presents the core loss, g_c is the conductance of the "stabilizer". Current transformer and potential transformer are used to measure the current and voltage. The merging units are GPS synchronized. The measurements are utilized in a dynamic state estimation. Any violation of the physical laws would imply an internal fault in the reactors. The protection logic would trip the reactor when either the physical laws are violated or the operating condition exceeds operating limits.

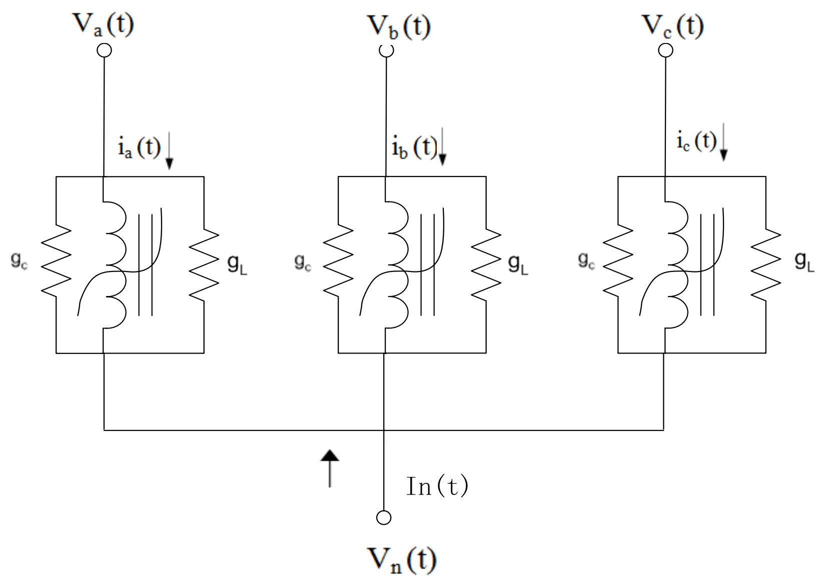

Figure 4.29: Three Phase Saturable Core Reactor Model

Mathematical Model: The mathematical model is presented for a specific three phase saturable core reactors. The example reactor ratings are: voltage rating is 115 kV, active power rating 100 MW, the nonlinear exponent is 11. The protection instrumentation includes the following measurements: terminal voltage measurements (phase to ground) and terminal current measurements.

The derivation of the SCAQCF device model for the saturable core reactor of Figure 4.29 and associated measurements is provided in Appendix G. The SCAQCF model is shown:

$$\begin{Bmatrix} I(\mathbf{x},\mathbf{u}) \\ \vdots \\ 0 \\ \vdots \end{Bmatrix} = Y_{eqx}\mathbf{x} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F^i_{eqx}\mathbf{x} \\ \vdots \end{Bmatrix} + Y_{equ}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F^i_{equ}\mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F^i_{eqxu}\mathbf{u} \\ \vdots \end{Bmatrix} - B_{eq}$$

$$B_{eq} = -N_{eqx}\mathbf{x}(t-h) - N_{equ}\mathbf{u}(t-h) - M_{eq}I(t-h) - K_{eq}$$

$$\mathbf{h}(\mathbf{x},\mathbf{u}) = Y_{opx}\mathbf{x} + Y_{opu}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F^i_{opx}\mathbf{x} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F^i_{opu}\mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F^i_{opxu}\mathbf{u} \\ \vdots \end{Bmatrix} - B_{op}$$

Scaling factors: $Iscale,\ Xscale\ and\ Uscale$

Connectivity: $TerminalNodeName$

$$subject\ to:\ \mathbf{h}_{\min} \le \mathbf{h}(\mathbf{x},\mathbf{u}) \le \mathbf{h}_{\max}$$

$$\mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max}$$

where:

$$Y_{eqx\,44\times 44} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix}$$

$$A_{11(7\times 7)} = \begin{bmatrix} g_L & 0 & 0 & -g_L & 0 & 0 & 0 \\ 0 & g_L & 0 & -g_L & 0 & 0 & 0 \\ 0 & 0 & g_L & -g_L & 0 & 0 & 0 \\ -g_L & -g_L & -g_L & 3g_L & 0 & 0 & 0 \\ -\frac{h}{6} & 0 & 0 & \frac{h}{6} & 1 & 0 & 0 \\ 0 & -\frac{h}{6} & 0 & \frac{h}{6} & 0 & 1 & 0 \\ 0 & 0 & -\frac{h}{6} & \frac{h}{6} & 0 & 0 & 1 \end{bmatrix}$$

$$A_{12(7\times15)}=\begin{bmatrix}0&0&0&0&\frac{i_0}{|\lambda_0|^{11}}&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&\frac{i_0}{|\lambda_0|^{11}}&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{i_0}{|\lambda_0|^{11}}\\0&0&0&0&-\frac{i_0}{|\lambda_0|^{11}}&0&0&0&0&-\frac{i_0}{|\lambda_0|^{11}}&0&0&0&0&-\frac{i_0}{|\lambda_0|^{11}}\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\end{bmatrix}$$

$$A_{13(7\times7)}=\begin{bmatrix}0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\-\frac{2h}{3}&0&0&\frac{2h}{3}&0&0&0\\0&-\frac{2h}{3}&0&\frac{2h}{3}&0&0&0\\0&0&-\frac{2h}{3}&\frac{2h}{3}&0&0&0\end{bmatrix}$$

$$A_{14}=[0]_{7\times15}$$
$$A_{21}=[0]_{15\times7}$$
$$A_{22}=I_{15\times15}$$
$$A_{23}=[0]_{15\times7}$$
$$A_{24}=[0]_{15\times15}$$

$$A_{31(7\times7)}=\begin{bmatrix}0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\\frac{h}{24}&0&0&-\frac{h}{24}&0&0&0\\0&\frac{h}{24}&0&-\frac{h}{24}&0&0&0\\0&0&\frac{h}{24}&-\frac{h}{24}&0&0&0\end{bmatrix}$$

$$A_{32} = [0]_{7\times 15}$$

$$A_{33(7\times 7)} = \begin{bmatrix} g_L & 0 & 0 & -g_L & 0 & 0 & 0 \\ 0 & g_L & 0 & -g_L & 0 & 0 & 0 \\ 0 & 0 & g_L & -g_L & 0 & 0 & 0 \\ -g_L & -g_L & -g_L & 3g_L & 0 & 0 & 0 \\ -\frac{h}{3} & 0 & 0 & \frac{h}{3} & 1 & 0 & 0 \\ 0 & -\frac{h}{3} & 0 & \frac{h}{3} & 0 & 1 & 0 \\ 0 & 0 & -\frac{h}{3} & \frac{h}{3} & 0 & 0 & 1 \end{bmatrix}$$

$$A_{34(7\times 15)} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{i_0}{|\lambda_0|^{11}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i_0}{|\lambda_0|^{11}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i_0}{|\lambda_0|^{11}} \\ 0 & 0 & 0 & 0 & -\frac{i_0}{|\lambda_0|^{11}} & 0 & 0 & 0 & 0 & -\frac{i_0}{|\lambda_0|^{11}} & 0 & 0 & 0 & 0 & -\frac{i_0}{|\lambda_0|^{11}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{41} = [0]_{15\times 7}$$

$$A_{42} = [0]_{15\times 15}$$

$$A_{43} = [0]_{15\times 7}$$

$$A_{44} = I_{15\times 15}$$

$$F_{eqx0} = \cdots = F_{eqx6} = 0_{44\times 44}$$

$$F_{eqx7}: f_{eqx[5\times 5]} = -1, \quad \textit{other elements are } 0$$

$$F_{eqx8}: f_{eqx[8\times 8]} = -1, \quad \textit{other elements are } 0$$

$$F_{eqx9}: f_{eqx[9\times 9]} = -1, \quad \textit{other elements are } 0$$

$$F_{eqx10}: f_{eqx[10\times 8]} = -1, \quad \textit{other elements are } 0$$

$$F_{eqx11}: f_{eqx[11\times 5]} = -1, \quad \textit{other elements are } 0$$

$$F_{eqx12}: f_{eqx[6\times 6]} = -1, \quad \textit{other elements are } 0$$

$$F_{eqx13}: f_{eqx[13\times 13]} = -1, \quad \textit{other elements are } 0$$

$$F_{eqx14}: f_{eqx[14\times14]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx15}: f_{eqx[15\times13]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx16}: f_{eqx[16\times6]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx17}: f_{eqx[7\times7]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx18}: f_{eqx[18\times18]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx19}: f_{eqx[19\times19]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx20}: f_{eqx[20\times18]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx21}: f_{eqx[21\times7]} = -1, \qquad other\ elements\ are\ 0$$

$$F_{eqx22} = \cdots = F_{eqx28} = 0_{44\times44}$$
$$F_{eqx29}: f_{eqx[27\times27]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx30}: f_{eqx[30\times30]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx31}: f_{eqx[31\times31]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx32}: f_{eqx[32\times30]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx33}: f_{eqx[33\times27]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx34}: f_{eqx[28\times28]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx35}: f_{eqx[35\times35]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx36}: f_{eqx[36\times36]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx37}: f_{eqx[37\times35]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx38}: f_{eqx[38\times28]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx39}: f_{eqx[29\times29]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx40}: f_{eqx[40\times40]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx41}: f_{eqx[41\times41]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx42}: f_{eqx[42\times40]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx43}: f_{eqx[43\times29]} = -1, \qquad other\ elements\ are\ 0$$

$$N_{eqx44X7} = \begin{bmatrix} 0_{4X7} \\ B_{1(3X7)} \\ 0_{19X7} \\ B_{2(3X7)} \\ 0_{15X7} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -\frac{h}{6} & 0 & 0 & \frac{h}{6} & -1 & 0 & 0 \\ 0 & -\frac{h}{6} & 0 & \frac{h}{6} & 0 & -1 & 0 \\ 0 & 0 & -\frac{h}{6} & \frac{h}{6} & 0 & 0 & -1 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} -\frac{5h}{24} & 0 & 0 & \frac{5h}{24} & -1 & 0 & 0 \\ 0 & -\frac{5h}{24} & 0 & \frac{5h}{24} & 0 & -1 & 0 \\ 0 & 0 & -\frac{5h}{24} & \frac{5h}{24} & 0 & 0 & -1 \end{bmatrix}$$

The scaling factors are:

$$X_{scale} = \begin{bmatrix} X_{scale\_1} & X_{scale\_2} & X_{scale\_3} & X_{scale\_4} \end{bmatrix}^T_{44}$$
$$X_{scale\_1} = \begin{bmatrix} 2.5e5 & 2.5e5 & 2.5e5 & 500 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$
$$X_{scale\_2} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$
$$X_{scale\_3} = X_{scale\_1}$$
$$X_{scale\_4} = X_{scale\_2}$$
$$I_{scale} = \begin{bmatrix} I_{scale\_1} & I_{scale\_2} & I_{scale\_3} & I_{scale\_4} \end{bmatrix}^T_{44}$$
$$I_{scale\_1} = \begin{bmatrix} 50 & 50 & 50 & 50 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$
$$I_{scale\_2} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$
$$I_{scale\_3} = I_{scale\_1}$$
$$I_{scale\_4} = I_{scale\_2}$$

An example events and the performance of the setting-less protective relay for this events are discussed below.

4.7.1 Reactor: Test case

This case involves an internal fault situation in three phase saturable core reactor system. The system is the shown in Figure 4.30. The single phase transformer is 115kV, 100 MW as described above. In this example, the protection of three phase saturable core reactor in 15% inter-turn fault situation is present. The entire simulation lasts 5 seconds. Several other events occur during this period but in this example we focus on the internal fault at time 0.35 seconds. The transformer connects to the system at t=0 s and suddenly an inter-turn fault occurs inside the transformer at t=0.35s. Since it is an internal fault so that transformer should be tripped.

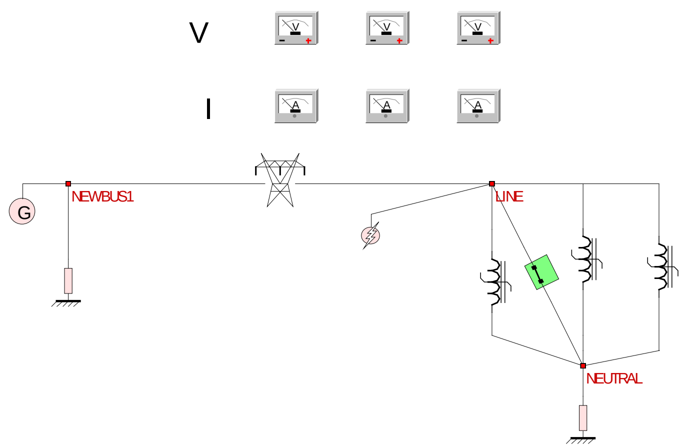

Figure 4.30: Test System for Three Phase Saturable Core Reactor

The results of DSE based protection for the period 0.25 to 0.56 seconds are shown in Figure 4.31. Specifically the figure shows eight sets of traces. The upper set of traces shows the actual and estimated phase A voltages of the reactor. The second set of traces shows the actual and estimated phase A currents of the transformer. The third trace shows the residual of the voltage while the forth trace shows the residual of the current. The fifth trace shows the normalized residual of the voltage while the sixth trace shows the normalized residual of current. The seventh trace shows the sum of the normalized residuals (chi-square variable). The eighth trace shows the confidence level computed by the state estimator analysis of the residuals.

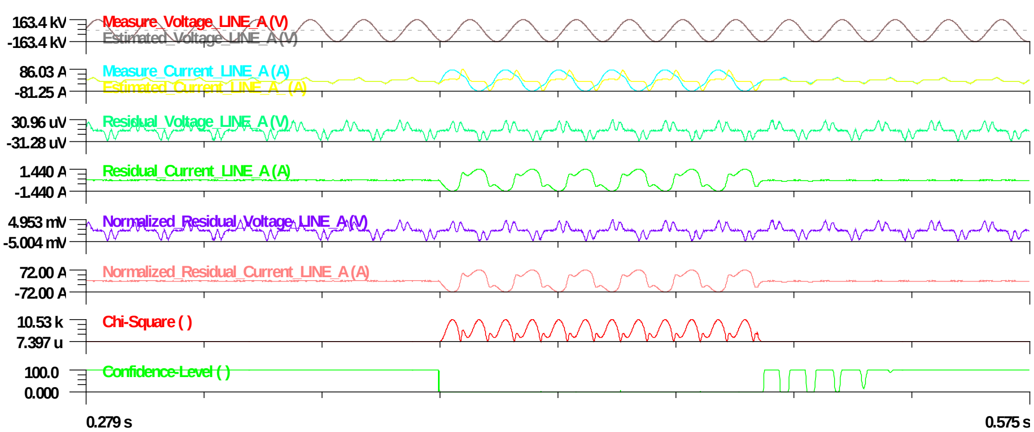

Figure 4.31: Results of Setting-less Protection for Three Phase Saturable Core Reactor

From the above figure, it can be seen that an internal fault happens at 0.35s and clears at 0.45s. Because of the fault, the residual becomes large and the confidence level drops to zero very quickly, which means the relays should send a trip signal to the breakers to prevent the damage of reactors.

5. References

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Appendix A: Overall Design of the Setting-less Relay

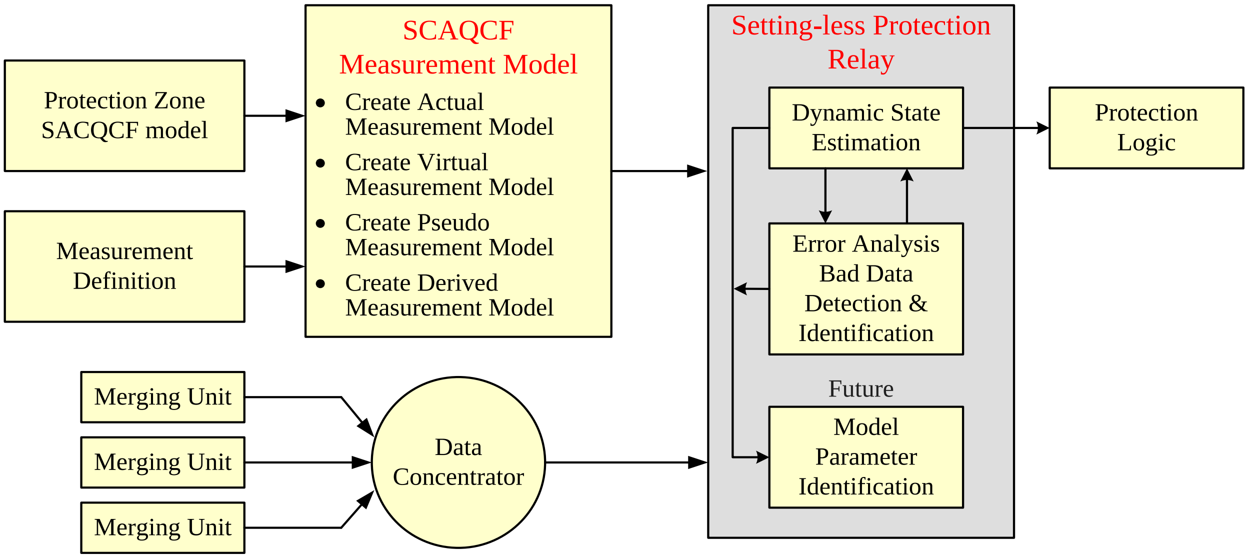

Figure A.1: Overall Design of Setting-less Protection Relay

Figure A1 illustrates the overall design of setting-less protection relay. The algorithms for the setting-less protection have been streamlined for the purpose of increasing efficiency. An object-oriented approach for the DSE based protection algorithm is developed by utilizing the State and Control Algebraic Quadratic Companion Form (SCAQCF). All the mathematical models of the protection zones in the power system are written in SCAQCF format so that the DSE based protection algorithm could be applied to any device. The setting-less protection relay has two kind of input data. One of the input data is the measurement model in the SCAQCF syntax and the other one is the real-time measurements data coming from the data concentrator. The measurement model in SCAQCF syntax is created by using the measurement pointers defined in the measurement definition file and getting the mathematical formulas for the measurements from the corresponding SCAQCF device model file, which describes the mathematical model of the protection zone. The details of how to formulate the SCAQCF measurement model can be found in Appendix B. There is a possibility that the real-time measurement data comes from multiple merging units, and a data concentrator is provided in the overall approach to align all data from different merging units with the same time stamp and feed them as one-way streaming data to the setting-less protection relay. The details of the data concentrator is described in section 4.2. The output of the setting-less protection relay is the protection logic, which determines the protection action according to the following criteria: if the real-time measurements fit the device measurement model well (the dynamic state estimation shows a high confidence level), then the protection zone is in a healthy status; otherwise, some abnormalities inside the protection zone have occurred, protection action should be acted. Note that the protection zone SCAQCF model and the measurement definition is automatically generated by the software WinIGS.

Inside the setting-less protection relay, dynamic state estimation performs whenever the data comes in. At the same time, error analysis and bad data detection are simultaneously operating in case there is some bad data or computation error which could both bring the incorrect protection results. In the near future, model parameter identification function will be added so that it can modify the protection zone model provide that the device parameters are changed.

Appendix B: Object-Oriented Implementation

B1. Time Domain SCAQCF Device Model Description

Each device mathematical model should be expressed in the generalized State and Control Algebraic Quadratic Companion Form (SCAQCF) so that each model is in the object-oriented manner. Most devices in the power system are nonlinear and for the differential equation, the quadratic integration method could be utilized to make each device model in quadratic form. The standard SCAQCF model is shown below:

$$\begin{Bmatrix} I(\mathbf{x},\mathbf{u}) \\ \vdots \\ 0 \\ \vdots \end{Bmatrix} = Y_{eqx}\mathbf{x} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{eqx}^i \mathbf{x} \\ \vdots \end{Bmatrix} + Y_{equ}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F_{equ}^i \mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{eqxu}^i \mathbf{u} \\ \vdots \end{Bmatrix} - B_{eq}$$

$$B_{eq} = -N_{eqx}\mathbf{x}(t-h) - N_{equ}\mathbf{u}(t-h) - M_{eq}I(t-h) - K_{eq}$$

$$\mathbf{h}(\mathbf{x},\mathbf{u}) = Y_{opx}\mathbf{x} + Y_{opu}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{opx}^i \mathbf{x} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F_{opu}^i \mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{opxu}^i \mathbf{u} \\ \vdots \end{Bmatrix} - B_{op}$$

Scaling factors: *Iscale, Xscale and Uscale*

Connectivity: *TerminalNodeName*

$$\textit{subject to}: \quad \mathbf{h}_{\min} \le \mathbf{h}(\mathbf{x},\mathbf{u}) \le \mathbf{h}_{\max}$$

$$\mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max}$$

where:

$I(\mathbf{x},\mathbf{u})$: the through variables of the device model, $I = [I(t), I(t_m)]$

$\mathbf{x}$: external and internal state variables of the device model, $\mathbf{x} = [\mathbf{x}(t), \mathbf{x}(t_m)]$

$\mathbf{u}$: control variables of the device model, i.e., transformer tap, etc. $\mathbf{u} = [\mathbf{u}(t), \mathbf{u}(t_m)]$

Y_{eqx} : matrix defining the linear part for state variables,

F_{eqx} : matrices defining the quadratic part for state variables,

Y_{equ} : matrix defining the linear part for control variables,

F_{equ} : matrices defining the quadratic part for control variables,

F_{eqxu} : matrices defining the quadratic part for the product of state and control variables,

B_{eq} : history dependent vector of the device model,

N_{eqx} : matrix defining the last integration step state variables part,

N_{equ} : matrix defining the last integration step control variables part,

M_{eq} : matrix defining the last integration step through variables part,

K_{eq} : constant vector of the device model,

$Iscale$: scaling factors for the through variables and zeros on the left side of the equations,

$Xscale$: scaling factors for the state variables $\mathbf{x}$,

$Uscale$: scaling factors for the control variables $\mathbf{u}$,

$TerminalNodeName$: terminal names defining the connectivity of the device model,

$\mathbf{h}_{\min} \le \mathbf{h}(\mathbf{x},\mathbf{u}) \le \mathbf{h}_{\max}$: operating constraints,

$\mathbf{u}_{\min}, \mathbf{u}_{\max}$: lower and upper bounds for the control variables.

Y_{opx} : constraint matrix defining the linear part for state variables,

F_{opx} : constraint matrices defining the quadratic part for state variables,

Y_{opu} : constraint matrix defining the linear part for control variables,

F_{opu} : constraint matrices defining the quadratic part for control variables,

F_{opxu} : constraint matrices defining the quadratic part for the product of state and control variables,

B_{op} : constraint history dependent vector of the device model.

B2. Time Domain SCAQCF Measurement Model Description

The measurement model is derived from the above SCAQCF device model. The primary data that define a measurement are pointers and the measurement error. Specifically a measurement is defined as follows:

Measurement type: number 14 stands for actual TORQUE measurement;
number 15 stands for actual SPEED measurement;
number 16 stands for actual VOLTAGE measurement;
number 17 stands for actual CURRENT measurement;
number 24 stands for the virtual measurement;
number 25 stands for voltage pseudo measurement;
number 27 stands for current pseudo measurement;
number 28 stands for voltage derived measurement;
number 29 stands for current derived measurement;
Measurement standard deviation: standard deviation in metric unit
Measurement Terminal: the terminal numbers where this measurement comes from

For derived measurements, the following three definitions are required:
Measurement Ratio: the ratio of the derived measurement to the actual measurement
Measurement Number: from which actual measurement this derived measurement can be derived

All details about the measurement definition are shown in the next section.

From the above definition and the SCAQCF the measurement model is extracted in the following form. All the measurements from the device are listed on the left side of the equations.

$$\mathbf{y}(\mathbf{x},\mathbf{u}) = Y_{m,x}\mathbf{x} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{m,x}^i \mathbf{x} \\ \vdots \end{Bmatrix} + Y_{m,u}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F_{m,u}^i \mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{m,xu}^i \mathbf{u} \\ \vdots \end{Bmatrix} + C_m$$

$$C_m = N_{m,x}\mathbf{x}(t-h) + N_{m,u}\mathbf{u}(t-h) + M_m I(t-h) + K_m$$

Measurement standard deviation: $sigma$ (metric unit)

where:

$\mathbf{y}(\mathbf{x},\mathbf{u})$: measurement variables at both time t and time t_m, $\mathbf{y} = [\mathbf{y}(t), \mathbf{y}(t_m)]$

$\mathbf{x}$: external and internal state variables of the measurement model, $\mathbf{x} = [\mathbf{x}(t), \mathbf{x}(t_m)]$

$\mathbf{u}$: control variables of the measurement model, i.e., transformer tap, etc. $\mathbf{u} = [\mathbf{u}(t), \mathbf{u}(t_m)]$

$Y_{m,x}$: matrix defining the linear part for state variables,

$F_{m,x}$: matrices defining the quadratic part for state variables,

$Y_{m,u}$: matrix defining the linear part for control variables,

$F_{m,u}$: matrices defining the quadratic part for control variables,

$F_{m,xu}$: matrices defining the quadratic part for the product of state and control variables,

C_m : history dependent vector of the measurement model,

$N_{m,x}$: matrix defining the last integration step state variables part,

$N_{m,u}$: matrix defining the last integration step control variables part,

M_m : matrix defining the last integration step through variables part,

K_m : constant vector of the measurement model,

$sigma$: matrix defining the standard deviation in metric unit.

The measurement model should be constructed from the device model and the definition of the measurements. The measurements include actual measurements, pseudo measurements, virtual measurements and derived measurements. The definition of all the measurements are shown in the next section and the creation of the SCAQCF time domain measurement model of different devices are presented in the following Appendices.

B3. Measurement Definition

This section shows how measurements are defined for the setting-less protection algorithm. There are four types of measurements: (1) actual measurement; (2) pseudo measurement; (3) virtual measurement and (4) derived measurement.

B3.1. Actual Measurement

Actual measurements are the measurements which can be actually measured. For each actual measurement, the following information is provided:

Measurement type: number 14 stands for TORQUE measurement; number 15 stands for SPEED measurement; number 16 stands for VOLTAGE measurement; number 17 stands for CURRENT measurement
Measurement standard deviation: standard deviation in metric unit
Measurement Terminal: the terminal numbers where this measurement comes from

MeasurementType, 16
MeasStdDev, 600
MeasTerminal, 2, 3
MeasurementEnd
MeasurementType, 17
MeasStdDev, 5
MeasTerminal, 0
MeasurementEnd

Some typical actual measurements are given above. From the measurement type it is easy to know the first measurement is a voltage actual measurement. Its standard deviation is 600V. The voltage is measured between device terminal 2 and 3, i.e., $v_{meas} = v_2 - v_3$.

The second measurement is a current actual measurement. Its standard deviation is 5A. The current is measurement at device terminal 0, i.e., $i_{meas} = i_0$. The expression of i_0 could be obtained from the SCAQCF device model.

B3.2. Virtual Measurement

Virtual measurements present the zeros on the left side of the internal equations. For each virtual measurement, the following information is provided:

Measurement type: number 24 stands for the virtual measurement
Measurement standard deviation: standard deviation in metric unit
Measurement Terminal: the device equation number where this measurement comes from

MeasurementType, 24
MeasStdDev, 0.0010000

MeasTerminal, 4
MeasurementEnd

A typical virtual measurement is given above. From the measurement type it is easy to know this measurement is a virtual measurement. Its standard deviation is 0.001. The value of this measurement is 0.0 and it is used for the 4^{th}equation (start from 0^{th}) of the device model.

B3.3. Pseudo Measurement

Pseudo measurements are the measurements which are normally not measured, like the voltage or current at the neutral terminal. For each pseudo measurement, the following information is provided:

Measurement type: number 25 stands for voltage pseudo measurement; number 27 stands for current pseudo measurement
Measurement standard deviation: standard deviation in metric unit
Measurement Terminal: the terminal number where this measurement comes from

MeasurementType, 25
MeasStdDev, 0.10000
MeasTerminal, 3
MeasurementEnd
MeasurementType, 27
MeasStdDev, 0.10000
MeasTerminal, 3
MeasurementEnd

Some typical pseudo measurements are given above. From the measurement type it is easy to know the first measurement is a voltage pseudo measurement. Its standard deviation is 0.1V. The pseudo measurement is for the voltage at device terminal 3, i.e., $0 = v_3$.

The second measurement is a current pseudo measurement. Its standard deviation is 0.1A. The pseudo measurement is for the current at device terminal 3, i.e., $0 = i_3$. The expression of i_0 could be obtained from the SCAQCF device model.

B3.4. Derived Measurement

Derived measurements are the measurements which can be derived from the actual measurements. For each derived measurement, the following information is provided:

Measurement type: number 28 stands for voltage derived measurement; number 29 stands for current derived measurement
Measurement standard deviation: standard deviation in metric unit
Measurement Terminal: the terminal numbers where this measurement comes from
Measurement Ratio: the ratio of the derived measurement to the actual measurement

Measurement Number: from which actual measurement this derived measurement can be derived

MeasurementType, 28
MeasStdDev, 600
MeasTerminal, 2, 3
MeasRatio, 1.0
MeasNumber, 0
MeasurementEnd
MeasurementType, 29
MeasStdDev, 5
MeasTerminal, 0
MeasRatio, 1.0
MeasNumber, 1
MeasurementEnd

Since the derived measurements come from the actual measurements, the standard deviation and the terminal information should be the same as the original measurement.

Some typical actual measurements are given above. From the measurement type it is easy to know the first measurement is a voltage derived measurement. Its standard deviation is 600V. This derived voltage measurement has the same value as the voltage measured between device terminal 2 and 3, i.e., $v_{dereived} = v_2 - v_3$.

The second measurement is a current derived measurement. Its standard deviation is 5A. This derived current measurement has the same value as the current measured at device terminal 0, i.e., $i_{derived} = i_0$. The expression of i_0 could be obtained from the SCAQCF device model.

B4. Sparsity Based State Estimation Algorithm

Given the SCAQCF measurement model and input matrices, the next step is the formation of the system Jacobian matrix and measurement vectors. A suitable sparse matrix library is used in order to store sparse matrices and execute sparse matrix operations.

The estimation algorithm is based on the Gauss-Newton iterative algorithm.

$$X^{\upsilon+1} = X^{\upsilon} - \left(H^T W H\right)^{-1} H^T W \left(h(X^{\upsilon}) - Z\right)$$

At each time step of the estimation algorithm, the contributions of each measurement to the information matrix $H^T W H$ and the vector $H^T W (h(X^{\nu}) - Z)$ are computed. For example assuming that the ith measurement has the following generic form:

$$z_i = c_i + a_{i1} \cdot x_{i1} + a_{i2} \cdot x_{i2} \cdot x_{i3} + \eta_i$$

Then the Jacobian matrix's ith row will be:

$$\begin{bmatrix} 0 & \cdots & a_{i1} & \cdots & a_{i2}\cdot x_{i2} & \cdots & a_{i2}\cdot x_{i3} & \cdots & 0 \end{bmatrix}$$

The contribution of this row to the information matrix $H^T WH$ is the following:

$$\begin{bmatrix}
0 & \cdots & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots \\
0 & \cdots & w_i a_{i1} a_{i1} & \cdots & w_i a_{i1} a_{i2} x_{i2} & \cdots & w_i a_{i1} a_{i2}\cdot x_{i3} & \cdots & 0 \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots \\
0 & \cdots & w_i a_{i1} a_{i2}\cdot x_{i2} & \cdots & w_i\left(a_{i2}\cdot x_{i2}\right)^2 & \cdots & w_i a_{i2}^2\cdot x_{i2} x_{i3} & \cdots & 0 \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots \\
0 & \cdots & w_i a_{i1} a_{i2}\cdot x_{i3} & \cdots & w_i a_{i2}^2\cdot x_{i2} x_{i3} & \cdots & w_i\left(a_{i2}\cdot x_{i3}\right)^2 & \cdots & 0 \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots \\
0 & \cdots & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0
\end{bmatrix}$$

The contribution of the measurement to the vector $H^T W(h(X^v)-Z)$ is the following:

$$\begin{bmatrix} 0 \\ \vdots \\ w_i a_{i1} b_i \\ \vdots \\ w_i a_{i2}\cdot x_{i2} b_i \\ \vdots \\ w_i a_{i2}\cdot x_{i3} b_i \\ \vdots \\ 0 \end{bmatrix}, \quad where \quad b_i = c_i + a_{i1}\cdot x_{i1} + a_{i2}\cdot x_{i2}\cdot x_{i3} - z_i$$

Based on the above formulas, it's possible to calculate the non-zero contributions of each measurement formula and insert the contributions to the information matrix $H^T WH$ and the vector $H^T W(h(X^v)-Z)$ .Once the reading of all the measurements is completed and their contribution is added to the corresponding matrix and vector, the formation of the information matrix $H^T WH$ and the vector $H^T W\left(h(X^\upsilon)-Z\right)$ is completed and stored in sparse form using a suitable Spartrix Library.

The flow chart of object-oriented setting-less protection is shown in Figure B1 and Figure B2. All the SCAQCF component models are stored in the standard matrices which are introduced in section 3. In the initialization step, the program reads all the matrices, calculates the linear part of the Jacobian matrix H and prepares the output COMTRADE channels. After the initialization, the dynamic station estimation based setting-less protection algorithm starts to check the health status of the component under protection. It performs the Chi-square test based on the calculated measurements from state estimation and the original measurements from the COMTRADE file. Meanwhile, bad data identification and operation limit monitoring are also done during the state estimation procedure. According to the result of Chi-square test, the protection logic decides the protection action for the component. The entire algorithm is real time and it iterates until no more measurements are available.

It is important to notice that most of device models are nonlinear, which means the Jacobian matrix needs to be updated at each iteration. To increase the speed, the linear part of matrix H can be pre-calculated at the SE initialization procedure. The program will identify whether the device model is linear or nonlinear. If the model is linear, then the Jacobian matrix H will stay constant. Otherwise, the program will only update the nonlinear part of H to minimize the calculation. The sparsity based state estimation algorithm mentioned before is applied and only the non-zero data will contribute to the information matrix H^TWH and the vector $H^TW\left(h(X^\upsilon)-Z\right)$.

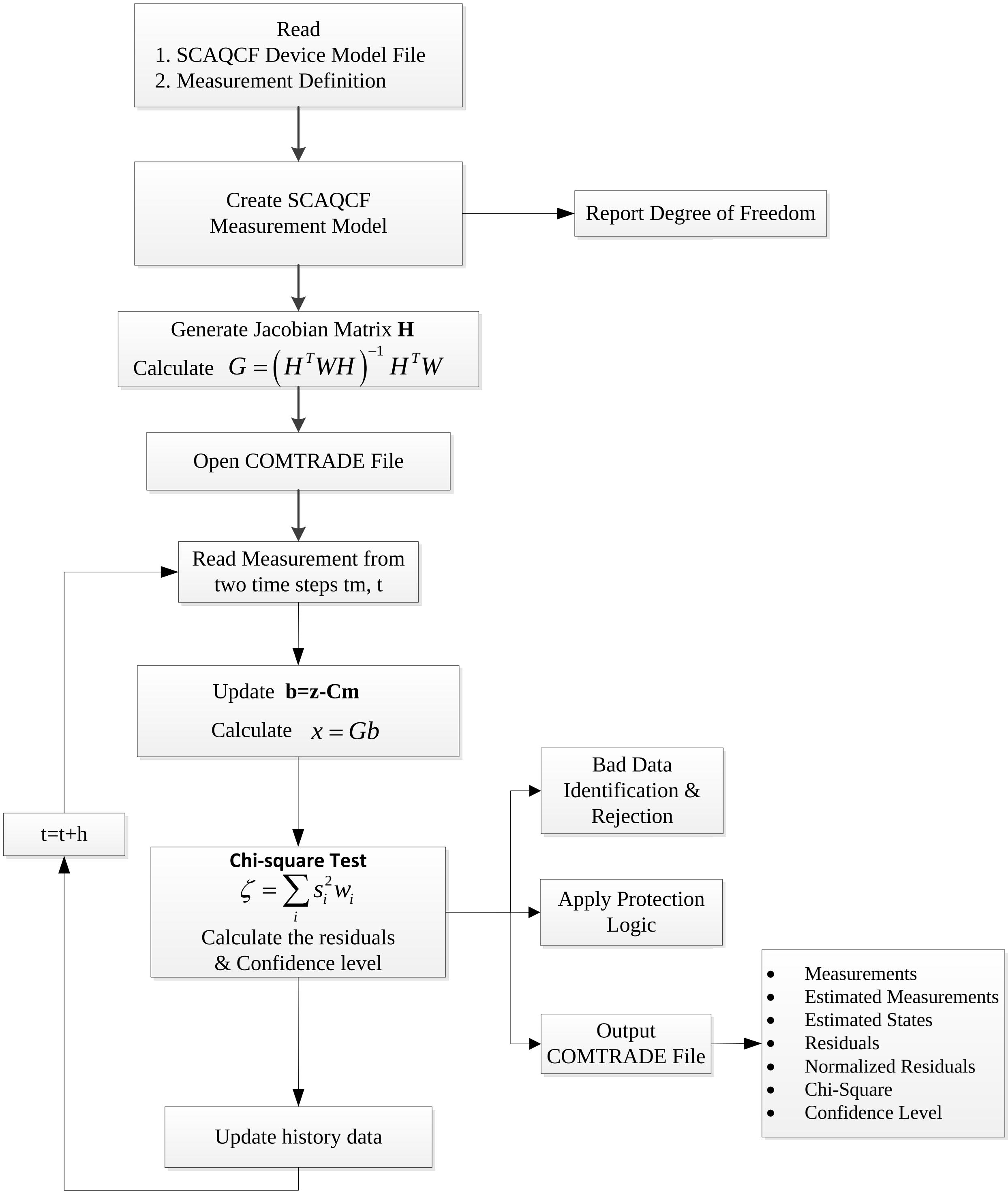

Figure B.1: Flow Chart of Object-Oriented Setting-less Protection for Linear Case

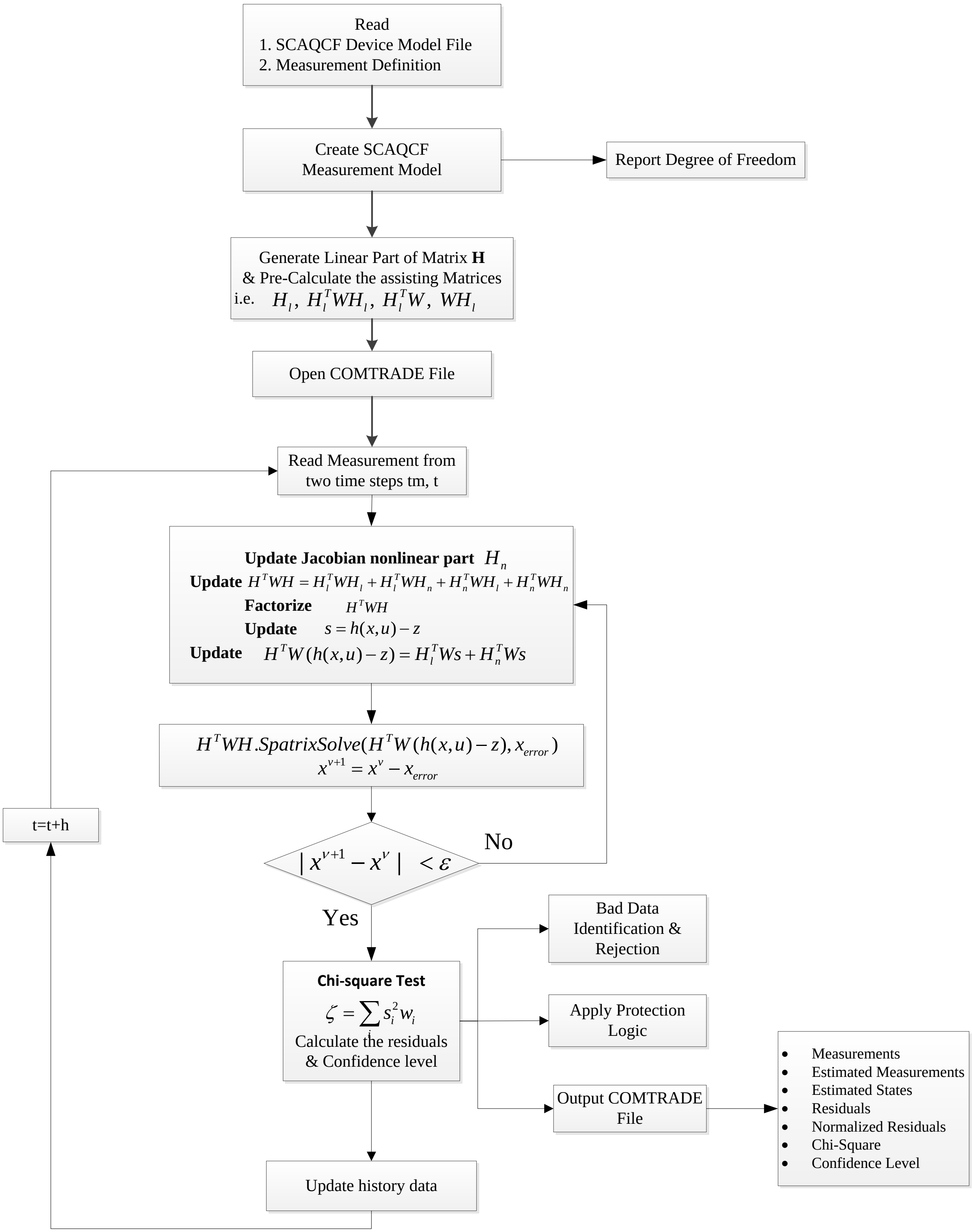

Figure B.2: Flow Chart of Object-Oriented Setting-less Protection for Nonlinear Case

B5. Frequency Domain SCAQCF Device Model Description

The frequency domain device model should be expressed in the generalized State and Control Algebraic Quadratic Companion Form (SCAQCF) as well. Compared with the time domain model, the frequency domain SCAQCF device model only has the states and control variables in the steady state. States in the frequency domain is usually expressed in the phasor format. Here in the SCAQCF model, each phasor is separated into real and imaginary parts for computation simplification. The frequency domain standard SCAQCF model is shown below:

$$\begin{Bmatrix} \mathbf{I} \\ \vdots \\ 0 \\ \vdots \end{Bmatrix} = Y_{eqx}\mathbf{X} + \begin{Bmatrix} \vdots \\ \mathbf{X}^T F_{eqx}^i \mathbf{X} \\ \vdots \end{Bmatrix} + Y_{equ}\mathbf{U} + \begin{Bmatrix} \vdots \\ \mathbf{U}^T F_{equ}^i \mathbf{U} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{X}^T F_{eqxu}^i \mathbf{U} \\ \vdots \end{Bmatrix} - B_{eq}$$

$$\mathbf{h}(\mathbf{X},\mathbf{U}) = Y_{opx}\mathbf{X} + Y_{opu}\mathbf{U} + \begin{Bmatrix} \vdots \\ \mathbf{X}^T F_{opx}^i \mathbf{X} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{U}^T F_{opu}^i \mathbf{U} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{X}^T F_{opxu}^i \mathbf{U} \\ \vdots \end{Bmatrix} - B_{op}$$

Scaling factors: $Iscale,\ Xscale\ and\ Uscale$

Connectivity: $TerminalNodeName$

$$subject\ to: \ \mathbf{h}_{\min} \le \mathbf{h}(\mathbf{X},\mathbf{U}) \le \mathbf{h}_{\max}$$

$$\mathbf{U}_{\min} \le \mathbf{U} \le \mathbf{U}_{\max}$$

where:

$\mathbf{I}$: the through variables of the device model,

$\mathbf{X}$: external and internal state variables of the device model,

$\mathbf{U}$: control variables of the device model, i.e., transformer tap, etc.

Y_{eqx} : matrix defining the linear part for state variables,

F_{eqx} : matrices defining the quadratic part for state variables,

Y_{equ} : matrix defining the linear part for control variables,

F_{equ} : matrices defining the quadratic part for control variables,

F_{eqxu} : matrices defining the quadratic part for the product of state and control variables,

B_{eq} : constant vector of the device model,

$Iscale$: scaling factors for the through variables and zeros on the left side of the equations,

$Xscale$: scaling factors for the state variables $\mathbf{X}$,

$Uscale$: scaling factors for the control variables $\mathbf{U}$,

$TerminalNodeName$: terminal names defining the connectivity of the device model

$\mathbf{h}_{\min} \le \mathbf{h}(\mathbf{X},\mathbf{U}) \le \mathbf{h}_{\max}$: operating constraints,

$\mathbf{U}_{\min}, \mathbf{U}_{\max}$: lower and upper bounds for the control variables.

Y_{opx} : constraint matrix defining the linear part for state variables,

F_{opx} : constraint matrices defining the quadratic part for state variables,

Y_{opu} : constraint matrix defining the linear part for control variables,

F_{opu} : constraint matrices defining the quadratic part for control variables,

F_{opxu} : constraint matrices defining the quadratic part for the product of state and control variables,

B_{op} : constraint history dependent vector of the device model.

B6. Frequency Domain SCAQCF Measurement Model Description

The frequency domain measurement model is derived from the above frequency domain SCAQCF device model. The primary data that define a measurement are pointers and the measurement error. Specifically a measurement is defined as follows:

Measurement type: number 14 stands for actual TORQUE measurement;
number 15 stands for actual SPEED measurement;
number 16 stands for actual VOLTAGE measurement;
number 17 stands for actual CURRENT measurement
number 24 stands for the virtual measurement;
number 25 stands for voltage pseudo measurement;
number 27 stands for current pseudo measurement;
number 28 stands for voltage derived measurement;
number 29 stands for current derived measurement
Measurement standard deviation: standard deviation in metric unit
Measurement Terminal: the terminal numbers where this measurement comes from

For derived measurements, the following three definitions are required:
Measurement Ratio: the ratio of the derived measurement to the actual measurement
Measurement Number: from which actual measurement this derived measurement can be derived

All details about the measurement definition are shown in the next section.

All the measurements from the device are listed on the left side of the equations.

$$\mathbf{Y} = Y_{mx}\mathbf{X} + \begin{Bmatrix} \vdots \\ \mathbf{X}^T F_{mx}^i \mathbf{X} \\ \vdots \end{Bmatrix} + Y_{mu}\mathbf{U} + \begin{Bmatrix} \vdots \\ \mathbf{U}^T F_{mu}^i \mathbf{U} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{X}^T F_{mxu}^i \mathbf{U} \\ \vdots \end{Bmatrix} + C_m$$

Measurement standard deviation: *sigma* (metric unit)

where:

$\mathbf{Y}$: measurement variables in the steady state,
$\mathbf{X}$: external and internal state variables of the component model,
$\mathbf{U}$: control variables of the component model, i.e., transformer tap, etc.
Y_{mx} : matrix defining the linear part for state variables,
F_{mx} : matrices defining the quadratic part for state variables,
Y_{mu} : matrix defining the linear part for control variables,
F_{mu} : matrices defining the quadratic part for control variables,
F_{mxu} : matrices defining the quadratic part for the product of state and control variables,
C_m : history dependent vector of the measurement model.
$sigma$: matrix defining the standard deviation in metric unit.

The frequency domain measurement model should be constructed from the frequency domain device model and the definition of the measurements. The measurements include actual measurements, pseudo measurements, virtual measurements and derived measurements. The definition of each measurement has already shown in Section 3.The creation of the frequency domain SCAQCF time domain measurement model are presented in Appendix B.

Appendix C: Time Domain SCAQCF Model – Capacitor Bank

This Appendix presents the derivation of the time domain model of a capacitor bank in the SCAQCF syntax. The model is first presented with what we call the compact form, which is the familiar standard notation model. We subsequently quadratized the model and then the quadratic model is integrated to provide the SCAQCF device model. A typical measurement channel list is also presented to show the procedure of creating the SCAQCF measurement model.

C1. Three Phase Capacitor Bank Compact Model

Figure C1 gives the circuit model of the three phase capacitor bank.

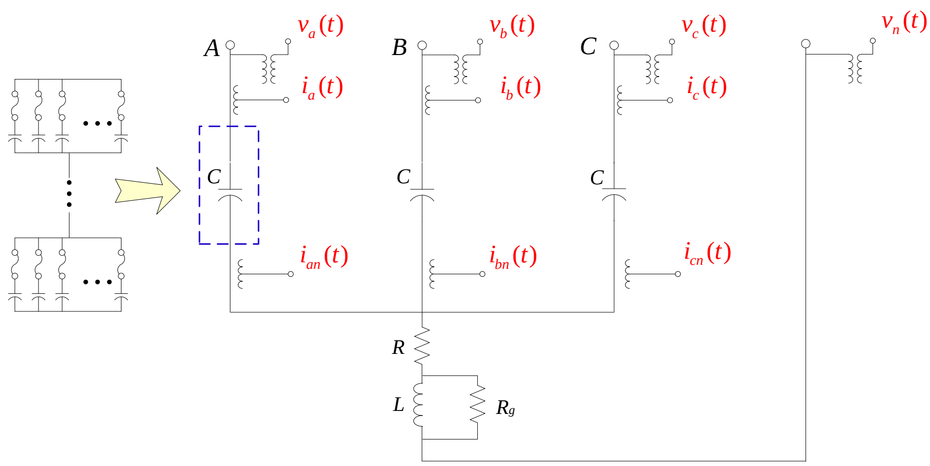

Figure C.1: The Three Phase Capacitor Bank Circuit Model

The compact capacitor bank model is derived from this circuit and it is given with the following equations:

$$i_a(t) = C \cdot \frac{d}{dt}(v_a(t) - v_m(t))$$

$$i_b(t) = C \cdot \frac{d}{dt}(v_b(t) - v_m(t))$$

$$i_c(t) = C \cdot \frac{d}{dt}(v_c(t) - v_m(t))$$

$$i_n(t) = -C \cdot \frac{d}{dt}(v_a(t) + v_b(t) + v_c(t) - 3v_m(t))$$

$$0 = G \cdot (v_m(t) - L \cdot \frac{di_L(t)}{dt} - v_n(t)) - C \cdot \frac{d}{dt}(v_a(t) + v_b(t) + v_c(t) - 3 \cdot v_m(t))$$

$$0 = g \cdot L \cdot \frac{di_L(t)}{dt} - C \cdot \frac{d}{dt}(v_a(t) + v_b(t) + v_c(t) - 3 \cdot v_m(t)) + i_L(t)$$

In compact matrix form, the model is:

$$\begin{bmatrix} i(t) \\ 0 \end{bmatrix} = A \cdot \begin{bmatrix} v(t) \\ i_L(t) \end{bmatrix} + B \cdot \frac{d}{dt} \begin{bmatrix} v(t) \\ i_L(t) \end{bmatrix}$$

where: $i(t) = \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \\ i_n(t) \end{bmatrix}$, $v(t) = \begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \\ v_n(t) \\ v_m(t) \end{bmatrix}$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -G & G & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathrm{B} = \begin{bmatrix} C & 0 & 0 & 0 & -C & 0 \\ 0 & C & 0 & 0 & -C & 0 \\ 0 & 0 & C & 0 & -C & 0 \\ C & C & C & 0 & -3C & 0 \\ -C & -C & -C & 0 & 3C & -GL \\ -C & -C & -C & 0 & 3C & gL \end{bmatrix}$$

C2. Three Phase Capacitor Bank Quadratized Model

All the terms in the three phase capacitor bank compact model are linear and quadratic terms. Therefore the quadratized model is identical to the compact model. In compact matrix form, the model is:

$$\begin{bmatrix} i(t) \\ 0 \end{bmatrix} = A \cdot \begin{bmatrix} v(t) \\ i_L(t) \end{bmatrix} + B \cdot \frac{d}{dt} \begin{bmatrix} v(t) \\ i_L(t) \end{bmatrix}$$

where: $i(t) = \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \\ i_n(t) \end{bmatrix}$, $v(t) = \begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \\ v_n(t) \\ v_m(t) \end{bmatrix}$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -G & G & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathrm{B} = \begin{bmatrix} C & 0 & 0 & 0 & -C & 0 \\ 0 & C & 0 & 0 & -C & 0 \\ 0 & 0 & C & 0 & -C & 0 \\ C & C & C & 0 & -3C & 0 \\ -C & -C & -C & 0 & 3C & -GL \\ -C & -C & -C & 0 & 3C & gL \end{bmatrix}$$

C3. Three Phase Capacitor Bank SCAQCF Device Model

The state and control algebraic quadratic companion form (SCAQCF)device model is derived after quadratic integration of the quadratized model with a time step h. The result is:

At time t,

$$i_a(t)=\frac{12}{h}\cdot C\cdot[v_a(t)-v_m(t)]-\frac{24}{h}\cdot C\cdot[v_a(t_m)-v_m(t_m)]+3i_a(t-h)+\frac{12}{h}\cdot C\cdot[v_a(t-h)-v_m(t-h)]$$

$$i_b(t)=\frac{12}{h}\cdot C\cdot[v_b(t)-v_m(t)]-\frac{24}{h}\cdot C\cdot[v_b(t_m)-v_m(t_m)]+3i_b(t-h)+\frac{12}{h}\cdot C\cdot[v_b(t-h)-v_m(t-h)]$$

$$i_c(t)=\frac{12}{h}\cdot C\cdot[v_c(t)-v_m(t)]-\frac{24}{h}\cdot C\cdot[v_c(t_m)-v_m(t_m)]+3i_c(t-h)+\frac{12}{h}\cdot C\cdot[v_c(t-h)-v_m(t-h)]$$

$$\begin{aligned}i_n(t)=&\frac{12}{h}\cdot C\cdot[-v_a(t)-v_b(t)-v_c(t)+3v_m(t)]-\frac{24}{h}\cdot C\cdot[-v_a(t_m)-v_b(t_m)-v_c(t_m)+3v_m(t_m)]\\&+3i_n(t-h)+\frac{12}{h}\cdot C\cdot[-v_a(t-h)-v_b(t-h)-v_c(t-h)+3v_m(t-h)]\end{aligned}$$

$$\begin{aligned}0=&\frac{h}{6}\cdot G\cdot(v_m(t)-v_n(t))+\frac{2h}{3}\cdot G\cdot(v_m(t_m)-v_n(t_m))-G\cdot L\cdot i_L(t)-C\cdot[v_a(t)+v_b(t)+v_c(t)-3v_m(t)]\\&+\frac{h}{6}\cdot G\cdot(v_m(t-h)-v_n(t-h))-G\cdot L\cdot i_L(t-h)-C\cdot[v_a(t-h)+v_b(t-h)+v_c(t-h)-3v_m(t-h)]\end{aligned}$$

$$\begin{aligned}0=&\frac{h}{6}\cdot i_L(t)+\frac{2h}{3}\cdot i_L(t_m)-g\cdot L\cdot i_L(t)-C\cdot[v_a(t)+v_b(t)+v_c(t)-3v_m(t)]\\&+\frac{h}{6}\cdot i_L(t-h)+g\cdot L\cdot i_L(t-h)-C\cdot[v_a(t-h)+v_b(t-h)+v_c(t-h)-3v_m(t-h)]\end{aligned}$$

At time t_m,

$$i_a(t_m)=\frac{1}{2h}\cdot C\cdot[v_a(t)-v_m(t)]+\frac{2}{h}\cdot C\cdot[v_a(t_m)-v_m(t_m)]-\frac{1}{2}i_a(t-h)-\frac{5}{2h}\cdot C\cdot[v_a(t-h)-v_m(t-h)]$$

$$i_b(t_m)=\frac{1}{2h}\cdot C\cdot[v_b(t)-v_m(t)]+\frac{2}{h}\cdot C\cdot[v_b(t_m)-v_m(t_m)]-\frac{1}{2}i_b(t-h)-\frac{5}{2h}\cdot C\cdot[v_b(t-h)-v_m(t-h)]$$

$$i_c(t_m)=\frac{1}{2h}\cdot C\cdot[v_c(t)-v_m(t)]+\frac{2}{h}\cdot C\cdot[v_c(t_m)-v_m(t_m)]-\frac{1}{2}i_c(t-h)-\frac{5}{2h}\cdot C\cdot[v_c(t-h)-v_m(t-h)]$$

$$i_n(t_m) = \frac{1}{2h}\cdot C\cdot[-v_a(t) - v_b(t) - v_c(t) + 3v_m(t)] + \frac{2}{h}\cdot C\cdot[-v_a(t_m) - v_b(t_m) - v_c(t_m) + 3v_m(t_m)]$$
$$-\frac{1}{2}i_n(t-h) - \frac{5}{2h}\cdot C\cdot[-v_a(t-h) - v_b(t-h) - v_c(t-h) + 3v_m(t-h)]$$
$$0 = -\frac{h}{24}\cdot G\cdot(v_m(t) - v_n(t)) + \frac{h}{3}\cdot G\cdot(v_m(t_m) - v_n(t_m)) - G\cdot L\cdot i_L(t_m) - C\cdot[v_a(t_m) + v_b(t_m) + v_c(t_m) - 3v_m(t_m)]$$
$$+\frac{5h}{24}\cdot G\cdot(v_m(t-h) - v_n(t-h)) - G\cdot L\cdot i_L(t-h) - C\cdot[v_a(t-h) + v_b(t-h) + v_c(t-h) - 3v_m(t-h)]$$
$$0 = -\frac{h}{24}\cdot i_L(t) + \frac{h}{3}\cdot i_L(t_m) - g\cdot L\cdot i_L(t_m) - C\cdot[v_a(t_m) + v_b(t_m) + v_c(t_m) - 3v_m(t_m)]$$
$$+\frac{5h}{24}\cdot i_L(t-h) + g\cdot L\cdot i_L(t-h) - C\cdot[v_a(t-h) + v_b(t-h) + v_c(t-h) - 3v_m(t-h)]$$

Write the above equations into the standard SCAQCF matrix form: (no control variable in this capacitor bank model)

$$I(\mathbf{x}) = Y_{eqx}\cdot\mathbf{x} - B_{eq}$$

where:

$$I(\mathbf{x}) = \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \\ i_n(t) \\ 0 \\ 0 \\ i_a(t_m) \\ i_b(t_m) \\ i_c(t_m) \\ i_n(t_m) \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \\ v_n(t) \\ v_m(t) \\ i_L(t) \\ v_a(t_m) \\ v_b(t_m) \\ v_c(t_m) \\ v_n(t_m) \\ v_m(t_m) \\ i_L(t_m) \end{bmatrix}$$

$$
Y_{eqx} = \begin{bmatrix}
\frac{12C}{h} & 0 & 0 & 0 & -\frac{12C}{h} & 0 & -\frac{24C}{h} & 0 & 0 & 0 & \frac{24C}{h} & 0 \\
0 & \frac{12C}{h} & 0 & 0 & -\frac{12C}{h} & 0 & 0 & -\frac{24C}{h} & 0 & 0 & \frac{24C}{h} & 0 \\
0 & 0 & \frac{12C}{h} & 0 & -\frac{12C}{h} & 0 & 0 & 0 & -\frac{24C}{h} & 0 & \frac{24C}{h} & 0 \\
-\frac{12C}{h} & -\frac{12C}{h} & -\frac{12C}{h} & 0 & \frac{36C}{h} & 0 & \frac{24C}{h} & \frac{24C}{h} & \frac{24C}{h} & 0 & -\frac{72C}{h} & 0 \\
-C & -C & -C & -\frac{h\cdot G}{6} & \frac{h\cdot G}{6}+3C & -G\cdot L & 0 & 0 & 0 & -\frac{2h\cdot G}{3} & \frac{2h\cdot G}{3} & 0 \\
-C & -C & -C & 0 & 3C & \frac{h}{6}-g\cdot L & 0 & 0 & 0 & 0 & 0 & \frac{2h}{3} \\
\frac{C}{2h} & 0 & 0 & 0 & -\frac{C}{2h} & 0 & \frac{2C}{h} & 0 & 0 & 0 & -\frac{2C}{h} & 0 \\
0 & \frac{C}{2h} & 0 & 0 & -\frac{C}{2h} & 0 & 0 & \frac{2C}{h} & 0 & 0 & -\frac{2C}{h} & 0 \\
0 & 0 & \frac{C}{2h} & 0 & -\frac{C}{2h} & 0 & 0 & 0 & \frac{2C}{h} & 0 & -\frac{2C}{h} & 0 \\
-\frac{C}{2h} & -\frac{C}{2h} & -\frac{C}{2h} & 0 & \frac{3C}{2h} & 0 & -\frac{2C}{h} & -\frac{2C}{h} & -\frac{2C}{h} & 0 & \frac{6C}{h} & 0 \\
0 & 0 & 0 & \frac{h\cdot G}{24} & -\frac{h\cdot G}{24} & 0 & -C & -C & -C & -\frac{h\cdot G}{3} & \frac{h\cdot G}{3}+3C & -G\cdot L \\
0 & 0 & 0 & 0 & 0 & -\frac{h}{24} & -C & -C & -C & 0 & 3C & \frac{h}{3}-g\cdot L
\end{bmatrix}
$$

$$
B_{eq} = \begin{bmatrix}
-3i_a(t-h) - \frac{12}{h}\cdot C\cdot[v_a(t-h) - v_m(t-h)] \\
-3i_b(t-h) - \frac{12}{h}\cdot C\cdot[v_b(t-h) - v_m(t-h)] \\
-3i_c(t-h) - \frac{12}{h}\cdot C\cdot[v_c(t-h) - v_m(t-h)] \\
-3i_n(t-h) - \frac{12}{h}\cdot C\cdot[-v_a(t-h) - v_b(t-h) - v_c(t-h) + 3v_m(t-h)] \\
-\frac{h}{6}\cdot G\cdot(v_m(t-h) - v_n(t-h)) + G\cdot L\cdot i_L(t-h) + C\cdot[v_a(t-h) + v_b(t-h) + v_c(t-h) - 3v_m(t-h)] \\
-\frac{h}{6}\cdot i_L(t-h) - g\cdot L\cdot i_L(t-h) + C\cdot[v_a(t-h) + v_b(t-h) + v_c(t-h) - 3v_m(t-h)] \\
-\frac{1}{2}i_a(t-h) + \frac{5}{2h}\cdot C\cdot[v_a(t-h) - v_m(t-h)] \\
-\frac{1}{2}i_b(t-h) + \frac{5}{2h}\cdot C\cdot[v_b(t-h) - v_m(t-h)] \\
-\frac{1}{2}i_c(t-h) + \frac{5}{2h}\cdot C\cdot[v_c(t-h) - v_m(t-h)] \\
-\frac{1}{2}i_n(t-h) + \frac{5}{2h}\cdot C\cdot[-v_a(t-h) - v_b(t-h) - v_c(t-h) + 3v_m(t-h)] \\
-\frac{5h}{24}\cdot G\cdot(v_m(t-h) - v_n(t-h)) + G\cdot L\cdot i_L(t-h) + C\cdot[v_a(t-h) + v_b(t-h) + v_c(t-h) - 3\cdot v_m(t-h)] \\
-\frac{5h}{24}\cdot i_L(t-h) - g\cdot L\cdot i_L(t-h) + C\cdot[v_a(t-h) + v_b(t-h) + v_c(t-h) - 3v_m(t-h)]
\end{bmatrix}
$$

The input matrices of the capacitor bank SCAQCF device model for the setting-less protection algorithm are shown below:

TerminalNodeName
CAPBANK_A
CAPBANK_B
CAPBANK_C
CAPBANK_N
-1

Xscale
0, 11500
1, 11500
2, 11500
3, 11500
4, 11500
5, 80
6, 11500

7, 11500
8, 11500
9, 11500
10, 11500
11, 80
-1

Iscale
0, 230
1, 230
2, 230
3, 230
4, 1
5, 1
6, 230
7, 230
8, 230
9, 230
10, 1
11, 1
-1

Yeqx
0, 0, 0.192
0, 4, -0.192
0, 6, -0.384
0, 10, 0.384
1, 1, 0.192
1, 4, -0.192
1, 7, -0.384
1, 10, 0.384
2, 2, 0.192
2, 4, -0.192
2, 8, -0.384
2, 10, 0.384
3, 0, -0.192
3, 1, -0.192
3, 2, -0.192
3, 4, 0.576
3, 6, 0.384
3, 7, 0.384
3, 8, 0.384
3, 10, -1.152
4, 0, -0.0000048
4, 1, -0.0000048
4, 2, -0.0000048
4, 3, -0.0000001667
4, 4, 0.0000145667

4, 9, -0.0000006667
4, 10, 0.0000006667
5, 0, -0.0000048
5, 1, -0.0000048
5, 2, -0.0000048
5, 4, 0.0000144
5, 5, 0.00001667
5, 11, 0.00006667
6, 0, 0.024
6, 4, -0.024
6, 6, 0.096
6, 10, -0.096
7, 1, 0.024
7, 4, -0.024
7, 7, 0.096
7, 10, -0.096
8, 2, 0.024
8, 4, -0.024
8, 8, 0.096
8, 10, -0.096
9, 0, -0.024
9, 1, -0.024
9, 2, -0.024
9, 4, 0.072
9, 6, -0.096
9, 7, -0.096
9, 8, -0.096
9, 10, 0.288
10, 3, 0.0000000416
10, 4, -0.0000000416
10, 6, -0.0000048
10, 7, -0.0000048
10, 8, -0.0000048
10, 9, -0.0000003333
10, 10, 0.0000147333
11, 5, -0.00000416
11, 6, -0.0000048
11, 7, -0.0000048
11, 8, -0.0000048
11, 10, 0.0000144
11, 11, 0.00003333
-1

Feqx
-1

Neqx
0, 0, 0.192

```
 0,   4,  -0.192
 1,   1,  0.192
 1,   4,  -0.192
 2,   2,  0.192
 2,   4,  -0.192
 3,   0,  -0.192
 3,   1,  -0.192
 3,   2,  -0.192
 3,   4,  0.576
 4,   0,  0.0000048
 4,   1,  0.0000048
 4,   2,  0.0000048
 4,   3,  -0.0000001667
 4,   4,  -0.000014233
 5,   0,  0.0000048
 5,   1,  0.0000048
 5,   2,  0.0000048
 5,   4,  -0.0000144
 5,   5,  0.00001667
 6,   0,  -0.12
 6,   4,  0.12
 7,   1,  -0.12
 7,   4,  0.12
 8,   2,  -0.12
 8,   4,  0.12
 9,   0,  0.12
 9,   1,  0.12
 9,   2,  0.12
 9,   4,  -0.36
10,   0,  0.0000048
10,   1,  0.0000048
10,   2,  0.0000048
10,   3,  -0.000000208
10,   4,  -0.000014192
11,   0,  0.0000048
11,   1,  0.0000048
11,   2,  0.0000048
11,   4,  -0.0000144
11,   5,  0.000000208
-1

Meq
 0,   0,  1.0
 1,   1,  1.0
 2,   2,  1.0
 3,   3,  1.0
 6,   0,  -0.5
 7,   1,  -0.5
```

8,   2,   -0.5
9,   3,   -0.5
-1

Keq
-1

C4. Three Phase Capacitor Bank SCAQCF Measurement Model

More specifically, for capacitor bank, the **actual measurements** are:
Three currents at time t (phase A, phase B, and phase C);
Three voltages at time t (phase A-G, phase B-G, phase C-G);
Three bottom currents at time t (bottom phase A, bottom phase B, and bottom phase C);
One neutral voltage at time t (phase N-G);
Three currents at time tm=t-h/2 (phase A, phase B, and phase C);
Three voltages at time tm=t-h/2 (phase A-G, phase B-G, phase C-G);
Three bottom currents at time tm=t-h/2 (bottom phase A, bottom phase B, and bottom phase C);
One neutral voltage at time tm=t-h/2 (phase N-G);

The **virtual measurements** are:
Two measurements with zero value at the left side of the equations at time t;
Two measurements with zero value at the left side of the equations at time tm=t-h/2;

The measurement channel list:
MeasurementType, 16
MeasStdDev, 1150
MeasTerminal,   0,   3
MeasurementEnd
MeasurementType, 17
MeasStdDev, 2
MeasTerminal,   0
MeasurementEnd
MeasurementType, 17
MeasStdDev, 2
MeasTerminal,   1
MeasurementEnd
MeasurementType, 17
MeasStdDev, 2
MeasTerminal,   2
MeasurementEnd
MeasurementType, 16
MeasStdDev, 1150
MeasTerminal,   1,   3
MeasurementEnd
MeasurementType, 16

MeasStdDev, 1150
MeasTerminal, 2, 3
MeasurementEnd
MeasurementType, 16
MeasStdDev, 1150
MeasTerminal, 3
MeasurementEnd
MeasurementType, 17
MeasStdDev, 2
MeasTerminal, 0
MeasurementEnd
MeasurementType, 17
MeasStdDev, 2
MeasTerminal, 1
MeasurementEnd
MeasurementType, 17
MeasStdDev, 2
MeasTerminal, 2
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal, 4
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal, 5
MeasurementEnd

Based on the device model and the measurement channel list, the equations for the measurement model are given below:

At time t,

$$v_{an}(t) = v_a(t) - v_n(t)$$
$$v_{bn}(t) = v_b(t) - v_n(t)$$
$$v_{cn}(t) = v_c(t) - v_n(t)$$
$$v_{nn}(t) = v_n(t)$$
$$i_a(t) = \frac{4}{h}\cdot C\cdot[v_a(t) - v_n(t)] - \frac{8}{h}\cdot C\cdot[v_a(t_m) - v_n(t_m)] + i_a(t-h) + \frac{4}{h}\cdot C\cdot[v_a(t-h) - v_n(t-h)]$$
$$i_b(t) = \frac{4}{h}\cdot C\cdot[v_b(t) - v_n(t)] - \frac{8}{h}\cdot C\cdot[v_b(t_m) - v_n(t_m)] + i_b(t-h) + \frac{4}{h}\cdot C\cdot[v_b(t-h) - v_n(t-h)]$$
$$i_c(t) = \frac{4}{h}\cdot C\cdot[v_c(t) - v_n(t)] - \frac{8}{h}\cdot C\cdot[v_c(t_m) - v_n(t_m)] + i_c(t-h) + \frac{4}{h}\cdot C\cdot[v_c(t-h) - v_n(t-h)]$$

$$i_{am}(t)=\frac{4}{h}\cdot C\cdot[v_a(t)-v_n(t)]-\frac{8}{h}\cdot C\cdot[v_a(t_m)-v_n(t_m)]+i_a(t-h)+\frac{4}{h}\cdot C\cdot[v_a(t-h)-v_n(t-h)]$$
$$i_{bm}(t)=\frac{4}{h}\cdot C\cdot[v_b(t)-v_n(t)]-\frac{8}{h}\cdot C\cdot[v_b(t_m)-v_n(t_m)]+i_b(t-h)+\frac{4}{h}\cdot C\cdot[v_b(t-h)-v_n(t-h)]$$
$$i_{cm}(t)=\frac{4}{h}\cdot C\cdot[v_c(t)-v_n(t)]-\frac{8}{h}\cdot C\cdot[v_c(t_m)-v_n(t_m)]+i_c(t-h)+\frac{4}{h}\cdot C\cdot[v_c(t-h)-v_n(t-h)]$$
$$0=v_n(t)$$
$$0=\frac{h}{6}\cdot G\cdot v_n(t)+\frac{2h}{3}\cdot G\cdot v_n(t_m)-G\cdot L\cdot i_L(t)-C\cdot[v_a(t)+v_b(t)+v_c(t)-3v_n(t)]$$
$$+\frac{h}{6}\cdot G\cdot v_n(t-h)+G\cdot L\cdot i_L(t-h)-C\cdot[v_a(t-h)+v_b(t-h)+v_c(t-h)-3v_n(t-h)]$$
$$0=\frac{h}{6}\cdot i_L(t)+\frac{2h}{3}\cdot i_L(t_m)-g\cdot L\cdot i_L(t)-C\cdot[v_a(t)+v_b(t)+v_c(t)-3v_n(t)]$$
$$+\frac{h}{6}\cdot i_L(t-h)+g\cdot L\cdot i_L(t-h)-C\cdot[v_a(t-h)+v_b(t-h)+v_c(t-h)-3v_n(t-h)]$$

At time t_m,

$$v_{an}(t_m)=v_a(t_m)-v_n(t_m)$$
$$v_{bn}(t_m)=v_b(t_m)-v_n(t_m)$$
$$v_{cn}(t_m)=v_c(t_m)-v_n(t_m)$$
$$v_{nn}(t_m)=v_n(t_m)$$
$$i_a(t_m)=\frac{1}{2h}\cdot C\cdot[v_a(t)-v_n(t)]+\frac{2}{h}\cdot C\cdot[v_a(t_m)-v_n(t_m)]-\frac{1}{2}i_a(t-h)-\frac{5}{2h}\cdot C\cdot[v_a(t-h)-v_n(t-h)]$$
$$i_b(t_m)=\frac{1}{2h}\cdot C\cdot[v_b(t)-v_n(t)]+\frac{2}{h}\cdot C\cdot[v_b(t_m)-v_n(t_m)]-\frac{1}{2}i_b(t-h)-\frac{5}{2h}\cdot C\cdot[v_b(t-h)-v_n(t-h)]$$
$$i_c(t_m)=\frac{1}{2h}\cdot C\cdot[v_c(t)-v_n(t)]+\frac{2}{h}\cdot C\cdot[v_c(t_m)-v_n(t_m)]-\frac{1}{2}i_c(t-h)-\frac{5}{2h}\cdot C\cdot[v_c(t-h)-v_n(t-h)]$$
$$i_{am}(t_m)=\frac{1}{2h}\cdot C\cdot[v_a(t)-v_n(t)]+\frac{2}{h}\cdot C\cdot[v_a(t_m)-v_n(t_m)]-\frac{1}{2}i_a(t-h)-\frac{5}{2h}\cdot C\cdot[v_a(t-h)-v_n(t-h)]$$
$$i_{bm}(t_m)=\frac{1}{2h}\cdot C\cdot[v_b(t)-v_n(t)]+\frac{2}{h}\cdot C\cdot[v_b(t_m)-v_n(t_m)]-\frac{1}{2}i_b(t-h)-\frac{5}{2h}\cdot C\cdot[v_b(t-h)-v_n(t-h)]$$
$$i_{cm}(t_m)=\frac{1}{2h}\cdot C\cdot[v_c(t)-v_n(t)]+\frac{2}{h}\cdot C\cdot[v_c(t_m)-v_n(t_m)]-\frac{1}{2}i_c(t-h)-\frac{5}{2h}\cdot C\cdot[v_c(t-h)-v_n(t-h)]$$
$$0=v_n(t_m)$$
$$0=-\frac{h}{24}\cdot G\cdot v_n(t)+\frac{h}{3}\cdot G\cdot v_n(t_m)-G\cdot L\cdot i_L(t)-C\cdot[v_a(t)+v_b(t)+v_c(t)-3v_n(t)]$$
$$+\frac{5h}{24}\cdot G\cdot v_n(t-h)+G\cdot L\cdot i_L(t-h)-C\cdot[v_a(t-h)+v_b(t-h)+v_c(t-h)-3v_n(t-h)]$$

$$
\begin{aligned}
0 = & -\frac{h}{24}\cdot i_L(t) + \frac{h}{3}\cdot i_L(t_m) - g\cdot L\cdot i_L(t) - C\cdot[v_a(t)+v_b(t)+v_c(t)-3v_n(t)] \\
& +\frac{5h}{24}\cdot i_L(t-h) + g\cdot L\cdot i_L(t-h) - C\cdot[v_a(t-h)+v_b(t-h)+v_c(t-h)-3v_n(t-h)]
\end{aligned}
$$

It is very easy to write the above equations in the standard SCAQCF format:

$$
\mathbf{y}(\mathbf{x},\mathbf{u}) = Y_{m,x}\mathbf{x} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{m,x}^i \mathbf{x} \\ \vdots \end{Bmatrix} + Y_{m,u}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F_{m,u}^i \mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{m,xu}^i \mathbf{u} \\ \vdots \end{Bmatrix} + N_{m,x}\mathbf{x}(t-h) + N_{m,u}\mathbf{u}(t-h) + M_m I(t-h) + K_m
$$

where:

$$
Y_{m,x} = \begin{bmatrix}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\frac{4C}{h} & 0 & 0 & 0 & -\frac{4C}{h} & 0 & -\frac{8C}{h} & 0 & 0 & 0 & \frac{8C}{h} & 0 \\
0 & \frac{4C}{h} & 0 & 0 & -\frac{4C}{h} & 0 & 0 & -\frac{8C}{h} & 0 & 0 & \frac{8C}{h} & 0 \\
0 & 0 & \frac{4C}{h} & 0 & -\frac{4C}{h} & 0 & 0 & 0 & -\frac{8C}{h} & 0 & \frac{8C}{h} & 0 \\
\frac{4C}{h} & 0 & 0 & 0 & -\frac{4C}{h} & 0 & -\frac{8C}{h} & 0 & 0 & 0 & \frac{8C}{h} & 0 \\
0 & \frac{4C}{h} & 0 & 0 & -\frac{4C}{h} & 0 & 0 & -\frac{8C}{h} & 0 & 0 & \frac{8C}{h} & 0 \\
0 & 0 & \frac{4C}{h} & 0 & -\frac{4C}{h} & 0 & 0 & 0 & -\frac{8C}{h} & 0 & \frac{8C}{h} & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
-C & -C & -C & -\frac{h \cdot G}{6} & \frac{h \cdot G}{6}+3C & -G \cdot L & 0 & 0 & 0 & -\frac{2h \cdot G}{3} & \frac{2h \cdot G}{3} & 0 \\
-C & -C & -C & 0 & 3C & \frac{h}{6}-g \cdot L & 0 & 0 & 0 & 0 & 0 & \frac{2h}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\frac{C}{2h} & 0 & 0 & 0 & -\frac{C}{2h} & 0 & \frac{2C}{h} & 0 & 0 & 0 & -\frac{2C}{h} & 0 \\
0 & \frac{C}{2h} & 0 & 0 & -\frac{C}{2h} & 0 & 0 & \frac{2C}{h} & 0 & 0 & -\frac{2C}{h} & 0 \\
0 & 0 & \frac{C}{2h} & 0 & -\frac{C}{2h} & 0 & 0 & 0 & \frac{2C}{h} & 0 & -\frac{2C}{h} & 0 \\
\frac{C}{2h} & 0 & 0 & 0 & -\frac{C}{2h} & 0 & \frac{2C}{h} & 0 & 0 & 0 & -\frac{2C}{h} & 0 \\
0 & \frac{C}{2h} & 0 & 0 & -\frac{C}{2h} & 0 & 0 & \frac{2C}{h} & 0 & 0 & -\frac{2C}{h} & 0 \\
0 & 0 & \frac{C}{2h} & 0 & -\frac{C}{2h} & 0 & 0 & 0 & \frac{2C}{h} & 0 & -\frac{2C}{h} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & \frac{h \cdot G}{24} & -\frac{h \cdot G}{24} & 0 & -C & -C & -C & -\frac{h \cdot G}{3} & \frac{h \cdot G}{3}+3C & -G \cdot L \\
0 & 0 & 0 & 0 & 0 & -\frac{h}{24} & -C & -C & -C & 0 & 3C & \frac{h}{3}-g \cdot L
\end{bmatrix}
$$

$$N_{m,x}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{4C}{h} & 0 & 0 & 0 & -\frac{4C}{h} & 0 \\ 0 & \frac{4C}{h} & 0 & 0 & -\frac{4C}{h} & 0 \\ 0 & 0 & \frac{4C}{h} & 0 & -\frac{4C}{h} & 0 \\ \frac{4C}{h} & 0 & 0 & 0 & -\frac{4C}{h} & 0 \\ 0 & \frac{4C}{h} & 0 & 0 & -\frac{4C}{h} & 0 \\ 0 & 0 & \frac{4C}{h} & 0 & -\frac{4C}{h} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ C & C & C & -\frac{h\cdot G}{6} & \frac{h\cdot G}{6}+3C & G\cdot L \\ C & C & C & 0 & -3C & \frac{h}{6}-g\cdot L \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{5C}{2h} & 0 & 0 & 0 & \frac{5C}{2h} & 0 \\ 0 & -\frac{5C}{2h} & 0 & 0 & \frac{5C}{2h} & 0 \\ 0 & 0 & -\frac{5C}{2h} & 0 & \frac{5C}{2h} & 0 \\ -\frac{5C}{2h} & 0 & 0 & 0 & \frac{5C}{2h} & 0 \\ 0 & -\frac{5C}{2h} & 0 & 0 & \frac{5C}{2h} & 0 \\ 0 & 0 & -\frac{5C}{2h} & 0 & \frac{5C}{2h} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ C & C & C & -\frac{5h\cdot G}{24} & \frac{5h\cdot G}{24}-3C & G\cdot L \\ C & C & C & 0 & -3C & \frac{5h}{24}-g\cdot L \end{bmatrix}$$

$$M_m = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Appendix D: Time Domain SCAQCF Model – Single Section Transmission Line Protection

This Appendix presents the derivation of the time domain model of a transmission line in the SCAQCF syntax. The model is first presented with what we call the compact form, which is the familiar standard notation model. We subsequently quadratize the model and then the quadratic model is integrated to provide the SCAQCF device model. A typical measurement channel list is also presented to show the procedure of creating the SCAQCF measurement model.

D1. Three Phase Transmission Line Compact Model

Figure D1 gives the circuit model of the three phase four wire transmission line.

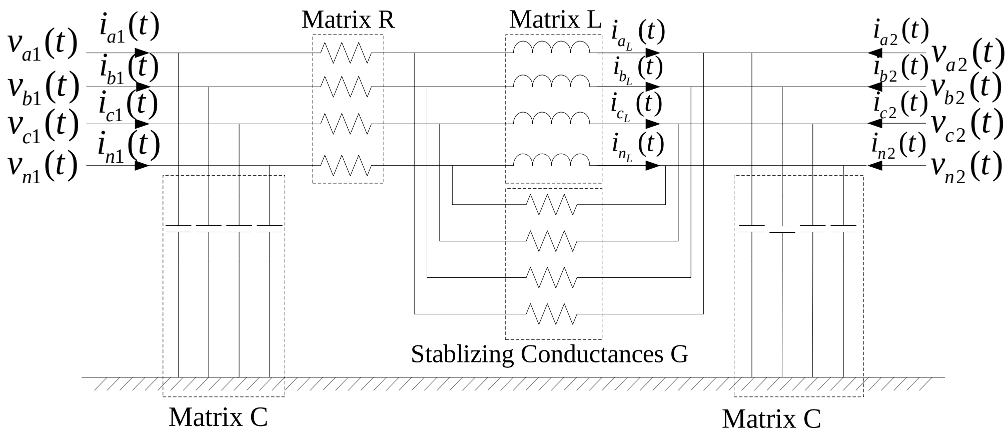

Figure D.1: The Three Phase Transmission Line Circuit Model

The compact transmission line model is derived from this circuit and it is given with the following equations:

$$i_1(t) = C\frac{dv_1(t)}{dt} + i_L(t) + GL\frac{di_L(t)}{dt}$$

$$i_2(t) = C\frac{dv_2(t)}{dt} - i_L(t) - GL\frac{di_L(t)}{dt}$$

$$0 = -v_1(t) + v_2(t) + R(i_L(t) + GL\frac{di_L(t)}{dt}) + L\frac{di_L(t)}{dt}$$

In compact matrix form, the model is:

$$\begin{bmatrix} i(t) \\ 0 \end{bmatrix} = A \cdot \begin{bmatrix} v(t) \\ i_L(t) \end{bmatrix} + B \cdot \frac{d}{dt}\begin{bmatrix} v(t) \\ i_L(t) \end{bmatrix}$$

where: $i(t) = \begin{bmatrix} i_{a1}(t) \\ i_{b1}(t) \\ i_{c1}(t) \\ i_{n1}(t) \\ i_{a2}(t) \\ i_{b2}(t) \\ i_{c2}(t) \\ i_{n2}(t) \end{bmatrix}$, $v(t) = \begin{bmatrix} v_{a1}(t) \\ v_{b1}(t) \\ v_{c1}(t) \\ v_{n1}(t) \\ v_{a2}(t) \\ v_{b2}(t) \\ v_{c2}(t) \\ v_{n2}(t) \end{bmatrix}$

$$A = \begin{bmatrix} 0 & 0 & I_{4\times4} \\ 0 & 0 & -I_{4\times4} \\ -I_{4\times4} & I_{4\times4} & R \end{bmatrix} \quad B = \begin{bmatrix} C & 0 & G\cdot L \\ 0 & C & -G\cdot L \\ 0 & 0 & R\cdot G\cdot L + L \end{bmatrix}$$

R, L, C, G are the resistance, inductance, capacitance and stabilizing conductance matrices of the transmission line (as shown in the above figure).

D2. Three Phase Transmission Line Quadratized Model

All the terms in the three phase transmission line compact model are linear and quadratic terms. Therefore the quadratized model is identical to the compact model. In compact matrix form, the model is:

$$\begin{bmatrix} i(t) \\ 0 \end{bmatrix} = A\cdot\begin{bmatrix} v(t) \\ i_L(t) \end{bmatrix} + B\cdot\frac{d}{dt}\begin{bmatrix} v(t) \\ i_L(t) \end{bmatrix}$$

where: $i(t) = \begin{bmatrix} i_{a1}(t) \\ i_{b1}(t) \\ i_{c1}(t) \\ i_{n1}(t) \\ i_{a2}(t) \\ i_{b2}(t) \\ i_{c2}(t) \\ i_{n2}(t) \end{bmatrix}$, $v(t) = \begin{bmatrix} v_{a1}(t) \\ v_{b1}(t) \\ v_{c1}(t) \\ v_{n1}(t) \\ v_{a2}(t) \\ v_{b2}(t) \\ v_{c2}(t) \\ v_{n2}(t) \end{bmatrix}$

$$A = \begin{bmatrix} 0 & 0 & I_{4\times4} \\ 0 & 0 & -I_{4\times4} \\ -I_{4\times4} & I_{4\times4} & R \end{bmatrix} \quad B = \begin{bmatrix} C & 0 & G\cdot L \\ 0 & C & -G\cdot L \\ 0 & 0 & R\cdot G\cdot L + L \end{bmatrix}$$

D3. Three Phase Transmission Line SCAQCF Device Model

The state and control algebraic quadratic companion form (SCAQCF) device model is derived after quadratic integration of the quadratized model with a time step h. The result is:

$$I(\mathbf{x}) = Y_{eqx} \cdot \mathbf{x} - B_{eq}$$

$$B_{eq} = -N_{eqx}\mathbf{x}(t-h) - N_{equ}\mathbf{u}(t-h) - M_{eq}I(t-h) - K_{eq}$$

where:

$$I(\mathbf{x}) = \begin{bmatrix} i(t) \\ 0 \\ 0 \\ 0 \\ 0 \\ i(t_m) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix} v(t) \\ i_L(t) \\ v(t_m) \\ i_L(t_m) \end{bmatrix}$$

$$i(t) = [i_{a1}(t), i_{b1}(t), i_{c1}(t), i_{n1}(t), i_{a2}(t), i_{b2}(t), i_{c2}(t), i_{n2}(t)]$$
$$i(t_m) = [i_{a1}(t_m), i_{b1}(t_m), i_{c1}(t_m), i_{n1}(t_m), i_{a2}(t_m), i_{b2}(t_m), i_{c2}(t_m), i_{n2}(t_m)]$$
$$v(t) = [v_{a1}(t), v_{b1}(t), v_{c1}(t), v_{n1}(t), v_{a2}(t), v_{b2}(t), v_{c2}(t), v_{n2}(t)]$$
$$v(t_m) = [v_{a1}(t_m), v_{b1}(t_m), v_{c1}(t_m), v_{n1}(t_m), v_{a2}(t_m), v_{b2}(t_m), v_{c2}(t_m), v_{n2}(t_m)]$$
$$i_L(t) = [i_{aL}(t), i_{bL}(t), i_{cL}(t), i_{nL}(t)]$$
$$i_L(t_m) = [i_{aL}(t_m), i_{bL}(t_m), i_{cL}(t_m), i_{nL}(t_m)]$$

$$Y_{eqx} = \begin{bmatrix} \frac{4}{h}C & 0 & I_4 + \frac{4}{h}GL & -\frac{8}{h}C & 0 & -\frac{8}{h}GL \\ 0 & \frac{4}{h}C & -I_4 - \frac{4}{h}GL & 0 & -\frac{8}{h}C & \frac{8}{h}GL \\ -I_4 & I_4 & R + \frac{4}{h}(RGL + L) & 0 & 0 & -\frac{8}{h}(RGL+L) \\ \frac{1}{2h}C & 0 & \frac{1}{2h}GL & \frac{2}{h}C & 0 & I_4 + \frac{2}{h}GL \\ 0 & \frac{1}{2h}C & -\frac{1}{2h}GL & 0 & \frac{2}{h}C & -I_4 - \frac{2}{h}GL \\ 0 & 0 & \frac{1}{2h}(RGL+L) & -I_4 & I_4 & R + \frac{2}{h}(RGL+L) \end{bmatrix}$$

$$N_{eqx} = \begin{bmatrix} \frac{8}{h}C & 0 & I_4 - \frac{4}{h}GL \\ 0 & \frac{8}{h}C & -I_4 + \frac{4}{h}GL \\ -I_4 & I_4 & R - \frac{4}{h}(RGL + L) \\ \frac{5}{2h}C & 0 & -\frac{1}{2}I_4 + \frac{5}{2h}GL \\ 0 & \frac{5}{2h}C & \frac{1}{2}I_4 - \frac{5}{2h}GL \\ \frac{1}{2}I_4 & -\frac{1}{2}I_4 & -\frac{1}{2}R + \frac{5}{2h}(RGL + L) \end{bmatrix}$$

$$M_{eq} = \begin{bmatrix} I_4 & 0 & 0 \\ 0 & I_4 & 0 \\ 0 & 0 & 0 \\ -\frac{1}{2}I_4 & 0 & 0 \\ 0 & -\frac{1}{2}I_4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The input matrices of the capacitor bank SCAQCF device model for the setting-less protection algorithm are shown below:

TerminalNodeName
YJLINE1_A
YJLINE1_B
YJLINE1_C
YJLINE1_N
YJLINE2_A
YJLINE2_B
YJLINE2_C
YJLINE2_N
-1

Xscale
0,115000.00
1,115000.00
2,115000.00
3,115000.00
4,115000.00
5,115000.00
6,115000.00
7,115000.00
8,500.00
9,500.00
10,500.00

11,500.00
12,115000.00
13,115000.00
14,115000.00
15,115000.00
16,115000.00
17,115000.00
18,115000.00
19,115000.00
20,500.00
21,500.00
22,500.00
23,500.00
-1

Iscale
0,500.00
1,500.00
2,500.00
3,500.00
4,500.00
5,500.00
6,500.00
7,500.00
8,115000.00
9,115000.00
10,115000.00
11,115000.00
12,500.00
13,500.00
14,500.00
15,500.00
16,500.00
17,500.00
18,500.00
19,500.00
20,115000.00
21,115000.00
22,115000.00
23,115000.00
-1

Yeqx
0,0,0.0059165000
0,1,-0.0014640000
0,2,-0.0007320000
0,3,-0.0011400000
0,8,1.6666666667
0,9,0.3422733078
0,10,0.3014048531
0,11,0.3346104725
0,12,-0.0118080000
0,13,0.0029280000
0,14,0.0014640000
0,15,0.0022800000

0,20,-1.3333333333
0,21,-0.6845466156
0,22,-0.6028097063
0,23,-0.6692209451
1,0,-0.0014640000
1,1,0.0060845000

-
-
-

23,22,0.0335621593
23,23,0.1158563363
-1

Feqx
-1

Neqx
0,0,0.0058915000
0,1,-0.0014640000
0,2,-0.0007320000

-
-
-

23,10,-0.0334556568
23,11,-0.1077480413
-1

Meq
0,0,1.0000000000
1,1,1.0000000000
2,2,1.0000000000
3,3,1.0000000000
4,4,1.0000000000
5,5,1.0000000000
6,6,1.0000000000
7,7,1.0000000000
12,0,-0.5000000000
13,1,-0.5000000000
14,2,-0.5000000000
15,3,-0.5000000000
16,4,-0.5000000000
17,5,-0.5000000000
18,6,-0.5000000000
19,7,-0.5000000000
-1

Keq
-1

D4. Three Phase Transmission Line SCAQCF Measurement Model

Specifically, for single section transmission line, the **actual measurements** are:
Three currents at left side at time t (phase A_1, phase B_1, and phase C_1);
Three currents at right side at time t (phase A_2, phase B_2, and phase C_2);
Three voltages at left side time t (phase A-G_1, phase B-G_1, phase C-G_1);
Three voltages at left side time t (phase A-G_2, phase B-G_2, phase C-G_2);
Three currents at left side at time tm (phase A_1, phase B_1, and phase C_1);
Three currents at right side at time tm (phase A_2, phase B_2, and phase C_2);
Three voltages at left side time tm (phase A-G_1, phase B-G_1, phase C-G_1);
Three voltages at left side time tm (phase A-G_2, phase B-G_2, phase C-G_2);

The **virtual measurements** are:
Four measurements with zero value at from KVL equation at time t;
Two measurements with zero value at from KVL equation at time tm=t-h/2;

The measurements definition is shown as follows:

MeasurementType, 16
MeasStdDev, 1080
MeasTerminal, 0, 3
MeasurementEnd
MeasurementType, 16
MeasStdDev, 1080
MeasTerminal, 1, 3
MeasurementEnd
MeasurementType, 16
MeasStdDev, 1080
MeasTerminal, 2, 3
MeasurementEnd
MeasurementType, 16
MeasStdDev, 1080
MeasTerminal, 4, 7
MeasurementEnd
MeasurementType, 16
MeasStdDev, 1080
MeasTerminal, 5, 7
MeasurementEnd
MeasurementType, 16
MeasStdDev, 1080
MeasTerminal, 6, 7
MeasurementEnd
MeasurementType, 17
MeasStdDev, 5.0
MeasTerminal, 0
MeasurementEnd
MeasurementType, 17
MeasStdDev, 5.0
MeasTerminal, 1
MeasurementEnd

MeasurementType, 17
MeasStdDev, 5.0
MeasTerminal, 2
MeasurementEnd
MeasurementType, 17
MeasStdDev, 5.0
MeasTerminal, 4
MeasurementEnd
MeasurementType, 17
MeasStdDev, 5.0
MeasTerminal, 5
MeasurementEnd
MeasurementType, 17
MeasStdDev, 5.0
MeasTerminal, 6
MeasurementEnd
MeasurementType, 24
MeasStdDev, 108
MeasTerminal, 8
MeasurementEnd
MeasurementType, 24
MeasStdDev, 108
MeasTerminal, 9
MeasurementEnd
MeasurementType, 24
MeasStdDev, 108
MeasTerminal, 10
MeasurementEnd
MeasurementType, 24
MeasStdDev, 108
MeasTerminal, 11
MeasurementEnd

Based on the device model and the measurement channel list, the equations for the measurement model are given below:

$$\mathbf{y}(\mathbf{x},\mathbf{u}) = Y_{m,x}\mathbf{x} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{m,x}^i \mathbf{x} \\ \vdots \end{Bmatrix} + Y_{m,u}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F_{m,u}^i \mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{m,xu}^i \mathbf{u} \\ \vdots \end{Bmatrix} + N_{m,x}\mathbf{x}(t-h) + N_{m,u}\mathbf{u}(t-h) + M_m I(t-h) + K_m$$

where:

$$Y_{mx} = \begin{bmatrix} K & 0 & 0 & 0 & 0 & 0 \\ 0 & K & 0 & 0 & 0 & 0 \\ \frac{4}{h}C & 0 & I_4 + \frac{4}{h}GL & -\frac{8}{h}C & 0 & -\frac{8}{h}GL \\ 0 & \frac{4}{h}C & -I_4 - \frac{4}{h}GL & 0 & -\frac{8}{h}C & \frac{8}{h}GL \\ -I_4 & I_4 & R + \frac{4}{h}(RGL + L) & 0 & 0 & -\frac{8}{h}(RGL + L) \\ 0 & 0 & 0 & K & 0 & 0 \\ 0 & 0 & 0 & 0 & K & 0 \\ \frac{1}{2h}C & 0 & \frac{1}{2h}GL & \frac{2}{h}C & 0 & I_4 + \frac{2}{h}GL \\ 0 & \frac{1}{2h}C & -\frac{1}{2h}GL & 0 & \frac{2}{h}C & -I_4 - \frac{2}{h}GL \\ 0 & 0 & \frac{1}{2h}(RGL + L) & -I_4 & I_4 & R + \frac{2}{h}(RGL + L) \end{bmatrix}$$

$$K = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$N_m = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{8}{h}C & 0 & I_4 - \frac{4}{h}GL \\ 0 & \frac{8}{h}C & -I_4 + \frac{4}{h}GL \\ -I_4 & I_4 & R - \frac{4}{h}(RGL + L) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{5}{2h}C & 0 & -\frac{1}{2}I_4 + \frac{5}{2h}GL \\ 0 & \frac{5}{2h}C & \frac{1}{2}I_4 - \frac{5}{2h}GL \\ \frac{1}{2}I_4 & -\frac{1}{2}I_4 & -\frac{1}{2}R + \frac{5}{2h}(RGL + L) \end{bmatrix}$$

$$M_{eq} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ I_4 & 0 & 0 \\ 0 & I_4 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{1}{2}I_4 & 0 & 0 \\ 0 & -\frac{1}{2}I_4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Appendix E: Time Domain SCAQCF Model – Multi-section Transmission Line Protection

This Appendix presents the derivation of the time domain model of a multi-section three phase transmission line in the SCAQCF syntax. The model is derived from integrating n sections of single-section transmission line model in Appendix D. A typical measurement channel list is also presented to show the procedure of creating the SCAQCF measurement model.

E1. Construction of Multi-section Transmission Line SCAQCF Model

Figure E1 shows the multi-section model of the three phase transmission line.

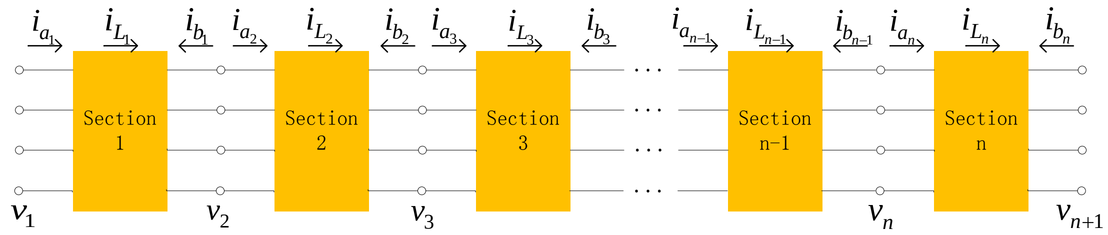

Figure E.1: Three Phase Multi-section Transmission Line Compact Model

Suppose Δt is the sampling time step of the system, and $h = 2\Delta t$. Similar to the single section model, three groups of values at time t, $t_m (= t - h/2)$ and $t - h$ are adopted.

For section i:

Parameters at time t are:

$i_{a_i}(t)$ and $i_{b_i}(t)$ represent 3-phase & neutral current at both sides of section i, at time t.

$v_i(t)$ and $v_{i+1}(t)$ represent 3-phase & neutral voltage at both sides of section i, at time t.

$i_{L_i}(t)$ represents three-phase & neutral current of the inductance in section i, at time t.

Parameters at time t_m are:

$i_{a_i}(t_m)$ and $i_{b_i}(t_m)$ represent 3-phase & neutral current at both sides of section i, at time t_m.

$v_i(t_m)$ and $v_{i+1}(t_m)$ represent 3-phase & neutral voltage at both sides of section i, at time t_m.

$i_{L_i}(t_m)$ represents three-phase & neutral current of the inductance in section i, at time t_m.

For each section, the formulation is similar to the single section model in Appendix D. For example, equations in section k are:

$$\begin{bmatrix} i_{a_k}(t) \\ i_{b_k}(t) \\ 0 \\ i_{a_k}(t_m) \\ i_{b_k}(t_m) \\ 0 \end{bmatrix} = E \cdot F_1 \begin{bmatrix} v_k(t) \\ v_{k+1}(t) \\ i_{L_k}(t) \\ v_k(t_m) \\ v_{k+1}(t_m) \\ i_{L_k}(t_m) \end{bmatrix} - \begin{bmatrix} b_{eq_k}(1) \\ b_{eq_k}(2) \\ b_{eq_k}(3) \\ b_{eq_k}(4) \\ b_{eq_k}(5) \\ b_{eq_k}(6) \end{bmatrix}$$

where:

$$\begin{bmatrix} b_{eq_k}(1) & b_{eq_k}(2) & b_{eq_k}(3) & b_{eq_k}(4) & b_{eq_k}(5) & b_{eq_k}(6) \end{bmatrix}^T = b_{eq_k}$$

and $b_{eq_k}(j)$ is a 4-element vector, $j = 1, 2, \cdots, 6$;

$$b_{eq_k} = E \cdot F_2 \cdot \begin{bmatrix} v_k(t-h) \\ v_{k+1}(t-h) \\ i_{L_k}(t-h) \end{bmatrix} + E \cdot F_3 \cdot \begin{bmatrix} i_{a_k}(t-h) \\ i_{b_k}(t-h) \\ 0 \end{bmatrix}$$

also,

$$E = \begin{bmatrix} \frac{h}{6} I_{12\times12} & \frac{2h}{3} I_{12\times12} \\ -\frac{h}{24} I_{12\times12} & \frac{h}{3} I_{12\times12} \end{bmatrix}^{-1} = \begin{bmatrix} \frac{4}{h} I_{12\times12} & -\frac{8}{h} I_{12\times12} \\ \frac{1}{2h} I_{12\times12} & \frac{2}{h} I_{12\times12} \end{bmatrix}$$

$$F_1 = \begin{bmatrix} \frac{h}{6} A + B & \frac{2h}{3} A \\ -\frac{h}{24} A & \frac{h}{3} A + B \end{bmatrix}, \quad F_2 = \begin{bmatrix} B - \frac{h}{6} A \\ B - \frac{5h}{24} A \end{bmatrix}, \quad F_3 = \begin{bmatrix} \frac{h}{6} I_{12\times12} \\ \frac{5h}{24} I_{12\times12} \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & I_{4\times4} \\ 0 & 0 & -I_{4\times4} \\ -I_{4\times4} & I_{4\times4} & R \end{bmatrix} \quad B = \begin{bmatrix} C & 0 & G \cdot L \\ 0 & C & -G \cdot L \\ 0 & 0 & R \cdot G \cdot L + L \end{bmatrix}$$

where R, L and C are the resistance, inductance and capacitance matrices of each section; G is the stabilizing conductance matrix of each section.

E2. Three Phase Multi-section Transmission Line SCAQCF Model

Divide $E \cdot F_1$ as follows, with each $M_{jk} : 4 \times 4$ $(j, k = 1, 2, ..., 6)$:

$$E \cdot F_1 = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} & M_{16} \\ M_{21} & M_{22} & M_{23} & M_{24} & M_{25} & M_{26} \\ M_{31} & M_{32} & M_{33} & M_{34} & M_{35} & M_{36} \\ M_{41} & M_{42} & M_{43} & M_{44} & M_{45} & M_{46} \\ M_{51} & M_{52} & M_{53} & M_{54} & M_{55} & M_{56} \\ M_{61} & M_{62} & M_{63} & M_{64} & M_{65} & M_{66} \end{bmatrix}$$

Divide $E \cdot F_2$ as follows, with each $P_{jk} : 4 \times 4 \ (j = 1,2,...,6, \ k = 1,2,3)$:

$$E \cdot F_2 = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \\ P_{41} & P_{42} & P_{43} \\ P_{51} & P_{52} & P_{53} \\ P_{61} & P_{62} & P_{63} \end{bmatrix}$$

Additionally,

$$E \cdot F_3 = \begin{bmatrix} \frac{4}{h} I_{12\times12} & -\frac{8}{h} I_{12\times12} \\ \frac{1}{2h} I_{12\times12} & \frac{2}{h} I_{12\times12} \end{bmatrix} \times \begin{bmatrix} \frac{h}{6} I_{12\times12} \\ \frac{5h}{24} I_{12\times12} \end{bmatrix} = \begin{bmatrix} -I_{12\times12} \\ 0.5 I_{12\times12} \end{bmatrix}$$

Thus, for time t:

Actual measurements:

$$\begin{aligned} i_{a_1}(t) &= M_{11}v_1(t) + M_{12}v_2(t) + M_{13}i_{L_1}(t) + M_{14}v_1(t_m) + M_{15}v_2(t_m) + M_{16}i_{L_1}(t_m) \\ &\quad -[P_{11}v_1(t-h) + P_{12}v_2(t-h) + P_{13}i_{L_1}(t-h)] - [-i_{a_1}(t-h)] \end{aligned}$$

$$\begin{aligned} i_{b_n}(t) &= M_{21}v_n(t) + M_{22}v_{n+1}(t) + M_{23}i_{L_n}(t) + M_{24}v_n(t_m) + M_{25}v_{n+1}(t_m) + M_{26}i_{L_n}(t_m) \\ &\quad -[P_{21}v_n(t-h) + P_{22}v_{n+1}(t-h) + P_{23}i_{L_n}(t-h)] - [-i_{b_n}(t-h)] \end{aligned}$$

Virtual current measurements:

$$\begin{aligned} & i_{b_1}(t) + i_{a_2}(t) = 0 \Rightarrow \\ & 0 = M_{21}v_1(t) + \left(M_{22} + M_{11}\right)v_2(t) + M_{12}v_3(t) + M_{23}i_{L_1}(t) + M_{13}i_{L_2}(t) \\ & +M_{24}v_1(t_m) + \left(M_{25} + M_{14}\right)v_2(t_m) + M_{15}v_3(t_m) + M_{26}i_{L_1}(t_m) + M_{16}i_{L_2}(t_m) \\ & -[P_{21}v_1(t-h) + \left(P_{22} + P_{11}\right)v_2(t-h) + P_{12}v_3(t-h) + P_{23}i_{L_1}(t-h) + P_{13}i_{L_2}(t-h)] \\ & -[-i_{b_1}(t-h) - i_{a_2}(t-h)] \end{aligned}$$

$$\begin{aligned} & i_{b_2}(t) + i_{a_3}(t) = 0 \Rightarrow \\ & 0 = M_{21}v_2(t) + \left(M_{22} + M_{11}\right)v_3(t) + M_{12}v_4(t) + M_{23}i_{L_2}(t) + M_{13}i_{L_3}(t) \\ & +M_{24}v_2(t_m) + \left(M_{25} + M_{14}\right)v_3(t_m) + M_{15}v_4(t_m) + M_{26}i_{L_2}(t_m) + M_{16}i_{L_3}(t_m) \\ & -[P_{21}v_2(t-h) + \left(P_{22} + P_{11}\right)v_3(t-h) + P_{12}v_4(t-h) + P_{23}i_{L_2}(t-h) + P_{13}i_{L_3}(t-h)] \\ & -[-i_{b_2}(t-h) - i_{a_3}(t-h)] \end{aligned}$$

$$\vdots$$

$$\begin{aligned} & i_{b_{n-1}}(t) + i_{a_n}(t) = 0 \Rightarrow \\ & 0 = M_{21}v_{n-1}(t) + \left(M_{22} + M_{11}\right)v_n(t) + M_{12}v_{n+1}(t) + M_{23}i_{L_{n-1}}(t) + M_{13}i_{L_n}(t) \\ & +M_{24}v_{n-1}(t_m) + \left(M_{25} + M_{14}\right)v_n(t_m) + M_{15}v_{n+1}(t_m) + M_{26}i_{L_{n-1}}(t_m) + M_{16}i_{L_n}(t_m) \\ & -[P_{21}v_{n-1}(t-h) + \left(P_{22} + P_{11}\right)v_n(t-h) + P_{12}v_{n+1}(t-h) + P_{23}i_{L_{n-1}}(t-h) + P_{13}i_{L_n}(t-h)] \\ & -[-i_{b_{n-1}}(t-h) - i_{a_n}(t-h)] \end{aligned}$$

Virtual voltage measurements:

$$\begin{aligned} 0 &= M_{31}v_1(t) + M_{32}v_2(t) + M_{33}i_{L_1}(t) + M_{34}v_1(t_m) + M_{35}v_2(t_m) + M_{36}i_{L_1}(t_m) \\ &\quad -[P_{31}v_1(t-h) + P_{32}v_2(t-h) + P_{33}i_{L_1}(t-h)] \end{aligned}$$

$$\begin{aligned} 0 &= M_{31}v_2(t) + M_{32}v_3(t) + M_{33}i_{L_2}(t) + M_{34}v_2(t_m) + M_{35}v_3(t_m) + M_{36}i_{L_2}(t_m) \\ &\quad -[P_{31}v_2(t-h) + P_{32}v_3(t-h) + P_{33}i_{L_2}(t-h)] \end{aligned}$$

$$\vdots$$

$$0 = M_{31}v_n(t) + M_{32}v_{n+1}(t) + M_{33}i_{L_n}(t) + M_{34}v_n(t_m) + M_{35}v_{n+1}(t_m) + M_{36}i_{L_n}(t_m)$$
$$-[P_{31}v_n(t-h) + P_{32}v_{n+1}(t-h) + P_{33}i_{L_n}(t-h)]$$

For time t_m:

Actual measurements:

$$i_{a_1}(t_m) = M_{41}v_1(t) + M_{42}v_2(t) + M_{43}i_{L_1}(t) + M_{44}v_1(t_m) + M_{45}v_2(t_m) + M_{46}i_{L_1}(t_m)$$
$$-[P_{41}v_1(t-h) + P_{42}v_2(t-h) + P_{43}i_{L_1}(t-h)] - [0.5i_{a_1}(t-h)]$$

$$i_{b_n}(t_m) = M_{51}v_n(t) + M_{52}v_{n+1}(t) + M_{53}i_{L_n}(t) + M_{54}v_n(t_m) + M_{55}v_{n+1}(t_m) + M_{56}i_{L_n}(t_m)$$
$$-[P_{51}v_n(t-h) + P_{52}v_{n+1}(t-h) + P_{53}i_{L_n}(t-h)] - [0.5i_{b_n}(t-h)]$$

Virtual current measurements:

$$i_{b_1}(t_m) + i_{a_2}(t_m) = 0 \Rightarrow$$
$$0 = M_{51}v_1(t) + (M_{52} + M_{41})v_2(t) + M_{42}v_3(t) + M_{53}i_{L_1}(t) + M_{43}i_{L_2}(t)$$
$$+M_{54}v_1(t_m) + (M_{55} + M_{44})v_2(t_m) + M_{45}v_3(t_m) + M_{56}i_{L_1}(t_m) + M_{46}i_{L_2}(t_m)$$
$$-[P_{51}v_1(t-h) + (P_{52} + P_{41})v_2(t-h) + P_{42}v_3(t-h) + P_{53}i_{L_1}(t-h) + P_{43}i_{L_2}(t-h)]$$
$$-[0.5i_{b_1}(t-h) + 0.5i_{a_2}(t-h)]$$

$$i_{b_2}(t_m) + i_{a_3}(t_m) = 0 \Rightarrow$$
$$0 = M_{51}v_2(t) + (M_{52} + M_{41})v_3(t) + M_{42}v_4(t) + M_{53}i_{L_2}(t) + M_{43}i_{L_3}(t)$$
$$+M_{54}v_2(t_m) + (M_{55} + M_{44})v_3(t_m) + M_{45}v_4(t_m) + M_{56}i_{L_2}(t_m) + M_{46}i_{L_3}(t_m)$$
$$-[P_{51}v_2(t-h) + (P_{52} + P_{41})v_3(t-h) + P_{42}v_4(t-h) + P_{53}i_{L_2}(t-h) + P_{43}i_{L_3}(t-h)]$$
$$-[0.5i_{b_2}(t-h) + 0.5i_{a_3}(t-h)]$$

$$\vdots$$

$$i_{b_{n-1}}(t_m) + i_{a_n}(t_m) = 0 \Rightarrow$$
$$0 = M_{51}v_{n-1}(t) + (M_{52} + M_{41})v_n(t) + M_{42}v_{n+1}(t) + M_{53}i_{L_{n-1}}(t) + M_{43}i_{L_n}(t)$$
$$+M_{54}v_{n-1}(t_m) + (M_{55} + M_{44})v_n(t_m) + M_{45}v_{n+1}(t_m) + M_{56}i_{L_{n-1}}(t_m) + M_{46}i_{L_n}(t_m)$$
$$-[P_{51}v_{n-1}(t-h) + (P_{52} + P_{41})v_n(t-h) + P_{42}v_{n+1}(t-h) + P_{53}i_{L_{n-1}}(t-h) + P_{43}i_{L_n}(t-h)]$$
$$-[0.5i_{b_{n-1}}(t-h) + 0.5i_{a_n}(t-h)]$$

Virtual voltage measurements:

$$0 = M_{61}v_1(t) + M_{62}v_2(t) + M_{63}i_{L_1}(t) + M_{64}v_1(t_m) + M_{65}v_2(t_m) + M_{66}i_{L_1}(t_m)$$
$$-[P_{61}v_1(t-h) + P_{62}v_2(t-h) + P_{63}i_{L_1}(t-h)]$$
$$0 = M_{61}v_2(t) + M_{62}v_3(t) + M_{63}i_{L_2}(t) + M_{64}v_2(t_m) + M_{65}v_3(t_m) + M_{66}i_{L_2}(t_m)$$
$$-[P_{61}v_2(t-h) + P_{62}v_3(t-h) + P_{63}i_{L_2}(t-h)]$$
$$\vdots$$
$$0 = M_{61}v_n(t) + M_{62}v_{n+1}(t) + M_{63}i_{L_n}(t) + M_{64}v_n(t_m) + M_{65}v_{n+1}(t_m) + M_{66}i_{L_n}(t_m)$$
$$-[P_{61}v_n(t-h) + P_{62}v_{n+1}(t-h) + P_{63}i_{L_n}(t-h)]$$

Write the above equations into the standard SCAQCF matrix form: (no control variable in this model)

$$I(x) = Y_{eqx} \cdot x - B_{eq}$$

where:

$$I(x) = \begin{bmatrix} i_{a_1}(t) \\ i_{b_n}(t) \\ [0 \ \ 0 \ \ \cdots \ \ 0]^T \\ [0 \ \ 0 \ \ \cdots \ \ 0]^T \\ i_{a_1}(t_m) \\ i_{b_n}(t_m) \\ [0 \ \ 0 \ \ \cdots \ \ 0]^T \\ [0 \ \ 0 \ \ \cdots \ \ 0]^T \end{bmatrix} \qquad x = \begin{bmatrix} v_1(t) \\ v_{n+1}(t) \\ [v_2(t) \ \ v_3(t) \ \ \cdots \ \ v_n(t)]^T \\ [i_{L_1}(t) \ \ i_{L_2}(t) \ \ \cdots \ \ i_{L_n}(t)]^T \\ v_1(t_m) \\ v_{n+1}(t_m) \\ [v_2(t_m) \ \ v_3(t_m) \ \ \cdots \ \ v_n(t_m)]^T \\ [i_{L_1}(t_m) \ \ i_{L_2}(t_m) \ \ \cdots \ \ i_{L_n}(t_m)]^T \end{bmatrix}$$

$$Y_{eqx} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \quad Y_{11} = \begin{bmatrix} M_{11} & 0 & M_{12} & 0 & 0 & \cdots & 0 & 0 \\ 0 & M_{22} & 0 & 0 & 0 & \cdots & 0 & M_{21} \\ M_{21} & 0 & M_{22}+M_{11} & M_{12} & 0 & \cdots & 0 & 0 \\ 0 & 0 & M_{21} & M_{22}+M_{11} & M_{12} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & M_{12} & 0 & 0 & 0 & \cdots & M_{21} & M_{22}+M_{11} \end{bmatrix}_{(4n+4)\times(4n+4)}$$

$$Y_{13} = \begin{bmatrix} M_{14} & 0 & M_{15} & 0 & 0 & \cdots & 0 & 0 \\ 0 & M_{25} & 0 & 0 & 0 & \cdots & 0 & M_{24} \\ M_{24} & 0 & M_{25}+M_{14} & M_{15} & 0 & \cdots & 0 & 0 \\ 0 & 0 & M_{24} & M_{25}+M_{14} & M_{15} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & M_{15} & 0 & 0 & 0 & \cdots & M_{24} & M_{25}+M_{14} \end{bmatrix}_{(4n+4)\times(4n+4)}$$

$$Y_{12} = \begin{bmatrix} M_{13} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & M_{23} \\ M_{23} & M_{13} & 0 & \cdots & 0 & 0 \\ 0 & M_{23} & M_{13} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & M_{23} & M_{13} \end{bmatrix}_{(4n+4)\times(4n)} \qquad Y_{14} = \begin{bmatrix} M_{16} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & M_{26} \\ M_{26} & M_{16} & 0 & \cdots & 0 & 0 \\ 0 & M_{26} & M_{16} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & M_{26} & M_{16} \end{bmatrix}_{(4n+4)\times(4n)}$$

$$Y_{21}=\begin{bmatrix} M_{31} & 0 & M_{32} & 0 & \cdots & 0 \\ 0 & 0 & M_{31} & M_{32} & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & M_{32} & 0 & 0 & \cdots & M_{31} \end{bmatrix}_{(4n)\times(4n+4)} \qquad Y_{22}=\begin{bmatrix} M_{33} & 0 & 0 & 0 \\ 0 & M_{33} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & M_{33} \end{bmatrix}_{(4n)\times(4n)}$$

$$Y_{23}=\begin{bmatrix} M_{34} & 0 & M_{35} & 0 & \cdots & 0 \\ 0 & 0 & M_{34} & M_{35} & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & M_{35} & 0 & 0 & \cdots & M_{34} \end{bmatrix}_{(4n)\times(4n+4)} \qquad Y_{24}=\begin{bmatrix} M_{36} & 0 & 0 & 0 \\ 0 & M_{36} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & M_{36} \end{bmatrix}_{(4n)\times(4n)}$$

$$Y_{31}=\begin{bmatrix} M_{41} & 0 & M_{42} & 0 & 0 & \cdots & 0 & 0 \\ 0 & M_{52} & 0 & 0 & 0 & \cdots & 0 & M_{51} \\ M_{51} & 0 & M_{52}+M_{41} & M_{42} & 0 & \cdots & 0 & 0 \\ 0 & 0 & M_{51} & M_{52}+M_{41} & M_{42} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & M_{42} & 0 & 0 & 0 & \cdots & M_{51} & M_{52}+M_{41} \end{bmatrix}_{(4n+4)\times(4n+4)}$$

$$Y_{33}=\begin{bmatrix} M_{44} & 0 & M_{45} & 0 & 0 & \cdots & 0 & 0 \\ 0 & M_{55} & 0 & 0 & 0 & \cdots & 0 & M_{54} \\ M_{54} & 0 & M_{55}+M_{44} & M_{45} & 0 & \cdots & 0 & 0 \\ 0 & 0 & M_{54} & M_{55}+M_{44} & M_{45} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & M_{45} & 0 & 0 & 0 & \cdots & M_{54} & M_{55}+M_{44} \end{bmatrix}_{(4n+4)\times(4n+4)}$$

$$Y_{32}=\begin{bmatrix} M_{43} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & M_{53} \\ M_{53} & M_{43} & 0 & \cdots & 0 & 0 \\ 0 & M_{53} & M_{43} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & M_{53} & M_{43} \end{bmatrix}_{(4n+4)\times(4n)} \qquad Y_{34}=\begin{bmatrix} M_{46} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & M_{56} \\ M_{56} & M_{46} & 0 & \cdots & 0 & 0 \\ 0 & M_{56} & M_{46} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & M_{56} & M_{46} \end{bmatrix}_{(4n+4)\times(4n)}$$

$$Y_{41}=\begin{bmatrix} M_{61} & 0 & M_{62} & 0 & \cdots & 0 \\ 0 & 0 & M_{61} & M_{62} & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & M_{62} & 0 & 0 & \cdots & M_{61} \end{bmatrix}_{(4n)\times(4n+4)} \qquad Y_{42}=\begin{bmatrix} M_{63} & 0 & 0 & 0 \\ 0 & M_{63} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & M_{63} \end{bmatrix}_{(4n)\times(4n)}$$

$$Y_{43}=\begin{bmatrix} M_{64} & 0 & M_{65} & 0 & \cdots & 0 \\ 0 & 0 & M_{64} & M_{65} & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & M_{65} & 0 & 0 & \cdots & M_{64} \end{bmatrix}_{(4n)\times(4n+4)} \qquad Y_{44}=\begin{bmatrix} M_{66} & 0 & 0 & 0 \\ 0 & M_{66} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & M_{66} \end{bmatrix}_{(4n)\times(4n)}$$

and,

$$B_{eq}=-N_{eqx}\begin{bmatrix} v_1(t-h) \\ v_{n+1}(t-h) \\ \begin{bmatrix} v_2(t-h) & v_3(t-h) & \cdots & v_n(t-h) \end{bmatrix}^T \\ \begin{bmatrix} i_{L_1}(t-h) & i_{L_2}(t-h) & \cdots & i_{L_n}(t-h) \end{bmatrix}^T \end{bmatrix} - M_{eq}\begin{bmatrix} i_{a_1}(t-h) \\ i_{b_n}(t-h) \\ \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}^T \\ \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}^T \end{bmatrix}$$

where:

$$N_{11}=\begin{bmatrix} P_{11} & 0 & P_{12} & 0 & 0 & \cdots & 0 & 0 \\ 0 & P_{22} & 0 & 0 & 0 & \cdots & 0 & P_{21} \\ P_{21} & 0 & P_{22}+P_{11} & P_{12} & 0 & \cdots & 0 & 0 \\ 0 & 0 & P_{21} & P_{22}+P_{11} & P_{12} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & P_{12} & 0 & 0 & 0 & \cdots & P_{21} & P_{22}+P_{11} \end{bmatrix}_{(4n+4)\times(4n+4)}$$

$$N_{31}=\begin{bmatrix} P_{41} & 0 & P_{42} & 0 & 0 & \cdots & 0 & 0 \\ 0 & P_{52} & 0 & 0 & 0 & \cdots & 0 & P_{51} \\ P_{51} & 0 & P_{52}+P_{41} & P_{42} & 0 & \cdots & 0 & 0 \\ 0 & 0 & P_{51} & P_{52}+P_{41} & P_{42} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & P_{42} & 0 & 0 & 0 & \cdots & P_{51} & P_{52}+P_{41} \end{bmatrix}_{(4n+4)\times(4n+4)}$$

$$N_{12}=\begin{bmatrix} P_{13} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & P_{23} \\ P_{23} & P_{13} & 0 & \cdots & 0 & 0 \\ 0 & P_{23} & P_{13} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & P_{23} & P_{13} \end{bmatrix}_{(4n+4)\times(4n)} \qquad N_{32}=\begin{bmatrix} P_{43} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & P_{53} \\ P_{53} & P_{43} & 0 & \cdots & 0 & 0 \\ 0 & P_{53} & P_{43} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & P_{53} & P_{43} \end{bmatrix}_{(4n+4)\times(4n)}$$

$$N_{21}=\begin{bmatrix} P_{31} & 0 & P_{32} & 0 & \cdots & 0 \\ 0 & 0 & P_{31} & P_{32} & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & P_{32} & 0 & 0 & \cdots & P_{31} \end{bmatrix}_{(4n)\times(4n+4)} \qquad N_{22}=\begin{bmatrix} P_{33} & 0 & 0 & 0 \\ 0 & P_{33} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & P_{33} \end{bmatrix}_{(4n)\times(4n)}$$

$$N_{41}=\begin{bmatrix} P_{61} & 0 & P_{62} & 0 & \cdots & 0 \\ 0 & 0 & P_{61} & P_{62} & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & P_{62} & 0 & 0 & \cdots & P_{61} \end{bmatrix}_{(4n)\times(4n+4)} \qquad N_{42}=\begin{bmatrix} P_{63} & 0 & 0 & 0 \\ 0 & P_{63} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & P_{63} \end{bmatrix}_{(4n)\times(4n)}$$

$$N_{eqx}=-\begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \\ N_{31} & N_{32} \\ N_{41} & N_{42} \end{bmatrix} \quad M_{eq}=\begin{bmatrix} I_{8\times8} & 0 \\ 0 & 0 \\ -0.5I_{8\times8} & 0 \\ 0 & 0 \end{bmatrix}$$

Use 2-section transmission line as an example. The input matrices of the three phase multi-section transmission line SCAQCF device model for the setting-less protection algorithm are shown below:

TerminalNodeName
TABL_A
TABL_B
TABL_C
TABL_N
VACA_A
VACA_B

VACA_C
VACA_N
-1
Xscale

0	,	*500000*
1	,	*500000*
2	,	*500000*
3	,	*500000*
4	,	*500000*
5	,	*500000*
6	,	*500000*
7	,	*500000*
8	,	*500000*
9	,	*500000*
10	,	*500000*
11	,	*500000*
12	,	*4041*
13	,	*4041*
14	,	*4041*
15	,	*4041*
16	,	*4041*
17	,	*4041*
18	,	*4041*
19	,	*4041*
20	,	*500000*
21	,	*500000*
22	,	*500000*
23	,	*500000*
24	,	*500000*
25	,	*500000*
26	,	*500000*
27	,	*500000*
28	,	*500000*
29	,	*500000*
30	,	*500000*
31	,	*500000*
32	,	*4041*
33	,	*4041*
34	,	*4041*
35	,	*4041*
36	,	*4041*
37	,	*4041*
38	,	*4041*
39	,	*4041*

-1

Iscale

0	*,*	*4041*
1	*,*	*4041*
2	*,*	*4041*
3	*,*	*4041*
4	*,*	*4041*
5	*,*	*4041*
6	*,*	*4041*
7	*,*	*4041*
8	*,*	*4041*
9	*,*	*4041*
10	*,*	*4041*
11	*,*	*4041*
12	*,*	*500000*
13	*,*	*500000*
14	*,*	*500000*
15	*,*	*500000*
16	*,*	*500000*
17	*,*	*500000*
18	*,*	*500000*
19	*,*	*500000*
20	*,*	*4041*
21	*,*	*4041*
22	*,*	*4041*
23	*,*	*4041*
24	*,*	*4041*
25	*,*	*4041*
26	*,*	*4041*
27	*,*	*4041*
28	*,*	*4041*
29	*,*	*4041*
30	*,*	*4041*
31	*,*	*4041*
32	*,*	*500000*
33	*,*	*500000*
34	*,*	*500000*
35	*,*	*500000*
36	*,*	*500000*
37	*,*	*500000*
38	*,*	*500000*
39	*,*	*500000*

-1

Yeqx ***(totally 640 non-zero entries)***

0	*,*	*0*	*,*	*0.0072096*
0	*,*	*1*	*,*	*-0.00168812*
0	*,*	*2*	*,*	*-0.00059576*
0	*,*	*3*	*,*	*-0.0010674*
0	*,*	*12*	*,*	*2*
0	*,*	*13*	*,*	*0.530225548*
0	*,*	*14*	*,*	*0.450162387*
0	*,*	*15*	*,*	*0.527372557*
0	*,*	*20*	*,*	*-0.0144192*
0	*,*	*21*	*,*	*0.00337624*
0	*,*	*22*	*,*	*0.00119152*
0	*,*	*23*	*,*	*0.0021348*
0	*,*	*32*	*,*	*-2*
0	*,*	*33*	*,*	*-1.060451095*
0	*,*	*34*	*,*	*-0.900324774*
0	*,*	*35*	*,*	*-1.054745115*
1	*,*	*0*	*,*	*-0.00168812*
1	*,*	*1*	*,*	*0.0076404*
1	*,*	*2*	*,*	*-0.00168812*
1	*,*	*3*	*,*	*-0.000751*
1	*,*	*12*	*,*	*0.530225548*
1	*,*	*13*	*,*	*2*
1	*,*	*14*	*,*	*0.530225548*
1	*,*	*15*	*,*	*0.506414742*
1	*,*	*20*	*,*	*0.00337624*
1	*,*	*21*	*,*	*-0.0152808*
		⋮		
39	*,*	*39*	*,*	*2288.734852*

-1

Feqx

-1

Neqx ***(totally 320 non-zero entries)***

0	*,*	*0*	*,*	*0.0072096*
0	*,*	*1*	*,*	*-0.00168812*
0	*,*	*2*	*,*	*-0.00059576*
0	*,*	*3*	*,*	*-0.0010674*
0	*,*	*12*	*,*	*0*
0	*,*	*13*	*,*	*0.530225548*
0	*,*	*14*	*,*	*0.450162387*

0 , 15 , 0.527372557
1 , 0 , -0.00168812
1 , 1 , 0.0076404
1 , 2 , -0.00168812
1 , 3 , -0.000751
1 , 12 , 0.530225548
1 , 13 , 0
1 , 14 , 0.530225548
1 , 15 , 0.506414742
2 , 0 , -0.00059576
2 , 1 , -0.00168812
2 , 2 , 0.0072096
2 , 3 , -0.000399056
2 , 12 , 0.450162387
2 , 13 , 0.530225548
2 , 14 , 0
2 , 15 , 0.451167214
3 , 0 , -0.0010674
3 , 1 , -0.000751
3 , 2 , -0.000399056
3 , 3 , 0.0044192
3 , 12 , 0.264879236
3 , 13 , 0.25435292
3 , 14 , 0.226604182
3 , 15 , 0
4 , 4 , 0.0072096
⋮
39 , 19 , -2781.311065
-1
Meq
0 , 0 , 1
1 , 1 , 1
2 , 2 , 1
3 , 3 , 1
4 , 4 , 1
5 , 5 , 1
6 , 6 , 1
7 , 7 , 1
20 , 0 , -0.5
21 , 1 , -0.5
22 , 2 , -0.5
23 , 3 , -0.5
24 , 4 , -0.5

25	*,*	*5*	*,*	*-0.5*
26	*,*	*6*	*,*	*-0.5*
27	*,*	*7*	*,*	*-0.5*

-1

Keq
-1

E3. Three Phase Multi-section Transmission Line SCAQCF Measurement Model

For multi-section transmission line, the **actual measurements** are:
Six currents at both side of the transmission line, at time t (phase A, phase B, and phase C);
Six voltages at both side of the transmission line, at time t (phase A-N, phase B-N, phase C-N);
Six currents at both side of the transmission line, at time tm=t-h/2 (phase A, phase B, and phase C);
Six voltages at both side of the transmission line, at time tm=t-h/2 (phase A-N, phase B-N, phase C-N);

The **pseudo measurements** are:
Two neutral currents at both side of the transmission line, at time t (phase N);
Two neutral voltages at both side of the transmission line, at time t (phase N);
Two neutral currents at both side of the transmission line, at time tm=t-h/2 (phase N);
Two neutral voltages at both side of the transmission line, at time tm=t-h/2 (phase N);

The **virtual measurements** are:
Twelve measurements with zero value at the left side of the equations at time t, including four virtual current measurements and eight virtual voltage measurements;
Twelve measurements with zero value at the left side of the equations at time tm=t-h/2, including four virtual current measurements and eight virtual voltage measurements;

The measurement channel list:
MeasurementType, 17
MeasStdDev, 40.41
MeasTerminal, 0
MeasurementEnd
MeasurementType, 17
MeasStdDev, 40.41
MeasTerminal, 1
MeasurementEnd
MeasurementType, 17
MeasStdDev, 40.41
MeasTerminal, 2
MeasurementEnd
MeasurementType, 17

MeasStdDev, 40.41
MeasTerminal, 4
MeasurementEnd
MeasurementType, 17
MeasStdDev, 40.41
MeasTerminal, 5
MeasurementEnd
MeasurementType, 17
MeasStdDev, 40.41
MeasTerminal, 6
MeasurementEnd
MeasurementType, 16
MeasStdDev, 5000
MeasTerminal, 0, 3
MeasurementEnd
MeasurementType, 16
MeasStdDev, 5000
MeasTerminal, 1, 3
MeasurementEnd
MeasurementType, 16
MeasStdDev, 5000
MeasTerminal, 2, 3
MeasurementEnd
MeasurementType, 16
MeasStdDev, 5000
MeasTerminal, 4, 7
MeasurementEnd
MeasurementType, 16
MeasStdDev, 5000
MeasTerminal, 5, 7
MeasurementEnd
MeasurementType, 16
MeasStdDev, 5000
MeasTerminal, 6, 7
MeasurementEnd
MeasurementType, 27
MeasStdDev, 404.1
MeasTerminal, 3
MeasurementEnd
MeasurementType, 27
MeasStdDev, 404.1
MeasTerminal, 7
MeasurementEnd
MeasurementType, 25
MeasStdDev, 50000
MeasTerminal, 3

MeasurementEnd
MeasurementType, 25
MeasStdDev, 50000
MeasTerminal, 7
MeasurementEnd
MeasurementType, 24
MeasStdDev, 4.041
MeasTerminal, 8
MeasurementEnd
MeasurementType, 24
MeasStdDev, 4.041
MeasTerminal, 9
MeasurementEnd
MeasurementType, 24
MeasStdDev, 4.041
MeasTerminal, 10
MeasurementEnd
MeasurementType, 24
MeasStdDev, 4.041
MeasTerminal, 11
MeasurementEnd
MeasurementType, 24
MeasStdDev, 500
MeasTerminal, 12
MeasurementEnd
MeasurementType, 24
MeasStdDev, 500
MeasTerminal, 13
MeasurementEnd
MeasurementType, 24
MeasStdDev, 500
MeasTerminal, 14
MeasurementEnd
MeasurementType, 24
MeasStdDev, 500
MeasTerminal, 15
MeasurementEnd
MeasurementType, 24
MeasStdDev, 500
MeasTerminal, 16
MeasurementEnd
MeasurementType, 24
MeasStdDev, 500
MeasTerminal, 17
MeasurementEnd
MeasurementType, 24

MeasStdDev, 500
MeasTerminal, 18
MeasurementEnd
MeasurementType, 24
MeasStdDev, 500
MeasTerminal, 19
MeasurementEnd

Based on the device model and the measurement channel list, the standard SCAQCF equations for the measurement model are given below:

$$\mathbf{y}(\mathbf{x},\mathbf{u}) = Y_{m,x}\mathbf{x} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{m,x}^i \mathbf{x} \\ \vdots \end{Bmatrix} + Y_{m,u}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F_{m,u}^i \mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{m,xu}^i \mathbf{u} \\ \vdots \end{Bmatrix} + N_{m,x}\mathbf{x}(t-h) + N_{m,u}\mathbf{u}(t-h) + M_m I(t-h) + K_m$$

where:

$$Y_{m,x} = \begin{bmatrix} T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & T & 0 & 0 & 0 & 0 & 0 & 0 \\ \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \end{bmatrix} \\ 0 & 0 & 0 & 0 & T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & T & 0 & 0 \\ \begin{bmatrix} Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \end{bmatrix} \quad N_{m,x} = -\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \begin{bmatrix} N_{31} & N_{32} \\ N_{41} & N_{42} \end{bmatrix} \end{bmatrix} \quad M_{m,x} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \begin{bmatrix} I_{8\times 8} & 0 \\ 0 & 0 \end{bmatrix} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \begin{bmatrix} -0.5I_{8\times 8} & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

And, the definitions of Y_{ij} and N_{ij} are given in Appendix E.2.

Appendix F: Time Domain SCAQCF Model – Single Phase Saturable Core Variable Tap Transformer Protection

This document describes the model of a single-phase, two-winding, saturable-core variable tap transformer. The model is based on a standard transformer equivalent circuit, but the core magnetizing reactance is modeled as a nonlinear inductor with a current characteristic equation of high degree of nonlinearity.

F1. Single Phase Saturable Core Transformer Compact Model

The overall physical model of a single phase, saturable core variable tap transformer is illustrated in Figure F1. Core losses are also considered in the model, expressed by the conductance g_c. The "numerical stabilizers" g_{s1}, g_{s2}, g_{s3} *and* g_{s4} are introduced to eliminate possible numerical problems. The compact model is based on the circuit analysis of the equivalent circuit of Figure F1.

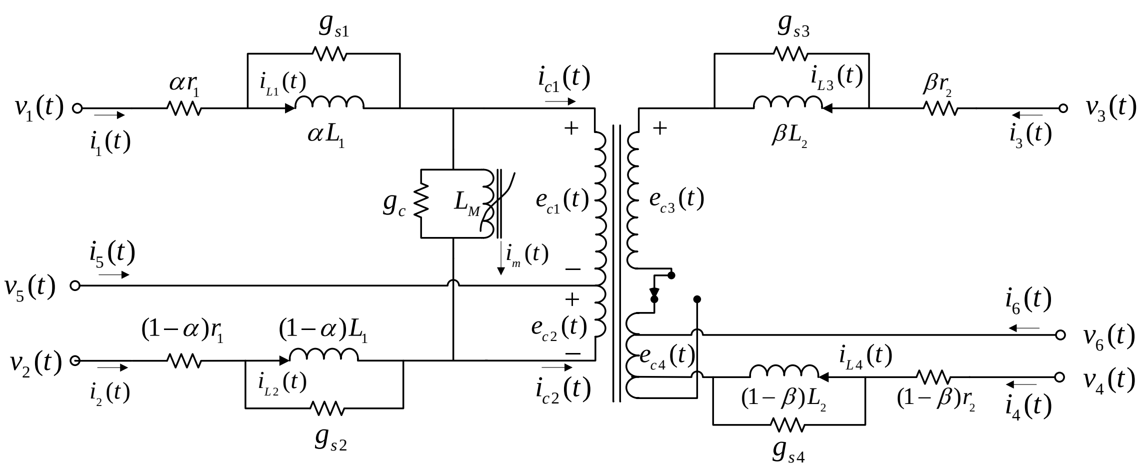

Figure F.1: Single Phase Transformer Equivalent Circuit

The model with nonlinear magnetizing inductor in the compact form is described by equations (3.1) through (3.18)

$$i_1(t) = i_{L1}(t) + g_{s1}\alpha L_1 \frac{di_{L1}(t)}{dt} \tag{3.1}$$

$$i_2(t) = i_{L2}(t) + g_{s2}(1-\alpha)L_1 \frac{di_{L2}(t)}{dt} \tag{3.2}$$

$$i_3(t) = i_{L3}(t) + g_{s3}\beta L_2 \frac{di_{L3}(t)}{dt} \tag{3.3}$$

$$i_4(t) = i_{L4}(t) + g_{s4}(1-\beta)L_2\frac{di_{L4}(t)}{dt} \tag{3.4}$$

$$i_5(t) = -i_1(t) - i_2(t) \tag{3.5}$$

$$i_6(t) = -i_3(t) - i_4(t) \tag{3.6}$$

$$0 = v_1(t) - v_5(t) - \alpha r_1 i_1(t) - \alpha L_1\frac{di_{L1}(t)}{dt} - e_{c1}(t) \tag{3.7}$$

$$0 = v_2(t) - v_5(t) - (1-\alpha)r_1 i_2(t) - (1-\alpha)L_1\frac{di_{L2}(t)}{dt} + e_{c2}(t) \tag{3.8}$$

$$0 = v_3(t) - v_6(t) - \beta r_2 i_3(t) - \beta L_2\frac{di_{L3}(t)}{dt} - e_{c3}(t) \tag{3.9}$$

$$0 = v_4(t) - v_6(t) - (1-\beta)r_2 i_4(t) - (1-\beta)L_2\frac{di_{L4}(t)}{dt} + e_{c4}(t) \tag{3.10}$$

$$0 = \alpha N_1 i_{c1}(t) - (1-\alpha)N_1 i_{c2}(t) + \beta N_2 i_3(t) - (1-\beta)N_2 i_4(t) \tag{3.11}$$

$$0 = e_{c1}(t) - \alpha\frac{d\lambda(t)}{dt} \tag{3.12}$$

$$0 = e_{c2}(t) - (1-\alpha)\frac{d\lambda(t)}{dt} \tag{3.13}$$

$$0 = e_{c3}(t) - \beta\frac{N_2}{N_1}\frac{d\lambda(t)}{dt} \tag{3.14}$$

$$0 = e_{c4}(t) - (1-\beta)\frac{N_2}{N_1}\frac{d\lambda(t)}{dt} \tag{3.15}$$

$$0 = -i_1(t) + i_{c1}(t) + i_m(t) + g_c\big(e_{c1}(t) + e_{c2}(t)\big) \tag{3.16}$$

$$0 = -i_2(t) + i_{c2}(t) - i_m(t) - g_c\big(e_{c1}(t) + e_{c2}(t)\big) \tag{3.17}$$

$$0 = i_m(t) - i_0\left|\frac{\lambda(t)}{\lambda_0}\right|^n sign(\lambda(t)) \tag{3.18}$$

$$0.9 < t(t) < 1.1$$

There are 18 states variables

$$X = \begin{bmatrix} v_1(t), v_2(t), v_3(t), v_4(t), v_5(t), v_6(t), \\ i_{L1}(t), i_{L2}(t), i_{L3}(t), i_{L4}(t), \lambda(t) \\ e_{c1}(t), e_{c2}(t), e_{c3}(t), e_{c4}(t), i_{c2}(t), i_{c2}(t), i_m(t) \end{bmatrix}$$

And 6 though variables

$$I = [i_1(t), i_2(t), i_3(t), i_4(t), i_5(t), i_6(t)]$$

And 1 control variable

$$U = [t(t)]$$

F2. Single Phase Saturable Core Transformer Quadratic Model

The model is quadratized by introducing additional internal state variables, so that the n^th exponent is replaced by equations of at most quadratic degree. Since the exact degree of nonlinearity is not known until the user specifies it, the model performs automatic quadratization of the equations. A special procedure is used, so that the model is quadratized using the minimum number of additional internal states, while also maintaining the scarcity of the resulting equations. The methodology is based on expressing the exponent in binary form. The binary representation provides all the information about the number of new variables and equations that need to be introduced and about the form of the equations (products of new variables). The procedure is described later. Following this procedure the model can be converted into the standard quadratized form:

$$i_1(t) = i_{L1}(t) + g_{s1}\alpha L_1 z_1(t) \tag{4.1}$$

$$i_2(t) = i_{L2}(t) + g_{s2}(1-\alpha)L_1 z_2(t) \tag{4.2}$$

$$i_3(t) = i_{L3}(t) + g_{s3}\beta L_2 z_3(t) \tag{4.3}$$

$$i_4(t) = i_{L4}(t) + g_{s4}(1-\beta)L_2 z_4(t) \tag{4.4}$$

$$i_5(t) = -i_{L1}(t) - i_{L2}(t) - g_{s1}\alpha L_1 z_1(t) - g_{s2}(1-\alpha)L_1 z_2(t) \tag{4.5}$$

$$i_6(t) = -i_{L3}(t) - i_{L4}(t) - g_{s3}\beta L_2 z_3(t) - g_{s4}(1-\beta)L_2 z_4(t) \tag{4.6}$$

$$0 = z_1(t) - \frac{di_{L1}(t)}{dt} \tag{4.7}$$

$$0 = z_2(t) - \frac{di_{L2}(t)}{dt} \tag{4.8}$$

$$0 = z_3(t) - \frac{di_{L3}(t)}{dt} \tag{4.9}$$

$$0 = z_4(t) - \frac{di_{L4}(t)}{dt} \tag{4.10}$$

$$0 = z_5(t) - \frac{d\lambda(t)}{dt} \tag{4.11}$$

$$0 = v_1(t) - v_5(t) - \alpha r_1(i_{L1}(t) + g_{s1}\alpha L_1 z_1(t)) - \alpha L_1 z_1(t) - e_{c1}(t) \tag{4.12}$$

$$\begin{aligned} 0 = v_2(t) - v_5(t) - (1-\alpha)r_1(i_{L2}(t) + g_{s2}(1-\alpha)L_1 z_2(t)) \\ - (1-\alpha)L_1 z_2(t) + e_{c2}(t) \end{aligned} \tag{4.13}$$

$$0 = v_3(t) - v_6(t) - \beta r_2(i_{L3}(t) + g_{s3}\beta L_2 z_3(t)) - \beta L_2 z_3(t) - e_{c3}(t) \tag{4.14}$$

$$\begin{aligned} 0 = v_4(t) - v_6(t) - (1-\beta)r_2(i_{L4}(t) + g_{s4}(1-\beta)L_2 z_4(t)) \\ - (1-\beta)L_2 z_4(t) + e_{c4}(t) \end{aligned} \tag{4.15}$$

$$0 = \alpha N_1 i_{c1}(t) - (1-\alpha)N_1 i_{c2}(t) + \beta N_2\big(i_{L3}(t) + g_{s3}\beta L_2 z_3(t)\big) - (1-\beta)N_2(i_{L4}(t) + g_{s4}(1-\beta)L_2 z_4(t)) \quad (4.16)$$

$$0 = e_{c1}(t) - \alpha z_5(t) \quad (4.17)$$

$$0 = e_{c2}(t) - (1-\alpha) z_5(t) \quad (4.18)$$

$$0 = e_{c3}(t) - \beta \frac{N_2}{N_1} z_5(t) \quad (4.19)$$

$$0 = e_{c4}(t) - (1-\beta) \frac{N_2}{N_1} z_5(t) \quad (4.20)$$

$$0 = -i_{L1}(t) - g_{s1}\alpha L_1 z_1(t) + i_{c1}(t) + i_m(t) + g_c\big(e_{c1}(t) + e_{c2}(t)\big) \quad (4.21)$$

$$0 = -i_{L2}(t) - g_{s2}(1-\alpha)L_1 z_2(t) + i_{c2}(t) - i_m(t) - g_c\big(e_{c1}(t) + e_{c2}(t)\big) \quad (4.22)$$

$$0 = i_m(t) - i_0 \cdot y_m(t) \cdot \big[sign(\lambda(t))\big]^{n+1} \quad (4.23)$$

$$0 = y_1(t) - \frac{\lambda(t)^2}{\lambda_0^2} \quad (4.24)$$

$$0 = y_2(t) - y_1(t)^2 \quad (4.25)$$

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$$0 = y_{m1}(t) - y_{m1-1}(t)^2 \quad (4.23+m1)$$

$$0 = y_{m1+1}(t) - y_{i1}(t) \cdot y_{j1}(t) \quad (4.23+m1+1)$$

$$0 = y_{m1+2}(t) - y_{m1+1}(t) \cdot y_{j2}(t) \quad (4.23+m1+2)$$

……

……

$$\begin{cases} 0 = y_m(t) - y_{m-1}(t) \cdot y_{jm2}(t) & , if\ n\ even \\ 0 = y_m(t) - y_{m-1}(t) \cdot \dfrac{\lambda(t)}{\lambda_0} & , if\ n\ odd \end{cases} \quad (4.23+m)$$

$$0.9 < t(t) < 1.1$$

There are 23+m states

$$X(t) = \begin{bmatrix} v_1(t), v_2(t), v_3(t), v_4(t), v_5(t), v_6(t), \\ i_{L1}(t), i_{L2}(t), i_{L3}(t), i_{L4}(t), \lambda(t), \\ z_1(t), z_2(t), z_3(t), z_4(t), z_5(t), \\ e_{c1}(t), e_{c2}(t), e_{c3}(t), e_{c4}(t), i_{c1}(t), i_{c2}(t), \\ i_m(t), y_1(t), y_2(t), \cdots\cdots, y_m(t) \end{bmatrix}$$

And 6 though variables

$$I = [i_1(t), i_2(t), i_3(t), i_4(t), i_5(t), i_6(t)]$$

And 1 control variable

$$U = [t(t)]$$

Based on the above formulation, the number of additional internal states and equations m is computed as follows:

$m = m_1 + m_2$

where:

$m_1 = \text{int}(\log_2(n))$

$m_2 =$ (# of ones in the binary representation of n) – 1

The sets of indices i and j in the last set of equations are provided by positions of ones in the binary representation of n. The values of these indices are equal to the values of the power of 2 corresponding to that position, meaning that the right most positions is indexed 0 and the left most indexed $\text{int}(\log_2(n))$. The variable y_0 is by definition equal to λ, so it is not used and appears as λ in the equations. If the symbol y_0 was used instead of λ then only the first case in equation (M4.23+m1+m2) would have been necessary, since this would include the second when the index becomes 0. However, since λ is physically related to flux, for the time being the use of this symbol is preferred instead.

F3. Single Phase Saturable Core Transformer SCAQCF Device Model

The differential equations in above model are integrated with the quadratic integration method and the equations that are algebraic are sufficed to be written at times t and t_m. The SCAQCF model yields the following model.

At Time t

$$i_1(t) = i_{L1}(t) + g_{s1}\alpha L_1 z_1(t)$$

$$i_2(t) = i_{L2}(t) + g_{s2}(1-\alpha)L_1 z_2(t)$$

$$i_3(t) = i_{L3}(t) + g_{s3}\beta L_2 z_3(t)$$

$$i_4(t) = i_{L4}(t) + g_{s4}(1-\beta)L_2 z_4(t)$$

$$i_5(t) = -i_{L1}(t) - i_{L2}(t) - g_{s1}\alpha L_1 z_1(t) - g_{s2}(1-\alpha)L_1 z_2(t)$$

$$i_6(t) = -i_{L3}(t) - i_{L4}(t) - g_{s3}\beta L_2 z_3(t) - g_{s4}(1-\beta)L_2 z_4(t)$$

$$0 = i_{L1}(t) - \frac{h}{6}z_1(t) - \frac{2h}{3}z_1(t_m) - i_{L1}(t-h) - \frac{h}{6}z_1(t-h)$$

$$0 = i_{L2}(t) - \frac{h}{6}z_2(t) - \frac{2h}{3}z_2(t_m) - i_{L2}(t-h) - \frac{h}{6}z_2(t-h)$$

$$0 = i_{L3}(t) - \frac{h}{6}z_3(t) - \frac{2h}{3}z_3(t_m) - i_{L3}(t-h) - \frac{h}{6}z_3(t-h)$$

$$0 = i_{L4}(t) - \frac{h}{6}z_4(t) - \frac{2h}{3}z_4(t_m) - i_{L4}(t-h) - \frac{h}{6}z_4(t-h)$$

$$0 = \lambda(t) - \frac{h}{6}z_5(t) - \frac{2h}{3}z_5(t_m) - \lambda(t-h) - \frac{h}{6}z_5(t-h)$$

$$0 = v_1(t) - v_5(t) - \alpha r_1(i_{L1}(t) + g_{s1}\alpha L_1 z_1(t)) - \alpha L_1 z_1(t) - e_{c1}(t)$$

$$\begin{aligned}0 = v_2(t) - v_5(t) - (1-\alpha)r_1(i_{L2}(t) + g_{s2}(1-\alpha)L_1 z_2(t))\\ - (1-\alpha)L_1 z_2(t) + e_{c2}(t)\end{aligned}$$

$$0 = v_3(t) - v_6(t) - \beta r_2(i_{L3}(t) + g_{s3}\beta L_2 z_3(t)) - \beta L_2 z_3(t) - e_{c3}(t)$$

$$\begin{aligned}0 = v_4(t) - v_6(t) - (1-\beta)r_2(i_{L4}(t) + g_{s4}(1-\beta)L_2 z_4(t))\\ - (1-\beta)L_2 z_4(t) + e_{c4}(t)\end{aligned}$$

$$\begin{aligned}0 = \alpha N_1 i_{c1}(t) - (1-\alpha)N_1 i_{c2}(t) + \beta N_2\big(i_{L3}(t) + g_{s3}\beta L_2 z_3(t)\big) -\\ (1-\beta)N_2(i_{L4}(t) + g_{s4}(1-\beta)L_2 z_4(t))\end{aligned}$$

$$0 = e_{c1}(t) - \alpha z_5(t)$$

$$0 = e_{c2}(t) - (1-\alpha)z_5(t)$$

$$0 = e_{c3}(t) - \beta\frac{N_2}{N_1}z_5(t)$$

$$0 = e_{c4}(t) - (1-\beta)\frac{N_2}{N_1}z_5(t)$$

$$0 = -i_{L1}(t) - g_{s1}\alpha L_1 z_1(t) + i_{c1}(t) + i_m(t) + g_c\big(e_{c1}(t) + e_{c2}(t)\big)$$

$$\begin{aligned}0 = -i_{L2}(t) - g_{s2}(1-\alpha)L_1 z_2(t) + i_{c2}(t) - i_m(t)\\ - g_c\big(e_{c1}(t) + e_{c2}(t)\big)\end{aligned}$$

$$0 = i_m(t) - i_0 \cdot y_m(t) \cdot \big[sign(\lambda(t))\big]^{n+1}$$

$$0 = y_1(t) - \frac{\lambda(t)^2}{\lambda_0^2}$$

$$0 = y_2(t) - y_1(t)^2$$

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$$0 = y_{m1}(t) - y_{m1-1}(t)^2$$

$$0 = y_{m1+1}(t) - y_{i1}(t) \cdot y_{j1}(t)$$

$$0 = y_{m1+2}(t) - y_{m1+1}(t) \cdot y_{j2}(t)$$

……

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$$\begin{cases} 0 = y_m(t) - y_{m-1}(t) \cdot y_{jm2}(t) & , if\ n\ even \\ 0 = y_m(t) - y_{m-1}(t) \cdot \dfrac{\lambda(t)}{\lambda_0} & , if\ n\ odd \end{cases}$$

$$0.9 < t(t) < 1.1$$

At Time t_m

$$i_1(t_m) = i_{L1}(t_m) + g_{s1}\alpha L_1 z_1(t_m)$$

$$i_2(t_m) = i_{L2}(t_m) + g_{s2}(1-\alpha)L_1 z_2(t_m)$$

$$i_3(t_m) = i_{L3}(t_m) + g_{s3}\beta L_2 z_3(t_m)$$

$$i_4(t_m) = i_{L4}(t_m) + g_{s4}(1-\beta)L_2 z_4(t_m)$$

$$i_5(t_m) = -i_{L1}(t_m) - i_{L2}(t_m) - g_{s1}\alpha L_1 z_1(t_m) - g_{s2}(1-\alpha)L_1 z_2(t_m)$$

$$i_6(t_m) = -i_{L3}(t_m) - i_{L4}(t_m) - g_{s3}\beta L_2 z_3(t_m) - g_{s4}(1-\beta)L_2 z_4(t_m)$$

$$0 = i_{L1}(t_m) + \frac{h}{24} z_1(t) - \frac{h}{3} z_1(t_m) - i_{L1}(t-h) - \frac{5h}{24} z_1(t-h)$$

$$0 = i_{L2}(t_m) + \frac{h}{24} z_2(t) - \frac{h}{3} z_2(t_m) - i_{L2}(t-h) - \frac{5h}{24} z_2(t-h)$$

$$0 = i_{L3}(t_m) + \frac{h}{24} z_3(t) - \frac{h}{3} z_3(t_m) - i_{L3}(t-h) - \frac{5h}{24} z_3(t-h)$$

$$0 = i_{L4}(t_m) + \frac{h}{24} z_4(t) - \frac{h}{3} z_4(t_m) - i_{L4}(t-h) - \frac{5h}{24} z_4(t-h)$$

$$0 = \lambda(t_m) + \frac{h}{24} z_5(t) - \frac{h}{3} z_5(t_m) - \lambda(t-h) - \frac{5h}{24} z_5(t-h)$$

$$0 = v_1(t_m) - v_5(t_m) - \alpha r_1(i_{L1}(t_m) + g_{s1}\alpha L_1 z_1(t_m)) - \alpha L_1 z_1(t_m) - e_{c1}(t_m)$$

$$0 = v_2(t_m) - v_5(t_m) - (1-\alpha) r_1(i_{L2}(t_m) + g_{s2}(1-\alpha) L_1 z_2(t_m)) - (1-\alpha) L_1 z_2(t_m) + e_{c2}(t_m)$$

$$0 = v_3(t_m) - v_6(t_m) - \beta r_2(i_{L3}(t_m) + g_{s3}\beta L_2 z_3(t_m)) - \beta L_2 z_3(t_m) - e_{c3}(t_m)$$

$$0 = v_4(t_m) - v_6(t_m) - (1-\beta) r_2(i_{L4}(t_m) + g_{s4}(1-\beta) L_2 z_4(t_m)) - (1-\beta) L_2 z_4(t_m) + e_{c4}(t_m)$$

$$0 = \alpha N_1 i_{c1}(t_m) - (1-\alpha) N_1 i_{c2}(t_m) + \beta N_2\big(i_{L3}(t_m) + g_{s3}\beta L_2 z_3(t_m)\big) - (1 - \beta) N_2(i_{L4}(t_m) + g_{s4}(1-\beta) L_2 z_4(t_m))$$

$$0 = e_{c1}(t_m) - \alpha z_5(t_m)$$

$$0 = e_{c2}(t_m) - (1-\alpha) z_5(t_m)$$

$$0 = e_{c3}(t_m) - \beta \frac{N_2}{N_1} z_5(t_m)$$

$$0 = e_{c4}(t_m) - (1-\beta) \frac{N_2}{N_1} z_5(t_m)$$

$$0 = -i_{L1}(t_m) - g_{s1}\alpha L_1 z_1(t_m) + i_{c1}(t_m) + i_m(t_m) + g_c\big(e_{c1}(t_m) + e_{c2}(t_m)\big)$$

$$0 = -i_{L2}(t_m) - g_{s2}(1-\alpha) L_1 z_2(t_m) + i_{c2}(t_m) - i_m(t_m) - g_c\big(e_{c1}(t_m) + e_{c2}(t_m)\big)$$

$$0 = i_m(t_m) - i_0 \cdot y_m(t_m) \cdot \big[sign(\lambda(t_m))\big]^{n+1}$$

$$0 = y_1(t_m) - \frac{\lambda(t_m)^2}{\lambda_0^2}$$

$$0 = y_2(t_m) - y_1(t_m)^2$$

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$$0 = y_{m1}(t_m) - y_{m1-1}(t_m)^2$$

$$0 = y_{m1+1}(t_m) - y_{i1}(t_m) \cdot y_{j1}(t_m)$$

$$0 = y_{m1+2}(t_m) - y_{m1+1}(t_m) \cdot y_{j2}(t_m)$$

$\cdots\cdots$

$\cdots\cdots$

$$\begin{cases} 0 = y_m(t_m) - y_{m-1}(t_m) \cdot y_{jm2}(t_m) & , if\ n\ even \\ 0 = y_m(t_m) - y_{m-1}(t_m) \cdot \dfrac{\lambda(t_m)}{\lambda_0} & , if\ n\ odd \end{cases}$$

$0.9 < t(t_m) < 1.1$

There are 46+2m states

$$X = [X(t) \quad X(t_m)]$$

$$X(t) = \begin{bmatrix} v_1(t), v_2(t), v_3(t), v_4(t), v_5(t), v_6(t), \\ i_{L1}(t), i_{L2}(t), i_{L3}(t), i_{L4}(t), \lambda(t), \\ z_1(t), z_2(t), z_3(t), z_4(t), z_5(t), \\ e_{c1}(t), e_{c2}(t), e_{c3}(t), e_{c4}(t), i_{c1}(t), i_{c2}(t), \\ i_m(t), y_1(t), y_2(t), \cdots\cdots, y_m(t) \end{bmatrix}$$

And 12 though variables

$$I = \begin{bmatrix} i_1(t), i_2(t), i_3(t), i_4(t), i_5(t), i_6(t) \\ i_1(t_m), i_2(t_m), i_3(t_m), i_4(t_m), i_5(t_m), i_6(t_m) \end{bmatrix}$$

And 2 control variable

$$U = [t(t) \quad t(t_m)]$$

Based on the above formulation, the number of additional internal states and equations m is computed as follows:

$m = m_1 + m_2$

where:

$m_1 = \mathrm{int}(\log_2(n))$
$m_2 = $ (# of ones in the binary representation of n) – 1

The sets of indices i and j in the last set of equations are provided by positions of ones in the binary representation of n. The values of these indices are equal to the values of the power of 2 corresponding to that position, meaning that the right most positions is indexed 0 and the left most indexed $\mathrm{int}(\log_2(n))$. The variable y_0 is by definition equal to λ, so it is not used and appears as λ in the equations. If the symbol y_0 was used instead of λ then only the first case in equation (M4.23+m1+m2) would have been necessary, since this would include the second when the index becomes 0. However, since λ is physically related to flux, for the time being the use of this symbol is preferred instead.

The input matrices of the single phase saturable core transformer SCAQCF device model for the setting-less protection algorithm are shown below:

The SCAQCF device model is shown below:

$$\begin{Bmatrix} I(\mathbf{x},\mathbf{u}) \\ \vdots \\ 0 \\ \vdots \end{Bmatrix} = Y_{eqx}\mathbf{x} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{eqx}^i \mathbf{x} \\ \vdots \end{Bmatrix} + Y_{equ}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F_{equ}^i \mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{eqxu}^i \mathbf{u} \\ \vdots \end{Bmatrix} - B_{eq}$$

$$B_{eq} = -N_{eqx}\mathbf{x}(t-h) - N_{equ}\mathbf{u}(t-h) - M_{eq}I(t-h) - K_{eq}$$

$$\mathbf{h}(\mathbf{x},\mathbf{u}) = Y_{opx}\mathbf{x} + Y_{opu}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{opx}^i \mathbf{x} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F_{opu}^i \mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{opxu}^i \mathbf{u} \\ \vdots \end{Bmatrix} - B_{op}$$

Scaling factors: $Iscale,\ Xscale\ and\ Uscale$

Connectivity: $TerminalNodeName$

$$subject\ to: \quad \mathbf{h}_{\min} \le \mathbf{h}(\mathbf{x},\mathbf{u}) \le \mathbf{h}_{\max}$$
$$\mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max}$$

where the matrices are:

$$Y_{eqx} = \begin{bmatrix} Y_{eqx\_11} & Y_{eqx\_12} \\ Y_{eqx\_21} & Y_{eqx\_22} \end{bmatrix}_{52\times 52}$$

$$
Y_{eqx\_11}=\begin{bmatrix}
0&0&0&0&0&0&1&0&0&0&0&\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&-1&-1&0&0&0&-\frac{h}{5}&-\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&-1&-1&0&0&0&-\frac{h}{5}&-\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&-\frac{h}{6}&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&-\frac{h}{6}&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&-\frac{h}{6}&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&-\frac{h}{6}&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&-\frac{h}{6}&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&-1&0&-\alpha r_1&0&0&0&0&-\alpha L_1-\frac{\alpha r_1 h}{5}&0&0&0&0&-1&0&0&0&0&0&0&0&0&0\\
0&1&0&0&-1&0&0&(1-\alpha)r_1&0&0&0&0&(1-\alpha)(L_1+\frac{r_1 h}{5})&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&1&0&0&-1&0&0&-\beta r_2&0&0&0&0&-\beta L_2-\frac{\beta r_2 h}{5}&0&0&0&0&-1&0&0&0&0&0&0&0\\
0&0&0&1&0&-1&0&0&0&(1-\beta)r_2&0&0&0&0&(1-\beta)(L_2+\frac{r_2 h}{5})&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&\beta N_2&(\beta-1)N_2&0&0&0&\frac{\beta N_2 h}{5}&\frac{(\beta-1)N_2 h}{5}&0&0&0&0&0&\alpha N_1&(\alpha-1)N_1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\alpha&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\alpha-1&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{\beta N_2}{N_1}&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{(\beta-1)N_2}{N_1}&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&-1&0&0&0&0&-\frac{h}{5}&0&0&0&0&g_c&g_c&0&0&1&0&1&0&0&0\\
0&0&0&0&0&0&0&-1&0&0&0&0&-\frac{h}{5}&0&0&0&-g_c&-g_c&0&0&0&1&-1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&-i_0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{bmatrix}
$$

$$
Y_{eqx\_12} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2h}{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2h}{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2h}{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2h}{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2h}{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
Y_{eqx\_21} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{h}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{h}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{h}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{h}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{h}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
Y_{eqx\_22} = \left[\begin{array}{cccccccccccccccccccccccccc}
0&0&0&0&0&0&1&0&0&0&0&\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&1&0&0&0&0&\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&1&0&0&0&0&\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&-1&-1&0&0&0&-\frac{h}{5}&-\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&-1&-1&0&0&0&-\frac{h}{5}&-\frac{h}{5}&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&1&0&0&0&0&-\frac{h}{3}&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&1&0&0&0&0&-\frac{h}{3}&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&1&0&0&0&0&-\frac{h}{3}&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&-\frac{h}{3}&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&-\frac{h}{3}&0&0&0&0&0&0&0&0&0&0 \\
1&0&0&0&-1&0&-\alpha r_1&0&0&0&0&-\alpha L_1-\frac{\alpha r_1 h}{5}&0&0&0&0&-1&0&0&0&0&0&0&0&0&0 \\
0&1&0&0&-1&0&0&(1-\alpha)r_1&0&0&0&0&(1-\alpha)(L_1+\frac{r_1 h}{5})&0&0&0&0&1&0&0&0&0&0&0&0&0 \\
0&0&1&0&0&-1&0&0&-\beta r_2&0&0&0&0&-\beta L_2-\frac{\beta r_2 h}{5}&0&0&0&0&-1&0&0&0&0&0&0&0 \\
0&0&0&1&0&-1&0&0&0&(1-\beta)r_2&0&0&0&0&(1-\beta)(L_2+\frac{r_2 h}{5})&0&0&0&0&1&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&\beta N_2&(\beta-1)N_2&0&0&0&\frac{\beta N_2 h}{5}&\frac{(\beta-1)N_2 h}{5}&0&0&0&0&0&\alpha N_1&(\alpha-1)N_1&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\alpha&1&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\alpha-1&0&1&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\frac{\beta N_2}{N_1}&0&0&1&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{(\beta-1)N_2}{N_1}&0&0&0&1&0&0&0&0&0&0 \\
0&0&0&0&0&0&-1&0&0&0&0&-\frac{h}{5}&0&0&0&0&g_c&g_c&0&0&1&0&1&0&0&0 \\
0&0&0&0&0&0&0&-1&0&0&0&0&-\frac{h}{5}&0&0&0&-g_c&-g_c&0&0&0&1&-1&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&-i_0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{array}\right]
$$

$$F_{eq0} = \cdots = F_{eq22} = 0_{52\times 52}$$

$$F_{eq23}: f_{eq[10\times 10]} = -\frac{1}{\lambda_0^2}, \qquad other\ elements\ are\ 0$$

$$F_{eq24}: f_{eq[23\times 23]} = -1, \qquad other\ elements\ are\ 0$$

$$F_{eq25}: f_{eq[24\times 20]} = -1, \qquad other\ elements\ are\ 0$$

$$F_{eq26} = \cdots = F_{eq48} = 0_{52\times 52}$$

$$F_{eq49}: f_{eq[36\times 36]} = -\frac{1}{\lambda_0^2}, \qquad other\ elements\ are\ 0$$

$$F_{eq50}: f_{eq[49\times 49]} = -1, \qquad other\ elements\ are\ 0$$

$$F_{eq51}: f_{eq[50\times 36]} = -1, \qquad other\ elements\ are\ 0$$

$$N_{eqx} = \begin{bmatrix} N_{eqx\_1} \\ N_{eqx\_2} \end{bmatrix}_{52\times 26}$$

$$
N_{eqx\_1} = \begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&-1&0&0&0&0&-\frac{h}{6}&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&-1&0&0&0&0&-\frac{h}{6}&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&-1&0&0&0&0&-\frac{h}{6}&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&-1&0&0&0&0&-\frac{h}{6}&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&-\frac{h}{6}&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}
$$

$$
N_{eqx\_1} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -\frac{5h}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -\frac{5h}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -\frac{5h}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -\frac{5h}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -\frac{5h}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

The scaling factors are:

$$X_{scale} = \begin{bmatrix} X_{scale\_1} & X_{scale\_2} & X_{scale\_3} & X_{scale\_4} \end{bmatrix}_{52}^{T}$$

$$X_{scale\_1} = \begin{bmatrix} 2.5e4 & 2.5e4 & 2.5e4 & 500 & 500 & 500 & 10 & 10 & 500 & 500 & 100 & 4.0e3 & 4.0e3 \end{bmatrix}$$

$$X_{scale\_2} = \begin{bmatrix} 4.0e4 & 4.0e4 & 2.5e4 & 2.5e4 & 2.5e4 & 500 & 500 & 10 & 10 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$X_{scale\_3} = X_{scale\_1}$$

$$X_{scale\_4} = X_{scale\_2}$$

$$I_{scale} = \begin{bmatrix} I_{scale\_1} & I_{scale\_2} & I_{scale\_3} & I_{scale\_4} \end{bmatrix}_{52}^{T}$$

$$I_{scale\_1} = \begin{bmatrix} 10 & 10 & 500 & 500 & 10 & 500 & 10 & 10 & 100 & 100 & 100 & 10000 & 10000 \end{bmatrix}$$

$$I_{scale\_2} = \begin{bmatrix} 500 & 500 & 10 & 10000 & 10000 & 500 & 500 & 100 & 100 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$I_{scale\_3} = I_{scale\_1}$$

$$I_{scale\_4} = I_{scale\_2}$$

TerminalNodeName
XFMRH_A
XFMRH_N
XFMRL_A
XFMRL_N
-1

Xscale
0, 25.0e3
1, 25.0e3
2, 25.0e3
3, 500.0
4, 500.0
5, 500.0
6, 10.00000
7, 10.00000
8, 500.0000
9, 500.0000
10, 100.0000
11, 4.0e3
12, 4.0e3
13, 4.0e4
14, 4.0e4
15, 25.0e3
16, 25.0e3
17, 25.0e3
18, 500.0
19, 500.0
20, 10.00000
21, 10.00000
22, 1.000000
23, 1.000000
24, 1.000000
25, 1.000000
26, 25.0e3
27, 25.0e3
28, 25.0e3
29, 500.000000
30, 500.000000
31, 500.000000
32, 10.000000
33, 10.000000
34, 500.0000
35, 500.0000
36, 100.0000

37, 4.0e3
38, 4.0e3
39, 4.0e4
40, 4.0e4
41, 25.0e3
42, 25.0e3
43, 25.0e3
44, 500.0
45, 500.0
46, 10.00000
47, 10.00000
48, 1.000000
49, 1.000000
50, 1.000000
51, 1.000000
-1

Iscale
0, 10.000000
1, 10.000000
2, 500.000000
3, 500.000000
4, 10.000000
5, 500.000000
6, 10.000000
7, 10.000000
8, 100.000000
9, 100.000000
10, 100.000000
11, 10000.0
12, 10000.0
13, 500.0
14, 500.0
15, 10.000000
16, 10000.0
17, 10000.0
18, 500.0
19, 500.0
20, 100.000000
21, 100.000000
22, 1.000000
23, 1.000000
24, 1.000000
25, 1.000000
26, 10.000000
27, 10.000000

28, 500.000000
29, 500.000000
30, 10.000000
31, 500.000000
32, 10.000000
33, 10.000000
34, 100.000000
35, 100.000000
36, 100.000000
37, 10000.0
38, 10000.0
39, 500.0
40, 500.0
41, 10.000000
42, 10000.0
43, 10000.0
44, 500.0
45, 500.0
46, 100.000000
47, 100.000000
48, 1.000000
49, 1.000000
50, 1.000000
51, 1.000000
-1

Yeqx
0, 6, 1.0000000000e+000
0, 11, 2.0000000000e-005
1, 7, 1.0000000000e+000
1, 12, 2.0000000000e-005
2, 8, 1.0000000000e+000
2, 13, 2.0000000000e-005
3, 9, 1.0000000000e+000
3, 14, 2.0000000000e-005
4, 6, -1.0000000000e+000
4, 7, -1.0000000000e+000
4, 11, -2.0000000000e-005
4, 12, -2.0000000000e-005
5, 8, -1.0000000000e+000
5, 9, -1.0000000000e+000
5, 13, -2.0000000000e-005
5, 14, -2.0000000000e-005
6, 6, 1.0000000000e+000
6, 11, -1.6666666667e-005
6, 37, -6.6666666667e-005

7,	*7,*	*1.0000000000e+000*
7,	*12,*	*-1.6666666667e-005*
7,	*38,*	*-6.6666666667e-005*
8,	*8,*	*1.0000000000e+000*
8,	*13,*	*-1.6666666667e-005*
8,	*39,*	*-6.6666666667e-005*
9,	*9,*	*1.0000000000e+000*
9,	*14,*	*-1.6666666667e-005*
9,	*40,*	*-6.6666666667e-005*
10,	*10,*	*1.0000000000e+000*
10,	*15,*	*-1.6666666667e-005*
10,	*41,*	*-6.6666666667e-005*
11,	*0,*	*1.0000000000e+000*
11,	*4,*	*-1.0000000000e+000*
11,	*6,*	*-3.4704150000e+000*
11,	*11,*	*-9.2125025272e-002*
11,	*16,*	*-1.0000000000e+000*
12,	*1,*	*1.0000000000e+000*
12,	*4,*	*-1.0000000000e+000*
12,	*17,*	*1.0000000000e+000*
13,	*2,*	*1.0000000000e+000*
13,	*5,*	*-1.0000000000e+000*
13,	*8,*	*-1.2788166667e-003*
13,	*13,*	*-3.3947241968e-005*
13,	*18,*	*-1.0000000000e+000*
14,	*3,*	*1.0000000000e+000*
14,	*5,*	*-1.0000000000e+000*
14,	*19,*	*1.0000000000e+000*
15,	*8,*	*1.9196119196e-002*
15,	*13,*	*3.8392238392e-007*
15,	*20,*	*1.0000000000e+000*
16,	*15,*	*-1.0000000000e+000*
16,	*16,*	*1.0000000000e+000*
17,	*17,*	*1.0000000000e+000*
18,	*15,*	*-1.9196119196e-002*
18,	*18,*	*1.0000000000e+000*
19,	*19,*	*1.0000000000e+000*
20,	*6,*	*-1.0000000000e+000*
20,	*11,*	*-2.0000000000e-005*
20,	*16,*	*7.2037494075e-006*
20,	*17,*	*7.2037494075e-006*
20,	*20,*	*1.0000000000e+000*
20,	*22,*	*1.0000000000e+000*
21,	*7,*	*-1.0000000000e+000*
21,	*12,*	*-2.0000000000e-005*
21,	*16,*	*-7.2037494075e-006*

21,	*17,*	*-7.2037494075e-006*
21,	*21,*	*1.0000000000e+000*
21,	*22,*	*-1.0000000000e+000*
22,	*22,*	*1.0000000000e+000*
22,	*25,*	*-1.0395010395*
23,	*23,*	*1.0000000000e+000*
24,	*24,*	*1.0000000000e+000*
25,	*25,*	*1.0000000000e+000*
26,	*32,*	*1.0000000000e+000*
26,	*37,*	*2.0000000000e-005*
27,	*33,*	*1.0000000000e+000*
27,	*38,*	*2.0000000000e-005*
28,	*34,*	*1.0000000000e+000*
28,	*39,*	*2.0000000000e-005*
29,	*35,*	*1.0000000000e+000*
29,	*40,*	*2.0000000000e-005*
30,	*32,*	*-1.0000000000e+000*
30,	*33,*	*-1.0000000000e+000*
30,	*37,*	*-2.0000000000e-005*
30,	*38,*	*-2.0000000000e-005*
31,	*34,*	*-1.0000000000e+000*
31,	*35,*	*-1.0000000000e+000*
31,	*39,*	*-2.0000000000e-005*
31,	*40,*	*-2.0000000000e-005*
32,	*11,*	*4.1666666667e-006*
32,	*32,*	*1.0000000000e+000*
32,	*37,*	*-3.3333333333e-005*
33,	*12,*	*4.1666666667e-006*
33,	*33,*	*1.0000000000e+000*
33,	*38,*	*-3.3333333333e-005*
34,	*13,*	*4.1666666667e-006*
34,	*34,*	*1.0000000000e+000*
34,	*39,*	*-3.3333333333e-005*
35,	*14,*	*4.1666666667e-006*
35,	*35,*	*1.0000000000e+000*
35,	*40,*	*-3.3333333333e-005*
36,	*15,*	*4.1666666667e-006*
36,	*36,*	*1.0000000000e+000*
36,	*41,*	*-3.3333333333e-005*
37,	*26,*	*1.0000000000e+000*
37,	*30,*	*-1.0000000000e+000*
37,	*32,*	*-3.4704150000e+000*
37,	*37,*	*-9.2125025272e-002*
37,	*42,*	*-1.0000000000e+000*
38,	*27,*	*1.0000000000e+000*
38,	*30,*	*-1.0000000000e+000*

38, 43, 1.0000000000e+000
39, 28, 1.0000000000e+000
39, 31, -1.0000000000e+000
39, 34, -1.2788166667e-003
39, 39, -3.3947241968e-005
39, 44, -1.0000000000e+000
40, 29, 1.0000000000e+000
40, 31, -1.0000000000e+000
40, 45, 1.0000000000e+000
41, 34, 1.9196119196e-002
41, 39, 3.8392238392e-007
41, 46, 1.0000000000e+000
42, 41, -1.0000000000e+000
42, 42, 1.0000000000e+000
43, 43, 1.0000000000e+000
44, 41, -1.9196119196e-002
44, 44, 1.0000000000e+000
45, 45, 1.0000000000e+000
46, 32, -1.0000000000e+000
46, 37, -2.0000000000e-005
46, 42, 7.2037494075e-006
46, 43, 7.2037494075e-006
46, 46, 1.0000000000e+000
46, 48, 1.0000000000e+000
47, 33, -1.0000000000e+000
47, 38, -2.0000000000e-005
47, 42, -7.2037494075e-006
47, 43, -7.2037494075e-006
47, 47, 1.0000000000e+000
47, 48, -1.0000000000e+000
48, 48, 1.0000000000e+000
48, 51, -1.0395010395
49, 49, 1.0000000000e+000
50, 50, 1.0000000000e+000
51, 51, 1.0000000000e+000
-1

Feqx
23, 10, 10, -3.412712e-4
24, 23, 23, -1.000000e+000
25, 24, 10, -0.01847353
49, 36, 36, -3.412712e-4
50, 49, 49, -1.000000e+000
51, 50, 36, -0.01847353
-1

Neqx
6, 6, -1.0000000000e+000
6, 11, -1.6666666667e-005
7, 7, -1.0000000000e+000
7, 12, -1.6666666667e-005
8, 8, -1.0000000000e+000
8, 13, -1.6666666667e-005
9, 9, -1.0000000000e+000
9, 14, -1.6666666667e-005
10, 10, -1.0000000000e+000
10, 15, -1.6666666667e-005
32, 6, -1.0000000000e+000
32, 11, -2.0833333333e-005
33, 7, -1.0000000000e+000
33, 12, -2.0833333333e-005
34, 8, -1.0000000000e+000
34, 13, -2.0833333333e-005
35, 9, -1.0000000000e+000
35, 14, -2.0833333333e-005
36, 10, -1.0000000000e+000
36, 15, -2.0833333333e-005
-1

Meq
-1

Keq
-1

F4. Single Phase Saturable Core Transformer SCAQCF Measurement Model

More specifically, for single phase saturable core transformer the **actual measurements** are:
Two currents at time t (Primary side and Secondary side);
Two voltages at time t (Primary side and Secondary side);
Two currents at time tm=t-h/2 (Primary side and Secondary side);
Two voltages at time tm=t-h/2 (Primary side and Secondary side);;

The **virtual measurements** are:
Twenty measurements with zero value at the left side of the equations at time t;
Twenty measurements with zero value at the left side of the equations at time tm=t-h/2;

The **derived measurements** are:
Two ground currents at time t (Primary side and Secondary side)
Two ground currents at time tm=t-h/2 (Primary side and Secondary side)

The **pseudo measurements** are:
Two ground voltages ate time t (Primary side and Secondary side)
Two ground voltages ate time tm=t-h/2 (Primary side and Secondary side)

The measurement channel list:

MeasurementType, 16
MeasStdDev, 300.0
MeasTerminal, 0, 1
MeasurementEnd
MeasurementType, 16
MeasStdDev, 6.0
MeasTerminal, 2, 3
MeasurementEnd
MeasurementType, 17
MeasStdDev, 0.2
MeasTerminal, 0
MeasurementEnd
MeasurementType, 17
MeasStdDev, 10.0
MeasTerminal, 2
MeasurementEnd
MeasurementType, 29
MeasStdDev, 0.2
MeasTerminal, 1
MeasRatio, -1.0
MeasNumber, 0
MeasurementEnd
MeasurementType, 29
MeasStdDev, 10.0
MeasTerminal, 3
MeasRatio, -1.0
MeasNumber, 2
MeasurementEnd
MeasurementType, 25
MeasStdDev, 0.10000
MeasTerminal, 1
MeasurementEnd
MeasurementType, 25
MeasStdDev, 0.10000
MeasTerminal, 3
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal, 6

MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal, 7
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal, 8
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal, 9
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal, 10
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal, 11
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal, 12
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal, 13
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal, 14
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal, 15
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal, 16
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal, 17
MeasurementEnd
MeasurementType, 24

MeasStdDev, 0.0010000
MeasTerminal, 18
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal, 19
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal, 20
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal, 21
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal, 22
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal, 23
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal, 24
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal, 25
MeasurementEnd

Based on the device model and the measurement channel list, the equations for the measurement model are given below:

At Time t

$$v_{12}(t) = v_1(t) - v_2(t)$$

$$0 = v_2(t)$$

$$v_{34}(t) = v_3(t) - v_4(t)$$

$$0 = v_4(t)$$

$$i_1(t) = i_{L1}(t) + g_{s1}\alpha L_1 z_1(t)$$

$$i_2(t) = i_{L2}(t) + g_{s2}(1-\alpha) L_1 z_2(t)$$

$$i_3(t) = i_{L3}(t) + g_{s3}\beta L_2 z_3(t)$$

$$i_4(t) = i_{L4}(t) + g_{s4}(1-\beta) L_2 z_4(t)$$

$$0 = i_{L1}(t) - \frac{h}{6} z_1(t) - \frac{2h}{3} z_1(t_m) - i_{L1}(t-h) - \frac{h}{6} z_1(t-h)$$

$$0 = i_{L2}(t) - \frac{h}{6} z_2(t) - \frac{2h}{3} z_2(t_m) - i_{L2}(t-h) - \frac{h}{6} z_2(t-h)$$

$$0 = i_{L3}(t) - \frac{h}{6} z_3(t) - \frac{2h}{3} z_3(t_m) - i_{L3}(t-h) - \frac{h}{6} z_3(t-h)$$

$$0 = i_{L4}(t) - \frac{h}{6} z_4(t) - \frac{2h}{3} z_4(t_m) - i_{L4}(t-h) - \frac{h}{6} z_4(t-h)$$

$$0 = \lambda(t) - \frac{h}{6} z_5(t) - \frac{2h}{3} z_5(t_m) - \lambda(t-h) - \frac{h}{6} z_5(t-h)$$

$$0 = v_1(t) - v_5(t) - \alpha r_1(i_{L1}(t) + g_{s1}\alpha L_1 z_1(t)) - \alpha L_1 z_1(t) - e_{c1}(t)$$

$$0 = v_2(t) - v_5(t) - (1-\alpha) r_1(i_{L2}(t) + g_{s2}(1-\alpha) L_1 z_2(t)) \\ - (1-\alpha) L_1 z_2(t) + e_{c2}(t)$$

$$0 = v_3(t) - v_6(t) - \beta r_2(i_{L3}(t) + g_{s3}\beta L_2 z_3(t)) - \beta L_2 z_3(t) - e_{c3}(t)$$

$$0 = v_4(t) - v_6(t) - (1-\beta) r_2(i_{L4}(t) + g_{s4}(1-\beta) L_2 z_4(t)) \\ - (1-\beta) L_2 z_4(t) + e_{c4}(t)$$

$$0 = \alpha N_1 i_{c1}(t) - (1-\alpha) N_1 i_{c2}(t) + \beta N_2\big(i_{L3}(t) + g_{s3}\beta L_2 z_3(t)\big) - \\ (1-\beta) N_2(i_{L4}(t) + g_{s4}(1-\beta) L_2 z_4(t))$$

$$0 = e_{c1}(t) - \alpha z_5(t)$$

$$0 = e_{c2}(t) - (1-\alpha) z_5(t)$$

$$0 = e_{c3}(t) - \beta \frac{N_2}{N_1} z_5(t)$$

$$0 = e_{c4}(t) - (1-\beta) \frac{N_2}{N_1} z_5(t)$$

$$0 = -i_{L1}(t) - g_{s1}\alpha L_1 z_1(t) + i_{c1}(t) + i_m(t) + g_c\big(e_{c1}(t) + e_{c2}(t)\big)$$

$$0 = -i_{L2}(t) - g_{s2}(1-\alpha) L_1 z_2(t) + i_{c2}(t) - i_m(t) \\ - g_c\big(e_{c1}(t) + e_{c2}(t)\big)$$

$$0 = i_m(t) - i_0 \cdot y_m(t) \cdot \big[sign(\lambda(t))\big]^{n+1}$$

$$0 = y_1(t) - \frac{\lambda(t)^2}{\lambda_0^2}$$

$$0 = y_2(t) - y_1(t)^2$$

$$\dots\dots$$

$$\dots\dots$$

$$0 = y_{m1}(t) - y_{m1-1}(t)^2$$

$$0 = y_{m1+1}(t) - y_{i1}(t) \cdot y_{j1}(t)$$

$$0 = y_{m1+2}(t) - y_{m1+1}(t) \cdot y_{j2}(t)$$

$$\dots\dots$$

$$\dots\dots$$

$$\begin{cases} 0 = y_m(t) - y_{m-1}(t) \cdot y_{jm2}(t) & ,if\ n\ even \\ 0 = y_m(t) - y_{m-1}(t) \cdot \dfrac{\lambda(t)}{\lambda_0} & ,if\ n\ odd \end{cases}$$

$$0.9 < t(t) < 1.1$$

At Time t_m

$$v_{12}(t_m) = v_1(t_m) - v_2(t_m)$$

$$0 = v_2(t_m)$$

$$v_{34}(t_m) = v_3(t_m) - v_4(t_m)$$

$$0 = v_4(t_m)$$

$$i_1(t_m) = i_{L1}(t_m) + g_{s1}\alpha L_1 z_1(t_m)$$

$$i_2(t_m) = i_{L2}(t_m) + g_{s2}(1-\alpha)L_1 z_2(t_m)$$

$$i_3(t_m) = i_{L3}(t_m) + g_{s3}\beta L_2 z_3(t_m)$$

$$i_4(t_m) = i_{L4}(t_m) + g_{s4}(1-\beta)L_2 z_4(t_m)$$

$$i_5(t_m) = -i_{L1}(t_m) - i_{L2}(t_m) - g_{s1}\alpha L_1 z_1(t_m) - g_{s2}(1 - \alpha)L_1 z_2(t_m)$$

$$i_6(t_m) = -i_{L3}(t_m) - i_{L4}(t_m) - g_{s3}\beta L_2 z_3(t_m) - g_{s4}(1 - \beta)L_2 z_4(t_m)$$

$$0 = i_{L1}(t_m) + \frac{h}{24} z_1(t) - \frac{h}{3} z_1(t_m) - i_{L1}(t-h) - \frac{5h}{24} z_1(t-h)$$

$$0 = i_{L2}(t_m) + \frac{h}{24} z_2(t) - \frac{h}{3} z_2(t_m) - i_{L2}(t-h) - \frac{5h}{24} z_2(t-h)$$

$$0 = i_{L3}(t_m) + \frac{h}{24} z_3(t) - \frac{h}{3} z_3(t_m) - i_{L3}(t-h) - \frac{5h}{24} z_3(t-h)$$

$$0 = i_{L4}(t_m) + \frac{h}{24} z_4(t) - \frac{h}{3} z_4(t_m) - i_{L4}(t-h) - \frac{5h}{24} z_4(t-h)$$

$$0 = \lambda(t_m) + \frac{h}{24} z_5(t) - \frac{h}{3} z_5(t_m) - \lambda(t-h) - \frac{5h}{24} z_5(t-h)$$

$$0 = v_1(t_m) - v_5(t_m) - \alpha r_1(i_{L1}(t_m) + g_{s1}\alpha L_1 z_1(t_m)) - \alpha L_1 z_1(t_m) - e_{c1}(t_m)$$

$$0 = v_2(t_m) - v_5(t_m) - (1-\alpha) r_1(i_{L2}(t_m) + g_{s2}(1-\alpha) L_1 z_2(t_m)) - (1-\alpha) L_1 z_2(t_m) + e_{c2}(t_m)$$

$$0 = v_3(t_m) - v_6(t_m) - \beta r_2(i_{L3}(t_m) + g_{s3}\beta L_2 z_3(t_m)) - \beta L_2 z_3(t_m) - e_{c3}(t_m)$$

$$0 = v_4(t_m) - v_6(t_m) - (1-\beta) r_2(i_{L4}(t_m) + g_{s4}(1-\beta) L_2 z_4(t_m)) - (1-\beta) L_2 z_4(t_m) + e_{c4}(t_m)$$

$$0 = \alpha N_1 i_{c1}(t_m) - (1-\alpha) N_1 i_{c2}(t_m) + \beta N_2\big(i_{L3}(t_m) + g_{s3}\beta L_2 z_3(t_m)\big) - (1 - \beta) N_2(i_{L4}(t_m) + g_{s4}(1-\beta) L_2 z_4(t_m))$$

$$0 = e_{c1}(t_m) - \alpha z_5(t_m)$$

$$0 = e_{c2}(t_m) - (1-\alpha) z_5(t_m)$$

$$0 = e_{c3}(t_m) - \beta \frac{N_2}{N_1} z_5(t_m)$$

$$0 = e_{c4}(t_m) - (1-\beta) \frac{N_2}{N_1} z_5(t_m)$$

$$0 = -i_{L1}(t_m) - g_{s1}\alpha L_1 z_1(t_m) + i_{c1}(t_m) + i_m(t_m) + g_c\big(e_{c1}(t_m) + e_{c2}(t_m)\big)$$

$$0 = -i_{L2}(t_m) - g_{s2}(1-\alpha) L_1 z_2(t_m) + i_{c2}(t_m) - i_m(t_m) - g_c\big(e_{c1}(t_m) + e_{c2}(t_m)\big)$$

$$0 = i_m(t_m) - i_0 \cdot y_m(t_m) \cdot \big[sign(\lambda(t_m))\big]^{n+1}$$

$$0 = y_1(t_m) - \frac{\lambda(t_m)^2}{\lambda_0^2}$$

$$0 = y_2(t_m) - y_1(t_m)^2$$

$\dots\dots$

$\dots\dots$

$$0 = y_{m1}(t_m) - y_{m1-1}(t_m)^2$$

$$0 = y_{m1+1}(t_m) - y_{i1}(t_m) \cdot y_{j1}(t_m)$$

$$0 = y_{m1+2}(t_m) - y_{m1+1}(t_m) \cdot y_{j2}(t_m)$$

$\dots\dots$

$\dots\dots$

$$\begin{cases} 0 = y_m(t_m) - y_{m-1}(t_m) \cdot y_{jm2}(t_m) & , if\ n\ even \\ 0 = y_m(t_m) - y_{m-1}(t_m) \cdot \dfrac{\lambda(t_m)}{\lambda_0} & , if\ n\ odd \end{cases}$$

$$0.9 < t(t_m) < 1.1$$

There are 46+2m states

$$X = [X(t) \quad X(t_m)]$$

$$X(t) = \begin{bmatrix} v_1(t), v_2(t), v_3(t), v_4(t), v_5(t), v_6(t), \\ i_{L1}(t), i_{L2}(t), i_{L3}(t), i_{L4}(t), \lambda(t), \\ z_1(t), z_2(t), z_3(t), z_4(t), z_5(t), \\ e_{c1}(t), e_{c2}(t), e_{c3}(t), e_{c4}(t), i_{c1}(t), i_{c2}(t), \\ i_m(t), y_1(t), y_2(t), \cdots\cdots, y_m(t) \end{bmatrix}$$

And 12 though variables

$$I = \begin{bmatrix} i_1(t), i_2(t), i_3(t), i_4(t), i_5(t), i_6(t) \\ i_1(t_m), i_2(t_m), i_3(t_m), i_4(t_m), i_5(t_m), i_6(t_m) \end{bmatrix}$$

And 2 control variable

$$U = [t(t) \quad t(t_m)]$$

Based on the above formulation, the number of additional internal states and equations m is computed as follows:

$$m = m_1 + m_2$$

where:

$m_1 = \mathrm{int}(\log_2(n))$
$m_2 =$ (# of ones in the binary representation of n) – 1

The sets of indices i and j in the last set of equations are provided by positions of ones in the binary representation of n. The values of these indices are equal to the values of the power of 2 corresponding to that position, meaning that the right most positions is indexed 0 and the left most indexed $\mathrm{int}(\log_2(n))$. The variable y_0 is by definition equal to λ, so it is not used and appears as λ in the equations. If the symbol y_0 was used instead of λ then only the first case in equation (M4.23+m1+m2) would have been necessary, since this would include the second when the index becomes 0. However, since λ is physically related to flux, for the time being the use of this symbol is preferred instead.

The input matrices of the single phase saturable core transformer SCAQCF device model for the setting-less protection algorithm are shown below:

It is very easy to write the above equations in the standard SCAQCF format:

$$\mathbf{y}(\mathbf{x},\mathbf{u}) = Y_{m,x}\mathbf{x} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{m,x}^i \mathbf{x} \\ \vdots \end{Bmatrix} + Y_{m,u}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F_{m,u}^i \mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{m,xu}^i \mathbf{u} \\ \vdots \end{Bmatrix} + N_{m,x}\mathbf{x}(t-h) + N_{m,u}\mathbf{u}(t-h) + M_m I(t-h) + K_m$$

where the matrices are:

$$F_{mx} = \begin{bmatrix} F_{mx\_11} & F_{mx\_12} \\ F_{mx\_21} & F_{mx\_22} \end{bmatrix}_{56\times 52}$$

$$
F_{mx\_11}=\left[\begin{array}{cccccccccccccccccccccccccc}
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{h}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{h}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{h}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{h}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -\frac{h}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -\frac{h}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -\frac{h}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -\frac{h}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -\frac{h}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & -1 & 0 & -\alpha r_1 & 0 & 0 & 0 & 0 & -\alpha L_1-\frac{\alpha r_1 h}{5} & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & 0 & 0 & (1-\alpha)r_1 & 0 & 0 & 0 & 0 & (1-\alpha)(L_1+\frac{r_1 h}{5}) & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -\beta r_2 & 0 & 0 & 0 & 0 & -\beta L_2-\frac{\beta r_2 h}{5} & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & (1-\beta)r_2 & 0 & 0 & 0 & 0 & (1-\beta)(L_2+\frac{r_2 h}{5}) & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta N_2 & (\beta-1)N_2 & 0 & 0 & 0 & \frac{\beta N_2 h}{5} & \frac{(\beta-1)N_2 h}{5} & 0 & 0 & 0 & 0 & 0 & \alpha N_1 & (\alpha-1)N_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\beta N_2}{N_1} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{(\beta-1)N_2}{N_1} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -\frac{h}{5} & 0 & 0 & 0 & 0 & g_c & g_c & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -\frac{h}{5} & 0 & 0 & 0 & -g_c & -g_c & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -i_0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]
$$

$$F_{mx_12} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2h}{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2h}{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2h}{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2h}{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2h}{-3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$
F_{mx\_21} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{h}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{h}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{h}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{h}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{h}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
F_{mx\_22}=\begin{bmatrix}
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{h}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{h}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{h}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{h}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -\frac{h}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -\frac{h}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -\frac{h}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -\frac{h}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -\frac{h}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & -1 & 0 & -\alpha r_1 & 0 & 0 & 0 & 0 & -\alpha L_1-\frac{\alpha r_1 h}{5} & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & 0 & 0 & (1-\alpha)r_1 & 0 & 0 & 0 & 0 & (1-\alpha)(L_1+\frac{r_1 h}{5}) & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -\beta r_2 & 0 & 0 & 0 & 0 & -\beta L_2-\frac{\beta r_2 h}{5} & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & (1-\beta)r_2 & 0 & 0 & 0 & 0 & (1-\beta)(L_2+\frac{r_2 h}{5}) & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta N_2 & (\beta-1)N_2 & 0 & 0 & 0 & \frac{\beta N_2 h}{5} & \frac{(\beta-1)N_2 h}{5} & 0 & 0 & 0 & 0 & 0 & \alpha N_1 & (\alpha-1)N_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\beta N_2}{N_1} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{(\beta-1)N_2}{N_1} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -\frac{h}{5} & 0 & 0 & 0 & 0 & g_c & g_c & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -\frac{h}{5} & 0 & 0 & 0 & -g_c & -g_c & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -i_0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

$$F_{m0} = \cdots = F_{m24} = 0_{52\times 52}$$

$$F_{m25}: f_{m[10\times 10]} = -\frac{1}{\lambda_0^2}, \qquad \textit{other elements are } 0$$

$$F_{m26}: f_{m[23\times 23]} = -1, \qquad \textit{other elements are } 0$$

$$F_{m27}: f_{m[24\times 20]} = -1, \qquad \textit{other elements are } 0$$

$$F_{m28} = \cdots = F_{m52} = 0_{52\times 52}$$

$$F_{m53}: f_{m[36\times 36]} = -\frac{1}{\lambda_0^2}, \qquad \textit{other elements are } 0$$

$$F_{m54}: f_{m[49\times 49]} = -1, \qquad \textit{other elements are } 0$$

$$F_{m55}: f_{m[50\times 36]} = -1, \qquad \textit{other elements are } 0$$

$$N_{mx} = \begin{bmatrix} N_{mx\_1} \\ N_{mx\_2} \end{bmatrix}_{56\times 26}$$

$$
N_{mx\_1} = \begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&-1&0&0&0&0&-\frac{h}{6}&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&-1&0&0&0&0&-\frac{h}{6}&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&-1&0&0&0&0&-\frac{h}{6}&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&-1&0&0&0&0&-\frac{h}{6}&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&-\frac{h}{6}&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
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0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}
$$

$$
N_{mx\_2} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -\frac{5h}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -\frac{5h}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Appendix G: Time Domain SCAQCF Model – Three Phase Saturable Core Reactor Protection

This section describes the three phase saturable reactor model for the state estimation based setting-less protection. The SCAQCF model is derived in 4 parts: (a) the reactor compact model, (b) the saturable core reactor quadratized model, (c) the SCAQCF device model and (d) the SCAQCF measurement model

G1. Three Phase Saturable Core Three Phase Reactor Compact Model Description

The circuit model of the three phase reactor is shown in Figure 1. Note that in Figure 1, g_L is the conductance that models the core losses. In this section, "numerical stabilizers" have been included to eliminate possible numerical problems caused by the numerical integration rule. In Figure G1, g_c is the conductance of the "stabilizer".

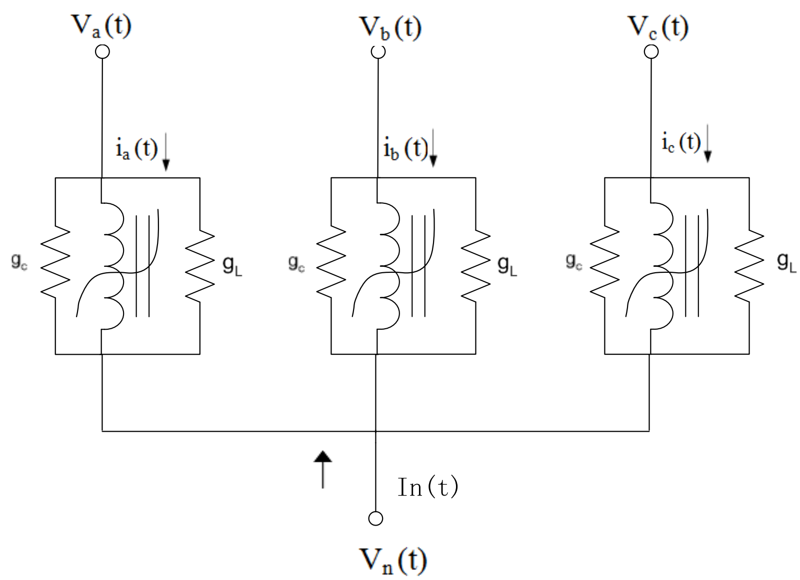

Figure G.1: Three Phase Reactor Model

In compact form the model becomes:

$$i_a(t) = i_0 \cdot \left|\frac{\lambda_a(t)}{\lambda_0}\right|^n \cdot sign\big(\lambda_a(t)\big) + (g_L + g_C)\cdot\big(v_a(t) - v_n(t)\big)$$

$$i_b(t) = i_0 \cdot \left|\frac{\lambda_b(t)}{\lambda_0}\right|^n \cdot sign\big(\lambda_b(t)\big) + (g_L + g_C)\cdot\big(v_b(t) - v_n(t)\big)$$

$$i_c(t) = i_0 \cdot \left|\frac{\lambda_c(t)}{\lambda_0}\right|^n \cdot sign\big(\lambda_c(t)\big) + (g_L + g_C)\cdot\big(v_c(t) - v_n(t)\big)$$

$$i_n(t) = -i_0 \cdot \left|\frac{\lambda_a(t)}{\lambda_0}\right|^n \cdot sign\left(\lambda_a(t)\right) - i_0 \cdot \left|\frac{\lambda_b(t)}{\lambda_0}\right|^n \cdot sign\left(\lambda_b(t)\right) - i_0 \cdot \left|\frac{\lambda_c(t)}{\lambda_0}\right|^n \cdot sign\left(\lambda_c(t)\right) - (g_L + g_C) \cdot \left(v_a(t) - v_n(t)\right) - (g_L + g_C) \cdot \left(v_b(t) - v_n(t)\right) - (g_L + g_C) \cdot \left(v_c(t) - v_n(t)\right)$$

$$0 = \frac{d\lambda_a(t)}{dt} - v_a(t) + v_n(t)$$

$$0 = \frac{d\lambda_b(t)}{dt} - v_b(t) + v_n(t)$$

$$0 = \frac{d\lambda_c(t)}{dt} - v_c(t) + v_n(t)$$

where the i_0 is the current constant and λ_0 is the flux constant.

G2. Three Phase Saturable Core Three Phase Reactor Quadratized Model Description

In this proposed formulation there are no nonlinearities in the dynamic part of the model. That is, all nonlinearities can be moved to the algebraic part of the model by the introduction of additional appropriate state variables. Also note that the nonlinear equations are of degree no more than two (at most quadratic equations) which is also achieved by the introduction of additional appropriate state variables. Assumption (just for now): n is odd. (It will be removed in the future).

The model is quadratized by introducing additional internal state variables, so that the n^{th} exponent is replaced by equations of at most quadratic degree. Since the exact degree of nonlinearity is not known until the user specifies it, the model performs automatic quadratization of the equations. A special procedure is used, so that the model is quadratized using the minimum number of additional internal states. The methodology is based on expressing the exponent in binary form. The binary representation provides all the information about the number of new variables and equations that need to be introduced and about the form of the equations (products of new variables). As an example, the exponent n is defined to be 11 so that the n+1-th exponent is even, which makes the term $\left[\mathrm{sign}\left(\lambda(\mathrm{t})\right)\right]^{\mathrm{n+1}}$ to be 1. Following this procedure the model is converted into the standard quadratized form:

$$i_a(t) = \frac{i_0}{|\lambda_0|^{11}} \cdot y_{a5}(t) + (g_L + g_C) \cdot \left(v_a(t) - v_n(t)\right)$$

$$i_b(t) = \frac{i_0}{|\lambda_0|^{11}} \cdot y_{b5}(t) + (g_L + g_C) \cdot \left(v_b(t) - v_n(t)\right)$$

$$i_c(t) = \frac{i_0}{|\lambda_0|^{11}} \cdot y_{c5}(t) + (g_L + g_C) \cdot \left(v_c(t) - v_n(t)\right)$$

$$i_n(t) = -\frac{i_0}{\left|\lambda_0\right|^{11}} \cdot y_{a5}(t) - \frac{i_0}{\left|\lambda_0\right|^{11}} \cdot y_{b5}(t) - \frac{i_0}{\left|\lambda_0\right|^{11}} \cdot y_{c5}(t) - (g_L + g_C) \cdot \big(v_a(t) - v_n(t)\big)$$
$$- (g_L + g_C) \cdot \big(v_b(t) - v_n(t)\big) - (g_L + g_C) \cdot \big(v_c(t) - v_n(t)\big)$$
$$0 = \frac{d\,\lambda_a(t)}{dt} - v_a(t) + v_n(t)$$
$$0 = \frac{d\,\lambda_b(t)}{dt} - v_b(t) + v_n(t)$$
$$0 = \frac{d\,\lambda_c(t)}{dt} - v_c(t) + v_n(t)$$
$$0 = y_{a1}(t) - \lambda_a(t)^2$$
$$0 = y_{a2}(t) - y_{a1}(t)^2$$
$$0 = y_{a3}(t) - y_{a2}(t)^2$$
$$0 = y_{a4}(t) - y_{a3}(t) \cdot y_{a1}(t)$$
$$0 = y_{a5}(t) - y_{a4}(t) \cdot \lambda_a(t)$$
$$0 = y_{b1}(t) - \lambda_b(t)^2$$
$$0 = y_{b2}(t) - y_{b1}(t)^2$$
$$0 = y_{b3}(t) - y_{b2}(t)^2$$
$$0 = y_{b4}(t) - y_{b3}(t) \cdot y_{b1}(t)$$
$$0 = y_{b5}(t) - y_{b4}(t) \cdot \lambda_b(t)$$
$$0 = y_{c1}(t) - \lambda_c(t)^2$$
$$0 = y_{c2}(t) - y_{c1}(t)^2$$
$$0 = y_{c3}(t) - y_{c2}(t)^2$$
$$0 = y_{c4}(t) - y_{c3}(t) \cdot y_{c1}(t)$$
$$0 = y_{c5}(t) - y_{c4}(t) \cdot \lambda_c(t)$$

Above equations are written in compact matrix form as:

$$\begin{bmatrix} i(t) \\ 0 \end{bmatrix} = Y_{eq} \cdot \begin{bmatrix} v(t) \\ y(t) \end{bmatrix} + B \cdot \frac{d}{dt} \cdot \begin{bmatrix} v(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} [v^T(t) \quad y^T(t)] \cdot F_{eq1(22\times22)} \cdot \begin{bmatrix} v(t) \\ y(t) \end{bmatrix} \\ \vdots \\ [v^T(t) \quad y^T(t)] \cdot F_{eq22(22\times22)} \cdot \begin{bmatrix} v(t) \\ y(t) \end{bmatrix} \end{bmatrix}$$

where: $i(t) = \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \\ i_n(t) \end{bmatrix}$, $v(t) = \begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \\ v_n(t) \end{bmatrix}$, $y(t)_{18X1} = \begin{bmatrix} \lambda_a(t) \\ \lambda_b(t) \\ \lambda_c(t) \\ y_{a1}(t) \\ y_{a2}(t) \\ y_{a3}(t) \\ y_{a4}(t) \\ y_{a5}(t) \\ y_{b1}(t) \\ y_{b2}(t) \\ y_{b3}(t) \\ y_{b4}(t) \\ y_{b5}(t) \\ y_{c1}(t) \\ y_{c2}(t) \\ y_{c3}(t) \\ y_{c4}(t) \\ y_{c5}(t) \end{bmatrix}$

$$Y_{eq} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$A_{11(7\times7)} = \begin{bmatrix} g_L + g_C & 0 & 0 & -g_L - g_C & 0 & 0 & 0 \\ 0 & g_L + g_C & 0 & -g_L - g_C & 0 & 0 & 0 \\ 0 & 0 & g_L + g_C & -g_L - g_C & 0 & 0 & 0 \\ -g_L - g_C & -g_L - g_C & -g_L - g_C & 3g_L + 3g_C & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{12(7\times15)}=\begin{bmatrix} 0&0&0&0&\frac{i_0}{|\lambda_0|^{11}}&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&\frac{i_0}{|\lambda_0|^{11}}&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{i_0}{|\lambda_0|^{11}}\\ 0&0&0&0&-\frac{i_0}{|\lambda_0|^{11}}&0&0&0&0&-\frac{i_0}{|\lambda_0|^{11}}&0&0&0&0&-\frac{i_0}{|\lambda_0|^{11}}\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$A_{21}=[0]_{15X7}$$
$$A_{22}=I_{15X15}$$

$$\mathrm{B}=\begin{bmatrix} 0_{4\times4} & 0_{4\times3} & 0_{4\times15}\\ 0_{3\times4} & I_{3\times3} & 0_{3\times15}\\ 0_{15\times4} & 0_{15\times3} & 0_{15\times15} \end{bmatrix}$$

$$F_{eq1}=\cdots=F_{eq10}=0_{22\times22}$$
$$F_{eq8}\colon f_{eq[5\times5]}=-1,\qquad other\ elements\ are\ 0$$
$$F_{eq9}\colon f_{eq[8\times8]}=-1,\qquad other\ elements\ are\ 0$$
$$F_{eq10}\colon f_{eq[9\times9]}=-1,\qquad other\ elements\ are\ 0$$
$$F_{eq11}\colon f_{eq[10\times8]}=-1,\qquad other\ elements\ are\ 0$$
$$F_{eq12}\colon f_{eq[11\times5]}=-1,\qquad other\ elements\ are\ 0$$
$$F_{eq13}\colon f_{eq[6\times6]}=-1,\qquad other\ elements\ are\ 0$$
$$F_{eq14}\colon f_{eq[13\times13]}=-1,\qquad other\ elements\ are\ 0$$
$$F_{eq15}\colon f_{eq[14\times14]}=-1,\qquad other\ elements\ are\ 0$$
$$F_{eq16}\colon f_{eq[15\times13]}=-1,\qquad other\ elements\ are\ 0$$
$$F_{eq17}\colon f_{eq[16\times6]}=-1,\qquad other\ elements\ are\ 0$$
$$F_{eq18}\colon f_{eq[7\times7]}=-1,\qquad other\ elements\ are\ 0$$
$$F_{eq19}\colon f_{eq[18\times18]}=-1,\qquad other\ elements\ are\ 0$$
$$F_{eq20}\colon f_{eq[19\times19]}=-1,\qquad other\ elements\ are\ 0$$
$$F_{eq21}\colon f_{eq[20\times18]}=-1,\qquad other\ elements\ are\ 0$$
$$F_{eq22}\colon f_{eq[21\times7]}=-1,\qquad other\ elements\ are\ 0$$

G3. Three Phase Saturable Core Three Phase Reactor SCAQCF Device Model Description

At time t,

$$i_a(t) = \frac{i_0}{|\lambda_0|^{11}} \cdot y_{a5}(t) + g_L \cdot \left(v_a(t) - v_n(t)\right)$$

$$i_b(t) = \frac{i_0}{|\lambda_0|^{11}} \cdot y_{b5}(t) + g_L \cdot \left(v_b(t) - v_n(t)\right)$$

$$i_c(t) = \frac{i_0}{|\lambda_0|^{11}} \cdot y_{c5}(t) + g_L \cdot \left(v_c(t) - v_n(t)\right)$$

$$i_n(t) = -\frac{i_0}{|\lambda_0|^{11}} \cdot y_{a5}(t) - g_L \cdot \left(v_a(t) - v_n(t)\right) - \frac{i_0}{|\lambda_0|^{11}} \cdot y_{b5}(t) - g_L \cdot \left(v_b(t) - v_n(t)\right) - \frac{i_0}{|\lambda_0|^{11}} \cdot y_{c5}(t) - g_L \cdot \left(v_c(t) - v_n(t)\right)$$

$$0 = \lambda_a(t) - \lambda_a(t-h) - \frac{h}{6}\left(v_a(t) - v_n(t)\right) - \frac{2h}{3}\left(v_a(t_m) - v_n(t_m)\right) - \frac{h}{6}\left(v_a(t-h) - v_n(t-h)\right)$$

$$0 = \lambda_b(t) - \lambda_b(t-h) - \frac{h}{6}\left(v_b(t) - v_n(t)\right) - \frac{2h}{3}\left(v_b(t_m) - v_n(t_m)\right) - \frac{h}{6}\left(v_b(t-h) - v_n(t-h)\right)$$

$$0 = \lambda_c(t) - \lambda_c(t-h) - \frac{h}{6}\left(v_c(t) - v_n(t)\right) - \frac{2h}{3}\left(v_c(t_m) - v_n(t_m)\right) - \frac{h}{6}\left(v_c(t-h) - v_n(t-h)\right)$$

$$0 = y_{a1}(t) - \lambda_a(t)^2$$
$$0 = y_{a2}(t) - y_{a1}(t)^2$$
$$0 = y_{a3}(t) - y_{a2}(t)^2$$
$$0 = y_{a4}(t) - y_{a3}(t) \cdot y_{a1}(t)$$
$$0 = y_{a5}(t) - y_{a4}(t) \cdot \lambda_a(t)$$
$$0 = y_{b1}(t) - \lambda_b(t)^2$$
$$0 = y_{b2}(t) - y_{b1}(t)^2$$
$$0 = y_{b3}(t) - y_{b2}(t)^2$$
$$0 = y_{b4}(t) - y_{b3}(t) \cdot y_{b1}(t)$$
$$0 = y_{b5}(t) - y_{b4}(t) \cdot \lambda_b(t)$$
$$0 = y_{c1}(t) - \lambda_c(t)^2$$
$$0 = y_{c2}(t) - y_{c1}(t)^2$$
$$0 = y_{c3}(t) - y_{c2}(t)^2$$
$$0 = y_{c4}(t) - y_{c3}(t) \cdot y_{c1}(t)$$

$$0 = y_{c5}(t) - y_{c4}(t) \cdot \lambda_c(t)$$

At time t_m,

$$i_a(t_m) = \frac{i_0}{|\lambda_0|^{11}} \cdot y_{a5}(t_m) + g_L \cdot \big(v_a(t_m) - v_n(t_m)\big)$$
$$i_b(t_m) = \frac{i_0}{|\lambda_0|^{11}} \cdot y_{b5}(t_m) + g_L \cdot \big(v_b(t_m) - v_n(t_m)\big)$$
$$i_c(t_m) = \frac{i_0}{|\lambda_0|^{11}} \cdot y_{b5}(t_m) + g_L \cdot \big(v_c(t_m) - v_n(t_m)\big)$$
$$i_n(t_m) = -i_{aL}(t_m) - g_L \cdot \big(v_a(t_m) - v_n(t_m)\big) - i_{bL}(t_m) - g_L \cdot \big(v_b(t_m) - v_n(t_m)\big) - i_{cL}(t_m) - g_L \cdot \big(v_c(t_m) - v_n(t_m)\big)$$
$$0 = \lambda_a(t_m) - \lambda_a(t-h) + \frac{h}{24}\big(v_a(t) - v_n(t)\big) - \frac{h}{3}\big(v_a(t_m) - v_n(t_m)\big) - \frac{5h}{24}\big(v_a(t-h) - v_n(t-h)\big)$$
$$0 = \lambda_b(t_m) - \lambda_b(t-h) + \frac{h}{24}\big(v_b(t) - v_n(t)\big) - \frac{h}{3}\big(v_b(t_m) - v_n(t_m)\big) - \frac{5h}{24}\big(v_b(t-h) - v_n(t-h)\big)$$
$$0 = \lambda_c(t_m) - \lambda_c(t-h) + \frac{h}{24}\big(v_c(t) - v_n(t)\big) - \frac{h}{3}\big(v_c(t_m) - v_n(t_m)\big) - \frac{5h}{24}\big(v_c(t-h) - v_n(t-h)\big)$$
$$0 = y_{a1}(t_m) - \lambda_a(t_m)^2$$
$$0 = y_{a2}(t_m) - y_{a1}(t_m)^2$$
$$0 = y_{a3}(t_m) - y_{a2}(t_m)^2$$
$$0 = y_{a4}(t_m) - y_{a3}(t_m) \cdot y_{a1}(t_m)$$
$$0 = y_{a5}(t_m) - y_{a4}(t_m) \cdot \lambda_a(t_m)$$
$$0 = y_{b1}(t_m) - \lambda_b(t_m)^2$$
$$0 = y_{b2}(t_m) - y_{b1}(t_m)^2$$
$$0 = y_{b3}(t_m) - y_{b2}(t_m)^2$$
$$0 = y_{b4}(t_m) - y_{b3}(t_m) \cdot y_{b1}(t_m)$$
$$0 = y_{b5}(t_m) - y_{b4}(t_m) \cdot \lambda_b(t_m)$$
$$0 = y_{c1}(t_m) - \lambda_c(t_m)^2$$
$$0 = y_{c2}(t_m) - y_{c1}(t_m)^2$$
$$0 = y_{c3}(t_m) - y_{c2}(t_m)^2$$
$$0 = y_{c4}(t_m) - y_{c3}(t_m) \cdot y_{c1}(t_m)$$
$$0 = y_{c5}(t_m) - y_{c4}(t_m) \cdot \lambda_c(t_m)$$

The SCAQCF device model is shown below:

$$\begin{Bmatrix} I(\mathbf{x},\mathbf{u}) \\ \vdots \\ 0 \\ \vdots \end{Bmatrix} = Y_{eqx}\mathbf{x} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{eqx}^i \mathbf{x} \\ \vdots \end{Bmatrix} + Y_{equ}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F_{equ}^i \mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{eqxu}^i \mathbf{u} \\ \vdots \end{Bmatrix} - B_{eq}$$

$$B_{eq} = -N_{eqx}\mathbf{x}(t-h) - N_{equ}\mathbf{u}(t-h) - M_{eq}I(t-h) - K_{eq}$$

$$\mathbf{h}(\mathbf{x},\mathbf{u}) = Y_{opx}\mathbf{x} + Y_{opu}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{opx}^i \mathbf{x} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F_{opu}^i \mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{opxu}^i \mathbf{u} \\ \vdots \end{Bmatrix} - B_{op}$$

Scaling factors: $Iscale,\ Xscale\ and\ Uscale$

Connectivity: *TerminalNodeName*

$$\begin{aligned} subject\ to: \quad & \mathbf{h}_{\min} \le \mathbf{h}(\mathbf{x},\mathbf{u}) \le \mathbf{h}_{\max} \\ & \mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max} \end{aligned}$$

where the matrices are:

$$Y_{eqx\,44\times 44} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix}$$

$$A_{11(7\times 7)} = \begin{bmatrix} g_L & 0 & 0 & -g_L & 0 & 0 & 0 \\ 0 & g_L & 0 & -g_L & 0 & 0 & 0 \\ 0 & 0 & g_L & -g_L & 0 & 0 & 0 \\ -g_L & -g_L & -g_L & 3g_L & 0 & 0 & 0 \\ -\frac{h}{6} & 0 & 0 & \frac{h}{6} & 1 & 0 & 0 \\ 0 & -\frac{h}{6} & 0 & \frac{h}{6} & 0 & 1 & 0 \\ 0 & 0 & -\frac{h}{6} & \frac{h}{6} & 0 & 0 & 1 \end{bmatrix}$$

$$A_{12(7\times15)}=\begin{bmatrix} 0&0&0&0&\frac{i_0}{|\lambda_0|^{11}}&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&\frac{i_0}{|\lambda_0|^{11}}&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{i_0}{|\lambda_0|^{11}}\\ 0&0&0&0&-\frac{i_0}{|\lambda_0|^{11}}&0&0&0&0&-\frac{i_0}{|\lambda_0|^{11}}&0&0&0&0&-\frac{i_0}{|\lambda_0|^{11}}\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$A_{13(7\times7)}=\begin{bmatrix} 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ -\frac{2h}{3}&0&0&\frac{2h}{3}&0&0&0\\ 0&-\frac{2h}{3}&0&\frac{2h}{3}&0&0&0\\ 0&0&-\frac{2h}{3}&\frac{2h}{3}&0&0&0 \end{bmatrix}$$

$$A_{14}=[0]_{7\times15}$$
$$A_{21}=[0]_{15\times7}$$
$$A_{22}=I_{15\times15}$$
$$A_{23}=[0]_{15\times7}$$
$$A_{24}=[0]_{15\times15}$$

$$A_{31(7\times7)}=\begin{bmatrix} 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0\\ \frac{h}{24}&0&0&-\frac{h}{24}&0&0&0\\ 0&\frac{h}{24}&0&-\frac{h}{24}&0&0&0\\ 0&0&\frac{h}{24}&-\frac{h}{24}&0&0&0 \end{bmatrix}$$

$$A_{32} = [0]_{7\times15}$$

$$A_{33(7\times7)} = \begin{bmatrix} g_L & 0 & 0 & -g_L & 0 & 0 & 0 \\ 0 & g_L & 0 & -g_L & 0 & 0 & 0 \\ 0 & 0 & g_L & -g_L & 0 & 0 & 0 \\ -g_L & -g_L & -g_L & 3g_L & 0 & 0 & 0 \\ -\frac{h}{3} & 0 & 0 & \frac{h}{3} & 1 & 0 & 0 \\ 0 & -\frac{h}{3} & 0 & \frac{h}{3} & 0 & 1 & 0 \\ 0 & 0 & -\frac{h}{3} & \frac{h}{3} & 0 & 0 & 1 \end{bmatrix}$$

$$A_{34(7\times15)} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{i_0}{|\lambda_0|^{11}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i_0}{|\lambda_0|^{11}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i_0}{|\lambda_0|^{11}} \\ 0 & 0 & 0 & 0 & -\frac{i_0}{|\lambda_0|^{11}} & 0 & 0 & 0 & 0 & -\frac{i_0}{|\lambda_0|^{11}} & 0 & 0 & 0 & 0 & -\frac{i_0}{|\lambda_0|^{11}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{41} = [0]_{15\times7}$$
$$A_{42} = [0]_{15\times15}$$
$$A_{43} = [0]_{15\times7}$$
$$A_{44} = I_{15\times15}$$

$$F_{eqx0} = \cdots = F_{eqx6} = 0_{44\times44}$$
$$F_{eqx7}: f_{eqx[5\times5]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx8}: f_{eqx[8\times8]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx9}: f_{eqx[9\times9]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx10}: f_{eqx[10\times8]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx11}: f_{eqx[11\times5]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx12}: f_{eqx[6\times6]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx13}: f_{eqx[13\times13]} = -1, \qquad other\ elements\ are\ 0$$

$$F_{eqx14}: f_{eqx[14\times14]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx15}: f_{eqx[15\times13]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx16}: f_{eqx[16\times6]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx17}: f_{eqx[7\times7]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx18}: f_{eqx[18\times18]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx19}: f_{eqx[19\times19]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx20}: f_{eqx[20\times18]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx21}: f_{eqx[21\times7]} = -1, \qquad other\ elements\ are\ 0$$

$$F_{eqx22} = \cdots = F_{eqx28} = 0_{44\times44}$$
$$F_{eqx29}: f_{eqx[27\times27]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx30}: f_{eqx[30\times30]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx31}: f_{eqx[31\times31]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx32}: f_{eqx[32\times30]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx33}: f_{eqx[33\times27]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx34}: f_{eqx[28\times28]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx35}: f_{eqx[35\times35]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx36}: f_{eqx[36\times36]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx37}: f_{eqx[37\times35]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx38}: f_{eqx[38\times28]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx39}: f_{eqx[29\times29]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx40}: f_{eqx[40\times40]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx41}: f_{eqx[41\times41]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx42}: f_{eqx[42\times40]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{eqx43}: f_{eqx[43\times29]} = -1, \qquad other\ elements\ are\ 0$$

$$N_{eqx44X7} = \begin{bmatrix} 0_{4X7} \\ B_{1(3X7)} \\ 0_{19X7} \\ B_{2(3X7)} \\ 0_{15X7} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -\frac{h}{6} & 0 & 0 & \frac{h}{6} & -1 & 0 & 0 \\ 0 & -\frac{h}{6} & 0 & \frac{h}{6} & 0 & -1 & 0 \\ 0 & 0 & -\frac{h}{6} & \frac{h}{6} & 0 & 0 & -1 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} -\frac{5h}{24} & 0 & 0 & \frac{5h}{24} & -1 & 0 & 0 \\ 0 & -\frac{5h}{24} & 0 & \frac{5h}{24} & 0 & -1 & 0 \\ 0 & 0 & -\frac{5h}{24} & \frac{5h}{24} & 0 & 0 & -1 \end{bmatrix}$$

The input matrices of the three phase reactor SCAQCF device model for the setting-less protection algorithm are shown below:

TerminalNodeName
LINE_A
LINE_B
LINE_C
NEUTRAL_N
-*1*

Iscale
0,50.0
1,50.0
2,50.0
3,50.0
4,1
5,1
6,1
7,1
8,1
9,1
10,1
11,1
12,1
13,1
14,1
15,1
16,1
17,1
18,1
19,1
20,1
21,1
22,50.0
23,50.0
24,50.0
25,50.0
26,1

27,1
28,1
29,1
30,1
31,1
32,1
33,1
34,1
35,1
36,1
37,1
38,1
39,1
40,1
41,1
42,1
43,1
-1

Xscale
0,250.0e3
1,250.0e3
2,250.0e3
3,250.0e3
4,500
5,1
6,1
7,1
8,1
9,1
10,1
11,1
12,1
13,1
14,1
15,1
16,1
17,1
18,1
19,1
20,1
21,1
22,250.0e3
23,250.0e3

24,250.0e3
25,250.0e3
26,500
27,1
28,1
29,1
30,1
31,1
32,1
33,1
34,1
35,1
36,1
37,1
38,1
39,1
40,1
41,1
42,1
43,1
-1

Yeqx
0,0,0.00005
0,3,-0.00005
0,11,0.5
1,1,0.00005
1,3,-0.00005
1,16,0.5
2,2,0.00005
2,3,-0.00005
2,21,0.5
3,0,-0.00005
3,1,-0.00005
3,2,-0.00005
3,3,0.00015
3,11,-0.5
3,16,-0.5
3,21,-0.5
4,0,-1.6666666667e-5
4,3,1.6666666667e-5
4,4,305.047
4,22,-6.6666666667e-5
4,25,6.6666666667e-5

5,1,-1.6666666667e-5
5,3,1.6666666667e-5
5,5,305.047
5,23,-6.6666666667e-5
5,25,6.6666666667e-5
6,2,-1.6666666667e-5
6,3,1.6666666667e-5
6,6,305.047
6,24,-6.6666666667e-5
6,25,6.6666666667e-5
7,7,1
8,8,1
9,9,1
10,10,1
11,11,1
12,12,1
13,13,1
14,14,1
15,15,1
16,16,1
17,17,1
18,18,1
19,19,1
20,20,1
21,21,1
22,22,0.00005
22,25,-0.00005
22,33,0.5
23,23,0.00005
23,25,-0.00005
23,38,0.5
24,24,0.00005
24,25,-0.00005
24,43,0.5
25,22,-0.00005
25,23,-0.00005
25,24,-0.00005
25,25,0.00015
25,33,-0.5
25,38,-0.5
25,43,-0.5
26,0,4.16666666667e-6
26,3,-4.16666666667e-6
26,22,-3.33333333333e-5

26,25,3.33333333333e-5
26,26,305.047
27,1,4.16666666667e-6
27,3,-4.16666666667e-6
27,23,-3.33333333333e-5
27,25,3.33333333333e-5
27,27,305.047
28,2,4.16666666667e-6
28,3,-4.16666666667e-6
28,24,-3.33333333333e-5
28,25,3.33333333333e-5
28,28,305.047
29,29,1
30,30,1
31,31,1
32,32,1
33,33,1
34,34,1
35,35,1
36,36,1
37,37,1
38,38,1
39,39,1
40,40,1
41,41,1
42,42,1
43,43,1
-1

Feqx
7,4,4,-1
8,7,7,-1
9,8,8,-1
10,9,7,-1
11,10,4,-1
12,5,5,-1
13,12,12,-1
14,13,13,-1
15,14,12,-1
16,15,5,-1
17,6,6,-1
18,17,17,-1
19,18,18,-1
20,19,17,-1

21,20,6,-1
29,26,26,-1
30,29,29,-1
31,30,30,-1
32,31,29,-1
33,32,26,-1
34,27,27,-1
35,34,34,-1
36,35,35,-1
37,36,34,-1
38,37,27,-1
39,28,28,-1
40,39,39,-1
41,40,40,-1
42,41,39,-1
43,42,28,-1
-1

Neqx
4,0,-1.6666666667e-5
4,3,1.6666666667e-5
4,4,-305.047
5,1,-1.6666666667e-5
5,3,1.6666666667e-5
5,5,-305.047
6,2,-1.6666666667e-5
6,3,1.6666666667e-5
6,6,-305.047
26,0,-2.083333333333e-5
26,3,2.083333333333e-5
26,4,-305.047
27,1,-2.083333333333e-5
27,3,2.083333333333e-5
27,5,-305.047
28,2,-2.083333333333e-5
28,3,2.083333333333e-5
28,6,-305.047
-1

Meq
-1

Keq
-1

G4. Three Phase Saturable Core Three Phase Reactor SCAQCF Measurement Model

Specifically, the **actual measurements** are:
Three currents at time t (phase A, phase B, and phase C);
Three voltages at time t (phase AN, phase BN, phase CN);
Three currents at time tm=t-h/2 (phase A, phase B, and phase C);
Three voltages at time tm=t-h/2 (phase AN, phase BN, phase CN);

The **virtual measurements** are:
18 measurements with zero value at the left side of the equations at time t;
18 measurements with zero value at the left side of the equations at time tm=t-h/2;

The **pseudo measurements** are
One measurement for voltage at time t (Vn)
One measurement for voltage at time tm=t-h/2 (Vn)

The measurement channel list:

```
MeasurementType,  16
MeasStdDev,  400
MeasTerminal,    0,   3
MeasurementEnd
MeasurementType,  16
MeasStdDev,  0400
MeasTerminal,    1,   3
MeasurementEnd
MeasurementType,  16
MeasStdDev,  400
MeasTerminal,    2,   3
MeasurementEnd
MeasurementType,  17
MeasStdDev,  05
MeasTerminal,    0
MeasurementEnd
MeasurementType,  17
MeasStdDev,  5
MeasTerminal,    1
MeasurementEnd
MeasurementType,  17
MeasStdDev,  5
MeasTerminal,    2
MeasurementEnd
MeasurementType,  25
MeasStdDev,  0.10000
```

MeasTerminal,   3
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal,   4
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal,   5
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal,   6
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal,   7
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal,   8
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal,   9
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal,   10
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal,   11
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal,   12
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000
MeasTerminal,   13
MeasurementEnd
MeasurementType, 24
MeasStdDev, 0.0010000

```
  MeasTerminal,    14
  MeasurementEnd
  MeasurementType,  24
  MeasStdDev,  0.0010000
  MeasTerminal,    15
  MeasurementEnd
  MeasurementType,  24
  MeasStdDev,  0.0010000
  MeasTerminal,    16
  MeasurementEnd
  MeasurementType,  24
  MeasStdDev,  0.0010000
  MeasTerminal,    17
  MeasurementEnd
  MeasurementType,  24
  MeasStdDev,  0.0010000
  MeasTerminal,    18
  MeasurementEnd
  MeasurementType,  24
  MeasStdDev,  0.0010000
  MeasTerminal,    19
  MeasurementEnd
  MeasurementType,  24
  MeasStdDev,  0.0010000
  MeasTerminal,    20
  MeasurementEnd
  MeasurementType,  24
  MeasStdDev,  0.0010000
  MeasTerminal,    21
 MeasurementEnd
```

Based on the device model and the measurement channel list, the equations for the measurement model are given below:

At time t,

$$v_{an}(t) = v_a(t) - v_n(t)$$
$$v_{bn}(t) = v_b(t) - v_n(t)$$
$$v_{cn}(t) = v_c(t) - v_n(t)$$
$$i_a(t) = \frac{i_0}{|\lambda_0|^{11}} \cdot y_{a5}(t) + g_L \cdot \big(v_a(t) - v_n(t)\big)$$
$$i_b(t) = \frac{i_0}{|\lambda_0|^{11}} \cdot y_{b5}(t) + g_L \cdot \big(v_b(t) - v_n(t)\big)$$

$$i_c(t) = \frac{i_0}{|\lambda_0|^{11}} \cdot y_{c5}(t) + g_L \cdot \left(v_c(t) - v_n(t)\right)$$
$$0 = v_n(t)$$
$$0 = \lambda_a(t) - \lambda_a(t-h) - \frac{h}{6}\left(v_a(t) - v_n(t)\right) - \frac{2h}{3}\left(v_a(t_m) - v_n(t_m)\right)$$
$$- \frac{h}{6}\left(v_a(t-h) - v_n(t-h)\right)$$
$$0 = \lambda_b(t) - \lambda_b(t-h) - \frac{h}{6}\left(v_b(t) - v_n(t)\right) - \frac{2h}{3}\left(v_b(t_m) - v_n(t_m)\right)$$
$$- \frac{h}{6}\left(v_b(t-h) - v_n(t-h)\right)$$
$$0 = \lambda_c(t) - \lambda_c(t-h) - \frac{h}{6}\left(v_c(t) - v_n(t)\right) - \frac{2h}{3}\left(v_c(t_m) - v_n(t_m)\right)$$
$$- \frac{h}{6}\left(v_c(t-h) - v_n(t-h)\right)$$
$$0 = y_{a1}(t) - \lambda_a(t)^2$$
$$0 = y_{a2}(t) - y_{a1}(t)^2$$
$$0 = y_{a3}(t) - y_{a2}(t)^2$$
$$0 = y_{a4}(t) - y_{a3}(t) \cdot y_{a1}(t)$$
$$0 = y_{a5}(t) - y_{a4}(t) \cdot \lambda_a(t)$$
$$0 = y_{b1}(t) - \lambda_b(t)^2$$
$$0 = y_{b2}(t) - y_{b1}(t)^2$$
$$0 = y_{b3}(t) - y_{b2}(t)^2$$
$$0 = y_{b4}(t) - y_{b3}(t) \cdot y_{b1}(t)$$
$$0 = y_{b5}(t) - y_{b4}(t) \cdot \lambda_b(t)$$
$$0 = y_{c1}(t) - \lambda_c(t)^2$$
$$0 = y_{c2}(t) - y_{c1}(t)^2$$
$$0 = y_{c3}(t) - y_{c2}(t)^2$$
$$0 = y_{c4}(t) - y_{c3}(t) \cdot y_{c1}(t)$$
$$0 = y_{c5}(t) - y_{c4}(t) \cdot \lambda_c(t)$$

At time t_m,

$$v_{an}(t_m) = v_a(t_m) - v_n(t_m)$$
$$v_{bn}(t_m) = v_b(t_m) - v_n(t_m)$$
$$v_{cn}(t_m) = v_c(t_m) - v_n(t_m)$$

$$i_a(t_m) = \frac{i_0}{|\lambda_0|^{11}} \cdot y_{a5}(t_m) + g_L \cdot \left(v_a(t_m) - v_n(t_m)\right)$$
$$i_b(t_m) = \frac{i_0}{|\lambda_0|^{11}} \cdot y_{b5}(t_m) + g_L \cdot \left(v_b(t_m) - v_n(t_m)\right)$$
$$i_c(t_m) = \frac{i_0}{|\lambda_0|^{11}} \cdot y_{b5}(t_m) + g_L \cdot \left(v_c(t_m) - v_n(t_m)\right)$$

$$0 = v_n(t_m)$$

$$0 = \lambda_a(t_m) - \lambda_a(t-h) + \frac{h}{24}\big(v_a(t) - v_n(t)\big) - \frac{h}{3}\big(v_a(t_m) - v_n(t_m)\big) - \frac{5h}{24}\big(v_a(t-h) - v_n(t-h)\big)$$

$$0 = \lambda_b(t_m) - \lambda_b(t-h) + \frac{h}{24}\big(v_b(t) - v_n(t)\big) - \frac{h}{3}\big(v_b(t_m) - v_n(t_m)\big) - \frac{5h}{24}\big(v_b(t-h) - v_n(t-h)\big)$$

$$0 = \lambda_c(t_m) - \lambda_c(t-h) + \frac{h}{24}\big(v_c(t) - v_n(t)\big) - \frac{h}{3}\big(v_c(t_m) - v_n(t_m)\big) - \frac{5h}{24}\big(v_c(t-h) - v_n(t-h)\big)$$

$$0 = y_{a1}(t_m) - \lambda_a(t_m)^2$$
$$0 = y_{a2}(t_m) - y_{a1}(t_m)^2$$
$$0 = y_{a3}(t_m) - y_{a2}(t_m)^2$$
$$0 = y_{a4}(t_m) - y_{a3}(t_m) \cdot y_{a1}(t_m)$$
$$0 = y_{a5}(t_m) - y_{a4}(t_m) \cdot \lambda_a(t_m)$$
$$0 = y_{b1}(t_m) - \lambda_b(t_m)^2$$
$$0 = y_{b2}(t_m) - y_{b1}(t_m)^2$$
$$0 = y_{b3}(t_m) - y_{b2}(t_m)^2$$
$$0 = y_{b4}(t_m) - y_{b3}(t_m) \cdot y_{b1}(t_m)$$
$$0 = y_{b5}(t_m) - y_{b4}(t_m) \cdot \lambda_b(t_m)$$
$$0 = y_{c1}(t_m) - \lambda_c(t_m)^2$$
$$0 = y_{c2}(t_m) - y_{c1}(t_m)^2$$
$$0 = y_{c3}(t_m) - y_{c2}(t_m)^2$$
$$0 = y_{c4}(t_m) - y_{c3}(t_m) \cdot y_{c1}(t_m)$$
$$0 = y_{c5}(t_m) - y_{c4}(t_m) \cdot \lambda_c(t_m)$$

It is very easy to write the above equations in the standard SCAQCF format:

$$\mathbf{y}(\mathbf{x},\mathbf{u}) = Y_{m,x}\mathbf{x} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{m,x}^i \mathbf{x} \\ \vdots \end{Bmatrix} + Y_{m,u}\mathbf{u} + \begin{Bmatrix} \vdots \\ \mathbf{u}^T F_{m,u}^i \mathbf{u} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \mathbf{x}^T F_{m,xu}^i \mathbf{u} \\ \vdots \end{Bmatrix} + N_{m,x}\mathbf{x}(t-h) + N_{m,u}\mathbf{u}(t-h) + M_m I(t-h) + K_m$$

where the matrices are:

$$F_{mx_{50\times 44}} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix}$$

$$A_{11(10\times 7)} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ g_L & 0 & 0 & -g_L & 0 & 0 & 0 \\ 0 & g_L & 0 & -g_L & 0 & 0 & 0 \\ 0 & 0 & g_L & -g_L & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -\frac{h}{6} & 0 & 0 & \frac{h}{6} & 1 & 0 & 0 \\ 0 & -\frac{h}{6} & 0 & \frac{h}{6} & 0 & 1 & 0 \\ 0 & 0 & -\frac{h}{6} & \frac{h}{6} & 0 & 0 & 1 \end{bmatrix}$$

$$A_{12(10\times 15)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{i_0}{\left|\lambda_0\right|^{11}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i_0}{\left|\lambda_0\right|^{11}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i_0}{\left|\lambda_0\right|^{11}} \\ 0 & 0 & 0 & 0 & -\frac{i_0}{\left|\lambda_0\right|^{11}} & 0 & 0 & 0 & 0 & -\frac{i_0}{\left|\lambda_0\right|^{11}} & 0 & 0 & 0 & 0 & -\frac{i_0}{\left|\lambda_0\right|^{11}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{13(10\times7)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{2h}{3} & 0 & 0 & \frac{2h}{3} & 0 & 0 & 0 \\ 0 & -\frac{2h}{3} & 0 & \frac{2h}{3} & 0 & 0 & 0 \\ 0 & 0 & -\frac{2h}{3} & \frac{2h}{3} & 0 & 0 & 0 \end{bmatrix}$$

$$A_{14} = [0]_{10\times15}$$

$$A_{21} = [0]_{15\times7}$$

$$A_{22} = I_{15\times15}$$

$$A_{23} = [0]_{15\times7}$$

$$A_{24} = [0]_{15\times15}$$

$$A_{31(10\times7)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{h}{24} & 0 & 0 & -\frac{h}{24} & 0 & 0 & 0 \\ 0 & \frac{h}{24} & 0 & -\frac{h}{24} & 0 & 0 & 0 \\ 0 & 0 & \frac{h}{24} & -\frac{h}{24} & 0 & 0 & 0 \end{bmatrix}$$

$$A_{32} = [0]_{7\times15}$$

$$
A_{33(10\times7)}=\begin{bmatrix}
1 & 0 & 0 & -1 & 0 & 0 & 0\\
0 & 1 & 0 & -1 & 0 & 0 & 0\\
0 & 0 & 1 & -1 & 0 & 0 & 0\\
g_L & 0 & 0 & -g_L & 0 & 0 & 0\\
0 & g_L & 0 & -g_L & 0 & 0 & 0\\
0 & 0 & g_L & -g_L & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & 0\\
-\frac{h}{3} & 0 & 0 & \frac{h}{3} & 1 & 0 & 0\\
0 & -\frac{h}{3} & 0 & \frac{h}{3} & 0 & 1 & 0\\
0 & 0 & -\frac{h}{3} & \frac{h}{3} & 0 & 0 & 1
\end{bmatrix}
$$

$$
A_{34(10\times15)}=\begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&\frac{i_0}{|\lambda_0|^{11}}&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&\frac{i_0}{|\lambda_0|^{11}}&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&\frac{i_0}{|\lambda_0|^{11}}\\
0&0&0&0&-\frac{i_0}{|\lambda_0|^{11}}&0&0&0&0&-\frac{i_0}{|\lambda_0|^{11}}&0&0&0&0&-\frac{i_0}{|\lambda_0|^{11}}\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}
$$

$$A_{41}=[0]_{15\times7}$$

$$A_{42}=[0]_{15\times15}$$

$$A_{43}=[0]_{15\times7}$$

$$A_{44}=I_{15\times15}$$

$$F_{mx0}=\cdots=F_{mx9}=0_{44\times44}$$

$$F_{mx10}: f_{mx[5\times5]}=-1,\qquad other\ elements\ are\ 0$$

$$F_{mx11}: f_{mx[8\times8]}=-1,\qquad other\ elements\ are\ 0$$

$$F_{mx12}: f_{mx[9\times9]}=-1,\qquad other\ elements\ are\ 0$$

$$F_{mx13}: f_{mx[10\times 8]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx14}: f_{mx[11\times 5]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx15}: f_{mx[6\times 6]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx16}: f_{mx[13\times 13]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx17}: f_{mx[14\times 14]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx18}: f_{mx[15\times 13]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx19}: f_{mx[16\times 6]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx20}: f_{mx[7\times 7]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx21}: f_{mx[18\times 18]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx22}: f_{mx[19\times 19]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx23}: f_{mx[20\times 18]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx24}: f_{mx[21\times 7]} = -1, \qquad other\ elements\ are\ 0$$

$$F_{mx25} = \cdots = F_{mx34} = 0_{44\times 44}$$
$$F_{mx35}: f_{mx[27\times 27]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx36}: f_{mx[30\times 30]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx37}: f_{mx[31\times 31]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx38}: f_{mx[32\times 30]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx39}: f_{mx[33\times 27]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx40}: f_{mx[28\times 28]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx41}: f_{mx[35\times 35]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx42}: f_{mx[36\times 36]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx43}: f_{mx[37\times 35]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx44}: f_{mx[38\times 28]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx45}: f_{mx[29\times 29]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx46}: f_{mx[40\times 40]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx47}: f_{mx[41\times 41]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx348}: f_{mx[42\times 40]} = -1, \qquad other\ elements\ are\ 0$$
$$F_{mx49}: f_{mx[43\times 29]} = -1, \qquad other\ elements\ are\ 0$$

$$N_{mx50X7} = \begin{bmatrix} 0_{7X7} \\ B_{1(3X7)} \\ 0_{22X7} \\ B_{2(3X7)} \\ 0_{15X7} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -\frac{h}{6} & 0 & 0 & \frac{h}{6} & -1 & 0 & 0 \\ 0 & -\frac{h}{6} & 0 & \frac{h}{6} & 0 & -1 & 0 \\ 0 & 0 & -\frac{h}{6} & \frac{h}{6} & 0 & 0 & -1 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} -\frac{5h}{24} & 0 & 0 & \frac{5h}{24} & -1 & 0 & 0 \\ 0 & -\frac{5h}{24} & 0 & \frac{5h}{24} & 0 & -1 & 0 \\ 0 & 0 & -\frac{5h}{24} & \frac{5h}{24} & 0 & 0 & -1 \end{bmatrix}$$